Studies in Economics
Part II

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PAPER – A
MONEY & BANKING

1664

1
MONEY

In the beginning of human existence, needs were simple and every individual produced all that was necessary to sustain life – he provided his own food, prepared his own clothing and found his own shelter. When the economy developed a little more, and when people settled down in different occupations, exchange came into existence. Initially exchange was direct, *i.e.*, exchange of goods for goods. Such exchange was known as Barter. The person who had plenty of food but no clothes exchanged a part of his food with the person who had plenty of clothes but not food. In those days wants were simple and the number of exchangeable goods was small. The system of barter, therefore, could continue for a long time indeed. It exists among the backward peoples of the world even to-day but in all organised communities it has been given up because of its inconveniences.

Difficulties of Barter System

1. Difficulty of Double Coincidence of Wants. Under the barter system, the first difficulty is that there has to be a double coincidence of wants. A must have what B wants and of the right amount. They must also require each other's goods simultaneously. A person who wants to exchange an article for another must find out another person who is prepared to part with the thing he wants and in exchange he is prepared to accept the article which the other person has. Supposing if a person has wheat and he wants cloth in exchange for wheat, he may find another person who has cloth to offer but he wants milk in exchange for cloth. Obviously, the transaction cannot be effected. In practice, it is always difficult to have such double coincidence of wants and therefore, there are delays in transactions and a considerable amount of time and effort is wasted in effecting an exchange. Sometimes, it may mean a large number of unnecessary intermediate deals. A man needs a horse against a cart. *Chandler* describes the situation in these words, "He might have to trade the horse for a cow, the cow for a boat, the boat for some sheep and the sheep for the desired cart." It is not possible to exchange a large amount of commodities like this in an organised market. Obviously, it hampers the growth of trade and business, thereby adversely affecting economic progress.

2. Difficulty of

metres of cloth and 5 litres of milk etc. All this is very confusing. And one has to remember a large number of cross relations of values in exchange for different goods which is physically impossible to do when there are an infinite number of commodities. Under such conditions, no meaningful accounting system can be evolved. In words of Shapiro, Soloman and White, "In a non-monetary (barter) society there would be no common denominator in terms of which the price of each item to be traded can be expressed. The price of each article in the market could not be stated as one quantity, but would have to be quoted in as many quantities as there were other goods and services in the market. If there were one million goods and services traded in the market, the price of each would have to be stated in terms of 99, 999 others. Book-keeping under such conditions, while not impossible, would be nearly so."

3. Difficulty of Indivisibility. A commodity can be small, big and valuable. If lemons are to be exchanged for mangoes, no difficulty arises because lemons and mangoes can be counted and exchange ratios can be decided. But there is a problem when an indivisible good has to be exchanged for a divisible good. Supposing a person has a cow and wants cloth and foodgrains in exchange. If he gives cow in exchange, he finds himself a looser. He cannot divide cow in parts so as to exchange one part for the foodgrains and another for cloth. Thus, in a society howsoever primitive it may be, a man has to satisfy his daily needs and for this he has to exchange his goods for others' goods. If he has such goods which can be divided, there is no problem but in reality there are many possessions which are indivisible and can never be exchanged for small things without loosing substantial value.

4. Difficulty of Deferred Payments. Another difficulty under barter system is that it lacks a standard of deferred payments so that contracts involving future payments or loan transactions cannot take place easily. In an exchange economy, often contracts have to be made for payments of wages, rent or interest at some future date. Such contracts become difficult. There may be controversy as to the specific commodity to be used for repayment. The specific commodity to be used may rise or fall much in value. In that case, borrowers or lenders are likely to lose. Borrowing and lending also becomes difficult because barter "would often invite controversy as to the quality of goods or services to be repaid." A person may borrow 100 cows, but the return of 100 cows may provoke violent diagreement. He may be returning dry, sick cows while he had borrowed healthy, young milk-giving cows.

5. Difficulty of Store of Value. Another difficulty relates to the storage of goods for the future. A person has to store all he requires for the future. Apart from the difficulties involved in physical storage, goods thus stored fluctuate in value. For example, non-durable goods like wheat may lose their value if stored for too long a period. Others like cattle multiply and increase in value over time. But much expenditure has to be incurred to store this type of wealth because cattle have to be

fed to keep the value stored in them thus, storage is expensive and disposal of goods at the end of storing may itself be difficult. These problems of storing wealth are serious enough to make it extremely inconvenient.

From the above discussion, it is clear that the barter system could suit only a primitive economy where the human needs were simple. As the population increased, civilisation progressed, production increased, exchange expanded, specialisation developed, the difficulties of barter were felt more and more. It became necessary to replace the barter system by some convenient method of exchange. This led to the invention of money.

The difficulties of barter made it essential for people to devise some means of overcoming them. The method devised was the use of something which served as a medium of exchange and a measure of value thing, which came to be known as money, has gradually become the central thing in an economy. The need for money was realised long ago and the idea originated in the very early stages of man's economic life. The difficulties of barter were felt more and more as production increased and exchange expanded. Specialisation developed exchange and the use of a medium and a standard became necessary. But money developed through a number of stages and its nature has changed from time to time. A large number of commodities was tried to serve the purpose of money at different times and at last we came down to coins and paper notes.

If we look back at the origin of money it will be found in the long complex lists of various commodities which were used as possible media of exchange in the early societies. Regarding the origin of money, J.M. Keynes wrote in his 'Treatise on Money' : "Money like certain other essential elements in civilization, is a far more ancient institution than we were brought to believe some few years ago. Its origins are lost in the mists when the ice was melting and may well stretch back into the paradisaic intervals in human history of the interglacial periods. When the wealth was delightful and the mind free to be fertile of new ideas in the islands of the Hesperides or Atlantic or some Eden of Central Asia."

So, ancient in origin,money carries with it the long history of social evolution. It was not miracle that money came into circulation. It was a gradual development of the media of exchange from hunting stage to the money economy stage. Whatever inconveniences and intricate problems people had to face in the barter economy, in fact they were at the root of the evolution or of invention of money.

Definition of Money. Everybody knows and recognises money but everybody does not know how to define money. Money has been defined differently by different economists. While some economists like Walker have defined money in terms of the functions; others like Keynes, Cole, Robertson etc. have emphasised the general acceptability aspect of it. To serve as money, the definition of money should be comprehensive enough to cover all the essential functions that money performs in the system. Before we arrive at the most suitable definition, it is essential on our part to study a few definitions of

money as given by some eminent economists. Robertson defines money as "Anything which is widely accepted in payments for goods, or in discharge of other kinds of business obligations." Lord J.M. Keynes has defined money as "that by delivery of which debt contracts and price contracts are discharged, and in the shape of which a store of general purchasing power is held." In the words of Dr. Marshall, "Money constitutes all those things which are at any time and place, generally current without doubt or special enquiry as a means of purchasing commodities and services, and of defraying expense." G.D.H. Cole writes, "Money is anything that is habitually and widely used as a means of payment, and is generally acceptable in the settlement of debts." According to Crowther, "*Money is anything that is generally acceptable as a means of exchange and that at the same time, acts as a measure and a store of value.*" Dr. Kent defines money as, "Anything which is commonly used and generally accepted as a medium of exchange or as a standard of value." Ely has defined money as, "Anything that passes freely from hand to hand as a medium of exchange and is generally received in final discharge of debts." Prof. Halm says, "The word money has been used to designate the medium of exchange as well as the standard of value."

Comparing all these, we find that the most suitable definition is that by Crowther, which covers the three important functions – those of means of exchange, measure of value and store of value.

DIFFERENT APPROACHES TO THE DEFINITION OF MONEY

With regard to the definition of money, there are broadly four main schools of thought. Harry G. Johnson and Edgar L. Fiege have indicated four important approaches to the definition of money; viz, (1) Conventional Approach, (2) Chicago Approach, (3) Gurley and Shaw Approach, and (4) Central Bank Approach.

(1) According to the traditional view or conventional approach, money is defined as currency and demand deposits and its most important function is to act as a medium of exchange. Keynes in his *General Theory* classified cash and bank demand deposits as money. Hicks in his *Critical Essay in Monetary Theory* points towards a threefold traditional classification of the nature of money, "to act as a unit of account (or measure of value as Wicksell put it), as a means of payment, and as a "store of value."

(2) The Chicago quantity theorists (Milton Friedman and others) define money as "a temporary abode of purchasing power" which comprises currency plus *total* commercial bank deposits. They take money in the sense of being an asset whose capital value is safe. They include in money time deposits, savings deposits or even such far-fetched items as borrowing power on life insurance policies. Time deposits and other such assets are not directly spendable and, therefore, do not function as a medium of exchange. The economists of the Chicago school, however, argue that time deposits can be converted into currency or demand deposits and should, therefore, be treated as money. The ease with which time deposits can be converted into currency or demand deposits together with the universally held notion that a savings deposit

account is "money in the bank" lends credibility to the close substitutability argument.

By money, Friedman means "the number of dollars people are carrying around in their pockets, the number of dollars they have to their credit at banks in the form of demand deposits, and also commercial bank time deposits." Thus, this school of thought emphasises the store of value function of money.

(3) J.G. Gurley and E.S. Shaw, in a series of contributions culminating in a major theoretical work, '*Money in a theory of Finance*' (1960) have held that money should include the liabilities of non-bank financial intermediaries as well, because they also constitute liquid assets closely substitutable for money. Gurley and Shaw approach includes in the list of close substitutes for the means of payment the deposits of and the claims against all types of financial intermediaries, of which commercial banks are only one variety. They emphasise the close substitution relationship between currency, demand deposits, time deposits, savingsbank deposits, credit institutions' shares, bonds etc. They define money supply as a weighted sum of all these assets, weights being assigned to each item on the basis of the degree of substitutability.

Gurley and Shaw have drawn an important distinction between what they call '*inside*' money and '*outside*' money. Outside money comes from outside the private sector and represents wealth to which there corresponds no debt. It is an asset for someone without being a debt for anyone else. Gold coins and currency notes may thus, be considered as outside money. Inside money, on the other hand, is created against private debt. It is typified by bank deposits and other assets created by financial intermediaries, the assets on the one side corresponding to liabilities on the other side of the Balance Sheet.

The Gurley and Shaw approach, based on the close substitutability argument like the Chicago school approach, differs from the latter in its coverage as well as analysis. It includes in the list of close substitutes for the means of payment,the deposits of and the claims against all types of financial intermediaries and, instead of lumping all the substitutes together, assigns weights to different assets on the basis of the degree of substitutability or closeness to the means of payment.

(4) The fourth approach, specially interested in monetary policy, takes money as a means of financing purchases in much broader concept, measurable or unmeasurable. In the measurable sense, the Federal Reserve Board takes the total amount of credit outstanding. The unmeasurable sense is exemplified by the Radcliffe Committee's concept of the liquidity of the economy. This broader concept of money implies that the economy is able to economise on money by substituting credit for it without limit.

These four schools of thought taken together seem to include both 'concrete' and 'abstract' money in the definition of money. Friedman and Schwartz are right when they say that "the definition of money is to be sought for not on grounds of principle but on grounds of usefulness in organising our knowledge of economic relationship.

Functions of Money. Money came to be introduced because of the difficulties of the barter system. The functions of money are such that they are

expected to remove the difficulties of barter. As already studied above, even the best definition of money cannot bring out the exact features of money. We must, therefore, like Prof. Walker, say, money is, what money does, *i.e.*, anything that performs the functions of money. A proper understanding of the term money, therefore, necessitates a discussion of the functions of money. It will not be out of place to quote a couplet which describes the functions of money :

"Money is a matter of functions four--

A medium, a measure, a standard and a store."

Kinely has classified the functions of money into three groups :

(I) Primary or essential; (II) Subsidiary or derived and (III) Contingent. Let us now discuss these functions :

(I) Primary or Essential Functions. These functions are also called as fundamental functions. These are functions which money has been performing since its inception; it is a different matter that the form of money has changed from time to time. In this group, the following functions of money are included.

Medium of Exchange. The most important economic function of money is to facilitate the buying and selling of goods and services, the common medium of exchange. The main difficulty of direct exchange of goods or barter is the necessity for double coincidence of wants. This is easily overcome by using money as a medium of exchange. Money has made exchange possible, through sale and purchase. The person who holds a commodity in surplus, sells it for money; and whenever he requires other commodities for his use, he purchases them with that money. In the opinion of Dr. Marshall, money is wanted not for its own sake but for the sake of giving possession over commodities and services. It is interesting to quote Dr. Benham in this connection, "A person will accept money in payment not because he necessarily wants money for its own sake, but because he knows that other people in turn will accept it from him in return for the goods and services which he himself requires." Money, functioning as a medium of exchange has, therefore, perfected the price mechanism and has considerably increased the productivity of labour. A Dutch economist N.G. Pierson compares money to, "a shunting locomotive at a railway station, at one moment it pulls one line of trucks, at the next it pushes another, its function being to bring each truck on the right rails in order that it may be able to reach its destination, but the locomotive never leaves the station."

Measure of Value. Money removes the difficulty of barter arising from the lack of common measures of value. Money is the yardstick used for measuring all values. Economic enterprise would be difficult without a unit of account. Today, all values are expressed in terms of monetary unit which makes it possible for exchange to be done easily and accounts to be kept properly. As the prices are expressed in terms of money, it becomes easy for the consumer to select and to pay.

What is true for international market is also true for international transactions. Every country has its own monetary unit e.g., rupee, pound, dollar, yen, mark etc. and the payment is made by it in its own currency after calculating the exchange rate of its currency with that of the other country. This facilitates

exports and imports. Money as a standard measuring rod of value has brought revolutionary changes in the economic life of an individual, of a family and of a country as a whole. One important point to be noted is that sometimes the medium of exchange may be different from the measure of value. But generally the same monetary unit is used both as a medium of exchange and a measure of value. In India, for example, rupee is used for both the purposes. However, the two functions of money should not be confused.

It may not be out of place to mention here that these two functions are closely related to each other. They are performed simultaneously and it cannot be said that such function is performed first while the other one is performed later. Unless the value of commodities is measured in terms of money, money itself cannot act as a collective medium of exchange.

Similarly, if money is not acceptable to people as a medium of exchange, it would be impossible for money to act as a collective measure of value.

(II) Subsidiary or Derived Functions. Under the second group falls the following functions of money :

Standard of Deferred Payments. If the payment is not made immediately after the conclusion of work or after the commodities are purchased or sold, but is made after sometime, it is called deferred payment. In an economic sphere,lending and borrowing are not unusual. Much of economic activity runs on the contract basis. Even in services payments are not made daily but after a fixed period. It may be a week or a month. For loans this period may extend to one year or more. This has become possible only because of money as a standard of payment. The system of deferred payment has in many ways helped in the economic progress of the individual and the country. In business and trade the system of deferred payments has been adopted as a mode of payment. In the business of export and import and loans from other countries or from international agencies deferred payment is the usual method of payment. As the value of money remains more stable than other commodities, it has made lending and borrowing easier and has stimulated economic activities. Its importance as a standard for deferred payments has increased in modern times.

Store of Value. It has been mentioned earlier that during the days of barter, value in the form of commodities could not be stored for any considerable period of time. As money is capable of discharging economic obligations, everybody wants to hold a reserve of this liquid purchasing power for future needs. People receive their incomes at one time but do not use them all at once. Expenditure can be spread over a period with the help of money. The satisfaction of future wants is quite important and money can be stored to be spent later.

Money is the most liquid of all assets, because it can be converted into anything at any time. That is why people would keep a part of their assets in the form of money. It does not perish nor does it lose its purchasing power. In keeping money safe, people actually keep their purchasing power intact and make their future secure. In service or in business or in any trade savings has its own importance. Savings is nothing but storing of value. It helps during difficult periods and helps in the promotion and expansion of business and trade. Thus,

as a store of value, money performs a very important function.

(III) Contingent Functions. In addition to the above functions of money, Prof. Kinley has enumerated some more functions of money. They are :

(a) It helps in the transference of value from one place to another. If anyone wants to transfer the stored value in the form of money to someone living in another city in the form of money, he can do so by various channels of communications without much difficulty. It is clear, therefore, that money has enabled people to transfer their assets and properties from one place to another.

(b) Production, in the modern economy is complex and roundabout. Too many heterogeneous factors cooperate in the production of the national dividend. One important problem is the distribution of the joint product Money simplifies this task as all the factors receive their remuneration in the form of money. Such a division was rather impossible in barter.

(c) Money has a unique property. It can be put to a number of uses *i.e.*, it can be spent on various goods and services. For getting maximum satisfaction, the utilities are to be equated at the margin. Money greatly simplifies this, and is, therefore, the giver of maximum satisfaction to the consumer and to the producer.

(d) Credit is the most important feature of modern business. But the main important supporting pillar of the credit and financial system is money. The supply of credit is linked with the supply of money. Thus in a modern economy money serves as the basis of credit.

(e) Money increases the productivity of capital; it is the most liquid type of capital. It can be put to any use. It is because of the liquidity of money tl capital can be easily transferred from less productive uses. It is because of t liquidity of money to more productive uses and it is because of this fact t mobility of money has enormously increased during recent times.

We have discussed above the various functions of money. Out of all these functions, the four are its main functions i.e., medium of exchange, measure of value, store of value and standard of deferred payments. But it would be more correct to say that each and every function of money is important in its own place and the modern economic system cannot do without it.

Classification of Money. We have noted earlier that money has taken various forms during the process of its development. Money has been classified in different ways. We give below some of the important classifications of money :

1. Metallic Money and Paper Money. Money can be classified on the basis of the material of which it is made. Money made of some metal is known as metallic money, e.g., coins made of gold or silver. Money made of paper is known as paper money, e.g., currency notes of different value.

2. Money and Money of Account. The term money refers to a medium of exchange which possesses general purchasing power. Notes and coins held by the people can be called money. Similarly, current deposits of bank deserve to be treated as money.

The payments for the goods and services bought, or in lieu of debts incurred have to be reckoned in terms of some unit. That common unit of money is called the money of account. For example, in India, money of account is the rupee; in America, it is the dollar and in England, it is the pound.

3. Money-Proper and Bank Money. The term money-proper or state money refers to money issued by the State. For instance, the notes and coins issued by the state constitute money-proper or state money. It has an advantage that its delivery discharges the contract or the debt.

Bank money is simply an acknowledgement of a debt expressed in money of account. Cheques, drafts, bills, hundies etc., serve as bank money. This type of money has become very important in the modern monetary economy.

4. Legal Tender Money and Non-Legal Tender Money. Legal tender money implies a universally acceptable money within a given political area. For example, notes issued by the government of India are legal tender within the political jurisdiction of India. A creditor is under compulsion by law to accept it as a means of payment by the debtor. The seal of the state authority is affixed on it. If a person refuses to accept a legal tender, he is supposed to commit an offence which is punishable under law.

But various forms of credit instruments also circulate along with money and are also used in discharge of business obligations e.g., bank drafts, cheques, hundies etc. They are accepted by a creditor only if he has faith in the credit worthiness of the debtor. Thus, their acceptance is optional and depends on the will of the person who is supposed to receive the payment. All these credit instruments are grouped under non-legal tender money.

5. Limited Legal Tender and Unlimited Legal Tender. The term unlimited legal tender refers to that kind of legal tender which can be given in payment in an unlimited quantity in discharge of a payment of debt. For instance, I may be required to make the payment for a machine costing Rs.2,000. I can give two thousand one rupee notes and the dealer shall have to accept them because they are unlimited legal tender. In India, therefore, 50 paise coin and above and paper notes of all denominations are unlimited legal tender.

Limited legal tender money refers to money that is accepted in payment upto a limited extent. In India, all the coins upto 25 paisa are limited legal tender upto Rs. 25/-. The dealer can refuse to accept the payment in small coins beyond this limit.

6. Standard Money and Token Money. Money can either be standard money or token money. Standard money is that money whose face value is equal to its real value or intrinsic value. Before the Second World War, the silver rupee in India was a standard coin. If it was melted, the value of silver was equal to one rupee.

As against it, token money refers to that type of money whose face value is more than its real value. The notes and coins in circulation in India today, are all examples of token money.

ROLE OF MONEY

Dudley G. Luckett remarks, "There is simply no adequate substitute for money, nor can we do without it." Money occupies a central position in our modern economy. Money has become the religion of the day in the ordinary business of life. In words of Prof. Marshall, "Money is the pivot around which economic science clusters." According tc Mc Connel, "Money betwitches people. They fret for money and they sweat for it. People will do almost anything for money and money will do almost anything for people." The importance of money in the modern world can hardly be overemphasized. It is admitted by all that money reigns supreme in the spheres of human life. May it be the horizon of economics or commerce, sociology or philosophy or politics, ethics or art, may it be peace or war, money is the guiding star. The whole world is at work for money. The entire world has been monetised. In words of Robertson, "The existence of a monetary economy helps society to discover what people want and how much they want it and so to decide what shall be produced and in what quantities, and to make the best use of its limited productive power. And it helps each member of society to ensure that the means of enjoyment to which he has access yield him the greatest amount of actual enjoyment which is within his reach."

Role of money can be discussed through the views of two schools of thought, (a) Classicals and (b) Modern economists.

I. Classical View. The classical economists of the 19th and early 20th centuries considered money to be a 'veil', colourless and purely passive. To classicals, money was neutral and ust a device for facilitating transactions in real things and it did not aff ct the essential nature of transactions. Money was important only i the sense that it was efficient than barter. It was merely a convenie ce. In words of J.S. Mill, "There cannot, in short be intrinsically a mor insignificant thing in the economy of society than money ; except in th character of a contrivance for sparing time and labour. It is a machine for doing quickly and commodiously, what would be done though less quickly and commodiously without it." Thus for classicists money acted merely as a medium of exchange and did not affect output and employment in any way. Adam Smith once compared money to a road over which all the produce of the country passes to the market but which does not produce itself even a single blade of anything." Thus he took money to be a sort of catalytic agent which quickened the chemical reaction but did not change the contents any way. To classicals money was a sort of wrapper or a garment in which goods came to the people—a veil behind which the functioning of the real economic forces were hidden. The volume of output, the kind of goods produced and their quantities, the distribution of wealth and incomes are normally the same in a monetary economy as under a highly developed and efficient barter economy. Money did not influence the real forces working in an economy.

Mill, however, admits that money "exerts a distinct and independent influence of its own when it gets out of order." But the classicals ignored these disorders because they were not frequent and non-existent in the long run. Classicals assign a neutral, passive and static role to money.

2. **Modern View.** According to modern economists money is very important. Money is not demanded only for buying goods and services but it plays an important role in all fields of economic activity. Money is not neutral, passive or static but it has a positive and dynamic function. In words of Benham, "Modern economic life which is founded on specialisation would not be possible without money." Amongst modern economists, there are two main views regarding the role of money :—

(a) **Keynesian View.** To Keynes, money is no more a veil, a convenient medium of exchange of goods, but something more vital, more pivotal, more crucial and more significant, the use of which as a medium of exchange, as a measure of value, as a standard of value, as a standard of deferred payments and more particularly "being a link between the present and the future" *i.e.* as a store of value along with contingent functions has the capacity of influencing the magnitude and trend of economic activity. To Keynes, the very presence of money in an economic system and more particularly its store of value function does influence the income, output and employment. According to Keynes, money affects the rate of interest and through it investment and hence the general level of economic activity *i.e.* income, output and employment. However, at the time of Great Depression, Keynes put greater reliance on fiscal policy because a fall in rate of interest may not increase investment if marginal efficiency of capital is low. The 60's and 70's saw the revival of monetary policy to check inflation and to provide more funds for investment.

Monetarist View. Monetary economists like Friedman also -tance to money. According to them, money does not income but monetary policy has a positive role in -blems of an economy. These economists -netary policy in different type of

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and price mechanism help the consumer to distribute his income over things in a manner as to get the maximum satisfaction. In words of Robertson. "Money helps each member of the society to ensure that the means of enjoyment to which he has access, yield him the greatest amount of actual enjoyment which is within his reach."

Money makes the consumer sovereign. Through price mechanism, a consumer gives an indication to society of what is to be produced and in what quantities. People's demands are reflected in the relative prices of commodities and production is organised to correspond to the consumers' scale of demand. A consumer is also free to distribute his income between savings and consumption.

Money and Production. Money has enabled the producers to organise productive factors in such a way as to obtain maximum gains. In words of Robertson, "The existence of a monetary economy helps the society to discover what and how much they want it and so to decide what shall be produced and in what quantities and to make best use of its limited productive power." Producers are concerned with planning their future production, with the cost of production and selling prices along with profit, all calculated in money.

Money has facilitated borrowing and lending and these are essential in present day production. A great deal of credit is used in production. Money is required by the producer in advance of the disposal of the final product. Raw material and equipment are purchased, labour is employed and paid long before the product is ready and sold. Thus loans of money have to be taken to enable work to go on. The magnitude of credit required today would be impossible without money. Money is the best standard of deferred payments and borrowing and lending are the easiest in a money economy.

The present industrial economy would be impossible without mon According to A.C. Pigou, "In the modern world, industry enfolded in the garment of money." Money has fa labour and specialisation and thus made large-sc

Kent says, "The use of money p energies upon the technical prob development of new processe mobility of capital. C more productive has

are adjusted. Money has facilitated exchange and widened the extent of market.

Money and Distribution. With money, distribution of national income has become easy. All production is the result of the joint effort by the four factors of production. They have to be paid their remuneration in the form of rent, wages, interest and profit. The entrepreneur distributes the remuneration in such a way that marginal productivity of a factor unit is equal to its marginal remuneration or marginal cost. It is with the help of money that the shares of different factors of production are properly adjusted. Accounting, receiving and storing of its share of income by any factor—unit in the form of money is also very convenient.

Money and Public Finance. Today state is no more a police state but a welfare state. The functions of the state have greatly widened. For discharging its functions the government receives payments in the form of taxes, fees and prices and these funds are used to provide administrative and developmental services to the community. The government spends money in such a way that marginal social benefit of public expenditure is equal to marginal social sacrifice of making payments to the government. Such large raising and spending of funds by the government cannot be possible without money.

Furthermore, the government has to budget its income and expenditure in advance and this would be impossible if no common measure of value is used. The role of public finance in the economic life of the community has become so important that the nature and size of public revenue, public expenditure, public borrowing and budgeting affects private expenditure, saving, investment and production. Fiscal policy and deficit financing have acquired great importance in economic life and this has been possible due to the existence of a unit of account—money.

Development of Credit Market. Money is essential for the development of a credit market. It is not easy to lend and borrow in terms of goods. Besides, there would be as many credit markets as different kinds of goods which may be lent or borrowed. Halm says, "The development of money (or credit) market, furthermore, is the precondition of the formation of a uniform price for the uniform service, that is the object of credit transactions. An integrated structure of rate of interest is born. Capital becomes more mobile. It is easy to remit capital in the form of money from one place to another.

Savings and Investments. Savings and investments are made in form of money—the rate of interest being the great stimulator. Money provides mobility to capital and capital formation is possible only with the help of money, for mobilisation of saving from the general public would have been impossible but for money. Furthermore, it is money which has facilitated foreign investments and has promoted monetary co-operation and monetary assistance through International Monetary Fund a[illegible] International Bank for Reconstruction and Development.

Index of Economic Development. There are few basic indicators of economic development like national income, per-capita income and distribution of income. We come to know about these indicators with the help of money.

Basis of Economic Development. The study of economic growth through Rostow's Stages of Growth—Traditional Society, Pre-Take off stage, Take off stage—shows that the basis of economic development has been money. Money only helped in development of technology, specialisation, investment, trade and optimum utilisation of resources.

Importance of Money in Non-Economic Fields

The importance of money is not limited to economic fields only. It is equally significant in non-economic fields. According to Prof. Davenport, "Almost all great political issues and almost all absorbing social problems and almost all international complications rest upon pecuniary standard." The importance of money in non-economic fields is as follows :—

1. Money and Political Hold. Money has helped people in achieving political hold and freedom. It is nothing but money that has strengthened the position of certain political parties and given birth to new organisations. Example of All India Congress is before us. Had it been financially weak, it would have never given so tough a fight to the British for the achievement of political freedom. Moreover, it is money which has facilitated the establishment of democratic institutions and national solidarity. Besides, payment of taxes makes people politically conscious. Government performs welfare functions with the help of money only.

2. Money and Social Reforms. In barter system many social evils existed. Serfdom prevailed, wages were paid in kind. But appearance of money on the socio-economic stage removed all social ills, eliminated serfdom, substituted competition and contract for customs and conventions. Thus, money is an extremely valuable social instrument which has largely contributed to economic welfare.

Money and Art. Money encourages the development of art and also evaluates art. The evaluation of a big artist or a famous dancer or a melodious singer or a first-class singer depends on what he or she earns in terms of money.

Thus, we have seen that money is the central pivot around which economic activity clusters. It affects the basic variables of our economic system. Money is also important in non-economic fields. In fact, we cannot think of a well-organised social, economic and political life in the present day world without money.

MONEY IN A CAPITALIST ECONOMY

A capitalist economy is one in which there is the institution of private property, profit motive, price mechanism, competition and economic freedom. The factors of production are owned by private individuals and there is no curb on the freedom of consumption, production and the choice of a profession. Under it, there is no state-interference. All the major economic decisions are taken by market forces. The importance of money in such an economy is as follows :—

1. Price-Mechanism. A capitalist economy has to solve the problems of what to produce, how to produce, and how much to produce. This is done by the price-mechanism. The scarce resources of the economy are allocated among their competing uses by the working of the price mechanism. The working of the price mechanism implies that changes in demand or preferences of consumers disturb the price-equilibrium which in turn leads to a process of change in the demand and supply position in the market and to adjustments in the price which tends to restore the equilibrium to a new point in a free market economy. Price mechanism is based on money.

2. Capital Accumulation. In a capitalistic economy there is institution of private property. There is infinite struggle for the acquisition and accumulation of money in view of the enormously formidable power which money vests in its master to command everything that is subject to exchange in terms of money.

3. Freedom of Enterprise. There is complete freedom of choice, contract and enterprise. He chooses that enterprise where he gets the maximum remuneration. Thus, choice of enterprise is also based on money.

4. Profit Motive. In this economy, producers try to get maximum profits, the consumers the maximum satisfaction. The factors are diverted in such a way that they get maximum money. Thus the entire productive mechanism of a capitalist society is organised with the hope of reaping a rich money reward and as such the pursuit of money becomes almost to be all and end-all of all economic activities.

5. Sovereignty of the Consumer. Sovereignty of the consumer depends on money. He can buy whatever he wants and the production is also guided by consumer preferences. In words of G.N. Halm, "If a wage earner receives a money income for selling labour, he is free to spend his earnings on whatever available commodities or services he decides to buy."

6. Control of Economic System. In a capitalistic economy, all economic activities like consumption, production, exchange and distribution are guided and controlled by money. In fact, money instils life into a capitalistic economy.

7. Instability. Keynesian analysis of the instability of the capitalist economy assigns a big weightage to the role of money and prescribes appropriate monetary measures to counter-act the ups and downs of business. He suggests a certain amount of state-intervention with a veiw to inject or draw money according to the needs of the economy and the adoption of appropriate contra-cyclical measures.

MONEY IN A SOCIALIST ECONOMY

A socialistic economy is one in which all the means of production are owned and controlled by the state. There is no right of private property and production is not done for profit but for welfare. There are two schools of thought about the role of money in a socialist economy. According to one school of thought, money is superfluous in a socialist economy while according to another school of thought, money has a significant role to play in a socialist economy.

Those socialist writers who contend that money has no role to play in a socialist economy, argue that in a socialistic economy, entire economic activity is planned, controlled and executed by the state, so the use of money can be dispensed with. Money, therefore, is considered superficial. Writers like Marx and Lenin have shown great hostility towards money. Money was looked down upon by these thinkers, for they believed that it was the fundamental cause of the exploitation of labour. Robert Owen contends that profit motive was the root cause of class conflict and exploitation. Maurice Dobb and Lange have to show that in a socialist economy where freedom of choice is abolished and the means of production were owned and operated by the state for the common good, money is superfluous as there is no necessity of pricing process. Under the impact of this thinking, the institution of money was abolished in Soviet Russia after the October Revolution in 1917.

However, the mistake was soon realised. It was admitted by Lenin in 1921 in his New Economic Policy that it was not possible to achieve communism without the use of the institution of money. L.D. Trotsky also maintains that the use of money in a socialistic economy is important. In this connection, he observes, "The blue prints produced by the offices must demonstrate their economic expediency through commercial calculations. Without a firm monetary unit commercial accounting can only increase the chaos." Prof. Lerner points out that a socialist economy cannot operate smoothly without money. He writes without money, "it is impossible for an economic system of any complexity to function with reasonable degree of efficiency." Prof. Halm says, "Even if the aims of production should be determined by a dictator, the allocation of resources according to these aims would have to be the result of the working of pricing process by means of which it is possible to compare the usefulness of the available resources in different fields of employment." Thus, the system of money and banking has been given its due place in the planning system of Russia. In Russia, two types of plans are prepared-physical plan and financial plan. Physical plan is

prepared in terms of real resources, whereas the financial plan is prepared in terms of money to serve as a guide for prices, taxes and government policies. Financial plan helps in the allocation of resources. Thus, even in an economy like Russia, many economic activities are carried in terms of money. Its importance is as follows :—

1. **Medium of Exchange.** Even in a socialist economy, money is essential as a medium of exchange. As a means of payment, a measure of value and a unit of account, it is the basis of the growth of commerce, trade and exchange. Even in a socialist economy, money is the means of circulation.

2. **Allocation of Resources.** Some system of money and prices is essential for allocation of resources and for fixing priorities even in a centrally planned economy. The system of pricing enables the planning authority to compare alternative schemes of allocation of productive resources and fix up the priorities. The pricing process is the only means through which the community's preferences can be known, exchange values of different goods can be expressed and compared. It is only with money that economic calculations can be made and planning is possible.

3. **Capital Formation.** In a socialist economy, capital intensive techniques are employed to have large-scale production. Capital formation is required to have capital-intensive techniques. Capital formation depends on savings and investment which can be made through money.

4. **Distribution of Income.** Money is also essential for distribution of income to the households and firms as reward for their productive services. In the absence of money, determination of remuneration to different factors of production is arbitrary and causes great dissatisfaction among people.

5. **Freedom of Choice.** Money enables the consumers to express their freedom of choice, though in a restricted manner, in a socialist economy. When income is received in money, individuals are at liberty to spend it on any consumption goods of their choice, within the range of goods produced under the plan.

Thus we agree with Prof. Halm when he says, "A socialist economy will remain a monetary economy." Thus money is important even in a socialist economy but it plays a minor role as compared with a capitalistic economy. To sum up, in a capitalist economy, money is the master, but in a socialist economy the servant ; nevertheless it is there in any form.

MONEY IN A MIXED AND DEVELOPING ECONOMY

A mixed economy is one where public sector and private sector exist side by side and work for the development of the economy. A developing economy is one which is making an effort to increase its per capita

income and is growing. The importance of money in a mixed and developing economy is as follows :—

1. **Economic Development**. Economic development requires full and proper use of existing of the economy. Money helps economic growth by encouraging savings, mobilising rural savings for investment, by increasing investment and by influencing the pattern of investment and production.

2. **Formulation of Plans**. In a mixed economy money plays an important role in formulation of plans. The planning authority has to make an estimate of the financial resources required to achieve physical targets. He has to study the sources from where the financial resources would be made available. All this requires calculation in terms of money.

3. **Monetisation of Non-monetised Sector**. A growing economy which has adopted mixed economy as the pattern of growth, more and more money is required for the rapid monetisation of the non-monetised sector of the economy. As the subsistence forming is commercialised, marketable surplus increases, more and more money is made use of. Thus the importance of money goes on increasing.

4. **Full Utilisation of Resources.** In developing countries more and more money is required to make full utilisation of resources. This money comes from taxes, loans and savings. If these sources are not enough, the government has to resort to deficit financing. Deficit financing has been possible only because of the existence of money.

Thus, we find that money is required for every type of economy. In words of Honoe Croome, "At all times, in any society, money will be needed for any complex economic society." We may conclude in words of A.C.L. Day, "The economies of all the countries which are of any significance in the modern world are monetary economies, whether they are capitalist or socialist, free enterprise or controlled, manufacturing or primary producing, they all have the common characteristic that they make extensive use of money."

EVILS OF MONEY

From the foregoing discussion one is bound to conclude that money is all prevading undoubtedly. It is a blessing but not an unmixed one. Admittedly money has helped humanity in solving many problems by facilitating exchange and lubricating the wheels of production, but at the same time it has given birth to many other problems. It is a 'good' servant but a bad master. It is good so far as our society can rid of its evils. Money can rightly be compared to an elephant who performs many services and whose presence in a circus show is highly attractive for many visitors. But if it goes mad, it causes problem to all concerned—circus owner, artists, spectators etc. Same is the case with money, if it goes out of control

of the monetary authorities it will naturally create disastrous conditions in any economy. The main evils of money are as follow :—

1. Instability in the Value of Money. One of the greatest demerits of money is the instability in the value. The fluctations in its value or purchasing power have far—reaching consequences. During inflation, wealth is concentrated in the hands of few businessmen and the poor suffer untold miseries due to high prices. This lowers propensity to consume and hence effective demand which in turn is bound to affect income, output and employment adversely. During deflation businessmen undergo large financial losses and workmen face the situatio n of serious unemployment. According to Prof Keynes, "unemployment, the precarious life of the worker, the disappointment of expectation, the sudden loss of savings, the excessive windfalls to individuals, the speculator, the profiteer—all proceed in large measure from the instability of the standard of value."

2. Inequality of Distribution of Income. Another evil of money is division of society into two main classes—the haves and the havenots. It generates the problem of monopoly and concentration of capital in a few hands. All the socialist writers are hostile towards money because they think that money is the chief source of exploitation of labour by capital and the possessor of money enjoys a superior power which is generally used for weakening the weaker links of the society. Thus, the gap between the rich and the poor is widened. The socialist always thought that money was "the very symbol of avarice, self-aggrandisement, capitalist exploitation and ruthless individualism."

3. Cause of Business and Credit Cycles. Money is held responsible for causing fluctuations in economic activity. A capitalist economy tends to be inherently unstable because of the role played by money through the medium of prices. Given a certain supply of goods, and increase in the quantity of money, either through the increase in the sum of currency (M) or by increase in its velocity of circulation (V) or by an increase in bank credit (M′) tends to push the prices upwards. This increase in prices creates in minds of the businessmen hopes of reaping a rich harvest of profits. The whole economy gets ahead booming and increasing employments, output and income. "The acquisition and accumulation of money during the early stage of a boom seems to intoxicate activity which leads to such a great over-supply of commodities that there is a glut of unsold stocks in the market." It ends in a 'crash'. The prices decline, profits shrink rather dwindle, workmen are sacked, unemployment increases, income fall, demand contracts and prices are pulled down still more resulting in mounting miseries to almost all the sections of the society. Thus we find that the force of money creates business and credit cycles.

4. Over-Capitalization. Credit is easily available and there may be over-capitalization of certain industries *i.e,* more than necessary

capital may be invested. There is over production resulting in fall in prices and instability.

5. **Evils of Factory System.** Money has given rise to the modern mass scale production and the evils of factory system become common. The workers live in slums. Women and child labour is exploited. The society is divided into 'haves' and 'havenots' and there is a constant conflict between the two. Strikes and lockouts disturb industrial peace.

6. **Misuse of Credit.** Money is the basis of credit. Sometimes this credit is mis-used. It is not utilised for productive purposes but for unproductive purposes. This has many bad effects.

7. **Hoarding.** Money becomes so important for people that they start hoarding it. This creates many problems because it reduces effective demand there by affecting income, output and employment adversely.

8. **Black-Marketing.** Money has also created the problem of unaccounted money. People start evading taxes by concealing income. This concealed income induces black marketing and speculative activities. Under such circumstances, monetary and fiscal policies have limited effectiveness.

9. **Moral Degeneration.** In the beginning money was thought to be a means but now it is regarded as an end in itself. All moral and ethical values seem to have been sacrificed at the shrine of money. To quote L.V. Mises, "Money is regarded as the cause of theft and murder, of deception and betrayal. Money is blamed when the prostitute sells her body, and when the bribed judge perverts the law. It is money against which moralist declaims when he wishes to excessive materialism." Thus money has given birth to many social evils—corruption, favourtism, elimination of free access to opportunities.

10. **Materialism and Money-Mindedness.** Money has encouraged materialism. The success or failure in life is judged in terms of money. To quote Ruskin, "The devil of money has come to possess their souls. No religion or philosophy seems to have the power of driving it out." The influence of money is so much that even values of life like friendship, goodness, love, affection and faith are measured in terms of money.

11. **Political Evils.** Money has corrupted political institutions. Democratic institutions and political organisations have become money-minded, changes in the value of money have caused political upheavals. In Germany, because of higher inflation people had lost faith in its currency, Mark. It became difficult for people even to buy the basic necessities of life. It resulted in political upheaval and the autocracy of Hitler in Germany. Inflation in China after Second World War led to communist Revolution and Maosetung became the dictator of China. Thus, money also has political evils.

According to Wadia and Joshi, "An institution like that of money which has in our times helped and stimulated production, and has offered to society the means of guaranteeing to every member, the physical requirements of life, releasing his energies for creative work, for social service, for the furtherance of the spiritual potentialities of human life, needs to be carefully guarded and preserved instead of being abolished, but also needs to be purified of the evils which have incidently resulted from its being regarded as an end in itself, instead of as a means of an end." Thus our beloved money is bound to live and live long along with modern civilisation. And as long as it exists, there is going to be expectations, uncertainity and attendant danger. If evil effects of money are minimised by rigid controls it will always be our obedient servant. If it is freed from such control it will be our dangerous master. In words of Prof. Robertson, "Money which is a source of so many blessings to mankind becomes also, unless we can control it a source of peril and confusion."

But as Walter Balghat remarks, "Money will not manage itself." Money, therefore, has to be controlled and regulated. Monetary policy can be used to regulate the demand for and supply of money. Combined with fiscal policy, monetary policy becomes more effective though it is no panacea for all evils.

Questions

1. Define money. What are the functions of money ?
2. What is money ? Discuss its importance and evil effects.
3. "Money which is a source of so many blessings to mankind, becomes also, unless we control it, a source of peril and confusion." Discuss the statement.
4. What do you mean by Barter economy ? Explain its importance in modern economic life.
5. Discuss the functions of money. Explain its importance in modern economic life.
6. Explain fully the functions of money and its advantages to consumers and producers.
7. Expalin the conventional approach and the Chicago school approach to the definition of money.
8. Describe the role of money in a capitalist economy.
9. Describible the role of money in a socialist economy.
10. How is money important in a mixed economy ?

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MONEY AND CAPITAL MARKETS

WHAT IS MONEY MARKET ?

The term 'money market' is interpreted in different senses. In a very broad sense, it is defined as a centre, where dealings in borrowed funds take place. Such definitions are given by writers, such as Lavington, Myers and others. In the narrow sense and in the sense in which the term is widely used in monetary and banking literature, it is restricted to denote the market, where only short period loans are lent and borrowed. Businessmen and many others need money for a short duration to carry on their transactions and money market provides them an organization for dealing in such a type (i.e. short period) of funds. The Reserve Bank of India defines money market as "the centre for dealings, mainly, of short term character in monetary assets; it meets the short term requirements of borrowers and provides liquidity or cash to the lenders. It is the place, where short term surplus investible funds at the disposal of financial and other institutions and individuals are bid by borrowers again comprising institutions and individuals and also the Government itself." Messrs Madden and Naddler explain it thus, "Money market is a mechanism through which a larger part of financial transactions of a particular country or of the world are cleared."

It is to be noted that the term money market is a misnomer. Money market does not deal in currency (the legal tender money) but only in the credit instruments. It is concerned with the lending and borrowing of near-moneys or money substitutes. Crowther rightly points out that money market is "the collective name given to the various forms and institutions that deal in the various grades of near-money." The word 'market' also does not refer to a particular bazar or place, where the transactions of short period loans take place. The transactions are carried on through various means of communications e.g., on phones or through mail or wire. Thus, the word money market is a misnomer. However, money market is associated with a particular place such as Bombay Money Market, London Money Market or New York Money Market. But London Money Market does not deal with the lending operations of London city only. On the other hand, London Money Market is an international money market. New York Money Market also stands upto the international reputation. Bombay Money Market is a national money market.

Dealers in the money market consist of borrowers and lenders of short period funds. Government, business houses and even individuals are in the need of short period loans. Government is the biggest borrower of such loans. Current deficits are covered by issuing treasury bills. Private business houses also demand short period loans to finance their transactions in working capital.

Other borrowers include commercial banks (who may borrow from other commercial banks or Central Bank), stock merchants, farmers, etc. Lending dealers are the Central Bank, commercial banks and other financial insitutions. Central Bank provides loans to Government and commercial banks. Commercial banks act as lenders to business houses and other needy individuals. Thus, we find that banks "form the nucleus of the whole money market."

So far we have said only that money market deals in short period funds. But, what is the duration of a short period ? Loans may be taken for a few hours, a few days, a few months or years. Money market deals in all such loans, whose duration varies from a few hours to a few months (say three or even upto six). Thus, money market deals not with 'one commodity' but with the whole range of near-money assets or relatively liquid assets, which can be readily converted into cash without the risk of much loss. Thus, money market is a composite term. It is composed of various sub-markets, each of them dealing with one particular kind of near-money asset. The near-money assets are arranged in the order of their maturity period and different types of sub-markets are developed for them. In fact, the soundness of a money market depends on the existence of various sub-markets as it is in the case of London Money Market.

Now, we are in a position to sum up certain essential features of money market : –

(*a*) Money market does not deal in money proper, but in near-money assets. Near-money assets are highly liquid assets.

(*b*) Activities of money market are not carried at a particular fixed place, but they are concentrated in some centre serving a particular region. The region served by the money market may even cover the whole of the world. Thus, money market may be local, national or international.

(*c*) The word 'money market' is a composite term. It refers to a group of specialised markets, which deal in various types of near money assets such as call money, bills of exchange or treasury bills.

(*d*) In its broader connotations, the term money market covers the whole complex of financial institutions, which cater to the monetary needs of people. However, the term is restricted to refer to only that group of related markets, which deals in short-period loans. Short period may vary from a few hours to a few months.

STRUCTURE OF MONEY MARKET

Money market, as noted earlier, is not a homogeneous one, but a group of related sub-markets, each of which deals with a different type of short term loans. The structure of money market differs from market to market and it is not possible to discuss the structure of money market in general. Madden and Nadler have mentioned six possible types of markets in money market in their famous book, "International Money Markets." However, these sub-markets are closely related to one another. Funds flow freely from one sub-market to another. Now, we mention a few important components of the money market.

1. Call Money Market. It refers to the market for extremely short- period loans. These loans are given only for up to seven days but more often from day to day or for overnights only. These are known as call loans or call money since the lending banks can call them up at the shortest possible notice. Such loans
1 are provided by the commercial banks to bill brokers and dealers in the stock exchange who require credit for short periods to finance their customer's tradings on margin and their own holdings of securities. When money is called by one bank it can usually be re-borrowed from another and a circulating fund is thereby kept in constant employment.

There may also be an inter-bank call money market where the demand on excess reserve balances of certain banks comes from other banks that need to command such balances in order to adjust their reserve positions to accord with existing statutory requirements.

2. The Acceptance Markets. The acceptance market refers to the market for bankers' acceptances which arise out of trade transactions – both domestic and foreign. A banker's acceptance may be described as a draft drawn by an individual or a firm upon a bank and accepted by the bank ordering it to pay to the bearer or to the order of the designated party a certain sum of money at a specified future date. The bankers' acceptances can be easily sold or discounted in the market called the acceptance market. In the London money market, there are specialist firms known as acceptance houses which accept bills drawn on them by traders. The acceptance market enjoyed a prominent place as a segment of money market in the past. However, its importance has declined considerably now.

3. Bill Market. The bill market or the discount market is that in which short-dated papers or bills are bought and sold. The bills of exchange and the treasury bills are the most important types of short dated papers.

The bill of exchange is an unconditional written order signed by the drawer requiring the party to whom it is addressed to pay on demand or at a specified future date a certain sum of money to the order of a specified person or to the bearer. It is a commercial paper which can be discounted by commercial banks to get financial accommodation.

Treasury bill is an IOU or the promissory note of the government to pay a specified sum after a specified period, generally 91 days from the date of issue. The treasury bills are generally sold by the central bank on behalf of the government. In view of their unparalleled excellence in liquidity and marketability treasury bills are heavily bought by commercial banks for the purpose of maintaining their secondary reserves.

4. Collateral Loan Market. It refers to the market for loans secured by stocks and bonds, or collateral loans. In its appropriate sense, the collateral market implies the market for collateral loans to brokers and dealers in securities, either on call or for a comparatively short- period of time. The importance of the collateral loan market has declined since the stock market crash of 1929.

Since discounting is the main process of exchange of credit in the segments

of money market described above, they may be taken as a whole and designated as the 'discount market.'

The bond market, the government long term-loan market or the treasury bonds, the stock exchange etc. relate to long term credit accommodation. These markets, therefore, belong to the capital market and are not to be regarded as components of the money market.

THE INSTITUTIONS OF THE MONEY MARKET

The institutions which generally deal in the money market are the commercial banks, non-bank financial institutions, acceptance houses, bill brokers, and the central bank as the apex institution.

1. Commercial Banks. The commercial banks are the most important constituents of the money market. Of all the functions of a modern bank, lending is by far the most important. Commercial banks usually use their funds obtained from deposits and other sources for providing short-term loans to the money market. A major part of their earnings comes from 'interest' and 'discount', that is to say, income derived from advances including bills discounted and bills purchased, and interest on investments. Advances comprise a large portion of a bank's total assets, and form the backbone of the bank's structure. The strength of a bank is thus primarily judged by the soundness of its advances. A wise and prudent policy in regard to advances is considered as an important factor inspiring confidence in the depositors and prospective customers of a bank.

A prudent banker's wish to reconcile the two conflicting motives of profitability and liquidity is materialsed through money market dealings. An ideal advance is one which is granted to a reliable customer for an approved purpose in which the customer has adequate experience. It is safe to ensure that the money will be used to advantage and repayment will be made within a reasonable period from trading receipts or known maturities due on or about given dates.

2. Non-banking Financial Institutions. Non-banking financial institutions also mobilise savings and allocate them. In this respect they perform the same functions in the money market as commercial banks. However, they differ from commercial banks. Since they cannot create credit or money as the banks do, their liabilities are not regarded as money while the liabilities (demand deposits) of commercial banks are regarded as money. Some of the financial institutions are, however, taken as 'near-banks' and their liabilities are taken as 'near money.' Non-banking financial institutions cover a very wide field ranging from highly specialised institutions such as development banks or insurance companies to simple institutions like mutual savings societies. Important non-banking financial institutions in India consist of investment trusts, hire-purchase finance institutions, chit funds, loans and finance companies and nidhis.

3. Acceptance Houses and Bill Brokers. Acceptance houses and bill brokers are the important institutions in the bill market. Acceptance houses (also known as merchant houses or merchant bankers) undertake to accept bills drawn upon them under arrangement, in return for an acceptance commission. They are an important constituent of the money market in the U.K. Bill brokers and discount houses also deal in the business of buying and selling bills of exchange and other bills. Bill brokers act as intermediaries between merchants who want to raise credit through selling bills and institutions who have short-term funds available for investment in bills. Discount houses specialise in discounting bills of exchange, and finance drawers of bills. They occupy an important place in the London Money Market.

4. The Central Bank. The Central bank of a country occupies the highest place in the money market. The central bank is the lender of the last resort and the ultimate reservoir of the funds. The central bank possesses power to create, expand and contract legal tender money and to regulate the credit business of banks. These powers can be fully effective if they cover a wide financial area and if all agencies dealing in money and credit are brought under the Bank's jurisdiction. The central bank, as the accepted leader of the money market, controls and guides the institutions of the money market and towards this end, it is armed with powers of credit control or monetary management.

The successful working of a money market in any economy is largely conditioned by the institutional pattern and structural factors which exist in the money market. It may be emphasised, however, that the various institutions which deal in the money market are intimately related to, and are dependent upon each other. The different segments of the money market cannot function without cooperation from each other. Further, some institutions like the commercial banks, investment banks and insurance companies might use their funds both for short- term as well as long-term and medium-term lendings and, thus, rank as institutions of both money and capital markets. The performance of the various institutions is largely conditioned by the character and composition of money market in which they operate.

Characteristics of an Undeveloped Money Market

The money markets in the majority of underdeveloped countries are mostly undeveloped or unorganised. In fact, they are dualistic, both developed and undeveloped money markets exist side by side. The developed money market consists of the central bank, the commercial banks, bill brokers, discount houses, acceptance houses, etc. On the other hand, the undeveloped money market consists of the money lenders, the indigenous bankers, traders, merchants, landlords, pawnbrokers, etc. Since the majority of the people in underdeveloped countries live in rural areas and are poor, the undeveloped market controls a major portion of the money market. The main characteristics of such a market are :

1. *Personal Touch.* The lenders have a personal touch with the borrowers. The lender knows every borrower personally in the village because the latter resides there.

2. *Flexibility in Loans.* There is no rigidity in loan transactions. The borrower can have more or less amount of loan according to his requirements depending upon the nature of security or his goodwill with the money lender.

3. *Multiplicity of Lending Activities.* Mostly people do not specialise in money lending alone. They combine money lending with other economic activities. A merchant may supply goods on loan instead of money in cash.

4. *Varied Interest Rates.* There is multiplicity of interest rates. Interest rates are much higher than rates in the developed sector of the money market. The interest rates are not even uniform. The rate of interest depends on the need of the borrower, the amount of loan, the time for which it is required and the nature of security. The greater the urgency, the higher will be the interest rate.

5. *Defective System of Accounting.* In the unorganised sector of the money market, the system of maintaining accounts is highly defective. Proper accounts are never maintained. Formal receipts are not issued for interest and the principal repaid by the borrowers. Besides, there is utmost secrecy in maintaining accounts and lending procedures in the undeveloped money market. The accounts of the money lenders are not liable to be checked by any higher authority.

6. *Absence of Link with the Developed Money Market.* The undeveloped sector is not linked with the developed sector of the money market in such countries. The former works independently of the latter and is also not under the control of the developed market. This has the effect of reducing the volume of monetary transactions and savings, and prevents their use in productive investments.

Characteristics of a Developed Money Market

The developed money market is a well organised market which has the following main features :–

1. *A Central Bank.* A developed money market has a central bank at the top which is the most powerful authority in monetary and banking matters. It controls, regulates and guides the entire money market. It provides liquidity to the money market, as it is the lender of the last resort to the various constituents of the money market.

2. *Organised Banking System.* An organised and integrated banking system is the second feature of a developed money market. In fact, it is the pivot around which the whole money market revolves. It is the commercial banks which supply short-term loans, and discount bills of exchange. They form an important link between the borrowers, brokers, discount houses and acceptance houses and the central bank in the money market.

3. *Specialised Sub-Markets.* A developed money market consists of a number of specialised sub-markets dealing in various types of credit instruments. There is the call loan market, the bill market, the treasury bill market, the collateral loan market, and the acceptance market and the foreign exchange market. The larger the number of sub-markets, the more developed is the money market. But the more number of sub-markets is not enough. What is required is that the various sub-markets,should have a number of dealers in

each market and the sub-markets should be properly integrated with one an other.

4. *Existence of large Near-Money Assets.* A developed money market has a large number of near-money assets of various types such as bills of exchange, promissory notes, treasury bills, securities, bonds, etc. The larger the number of near-money assets, the more developed is the money market.

5. *Integrated Interest-rate Structure.* Another important characteristic of a developed money market is that it has an integrated interest-rate structure. The interest rates prevailing in the various sub-markets are integrated to each other. A change in the bank rate leads to proportional changes in the interest rates prevailing in the sub-markets.

6. *Adequate Financial Resources.* A developed money market has easy access to financial sources from both within and outside the country. in fact, such a market attracts adequate funds from both sources, as is the case with the London Money Market.

7. *Remittance Facilities.* A developed money market provides easy and cheap remittance facilities for transferring funds from one market to the other. The London Money Market provides such remittance facilities throughout the world.

8. *Miscellaneous Factors.* Besides the above noted factors, a developed money market is highly influenced by such factors as restrictions on international transactions, crisis, boom, depression, war, political instability, etc.

SIGNIFICANCE OF THE MONEY MARKET

The money market brings together the lenders and the borrowers and provides an arrangement to make transactions in short-term funds. The services of money market are, therefore, essential to the smooth day-to-day working of the banking and financial institutions, business and government houses, and thus for the economy as a whole. Burgess has rightly remarked, "What a bank balance is to the individual, the money market is to the country's credit system."

The authors of the Radcliffe Committee's Report have emphasised the role of the 'wider structure of liquidity in the economy'. To the extent that liquidity means the 'ease with which money can be raised,' the lending institutions, particularly commercial banks, make a special contribution. The principal business of the commercial banks is to receive their depositors' money and pay it on demand, but they can earn income by lending it to customers in such a way that the liquidity of the funds suffers least impairment. A developed money market helps the commercial banks to employ their cash reserves more economically and thus to reconcile the conflicting principles of liquidity and profitability to their maximum benefit.

The money market provides an investment outlet to the commercial banks for any temporarily surplus funds they may have available with them. If the several banks operate rather dissimilar types of business, they may well be able to accommodate each other by organising a 'call loan market.' On the other

hand, if their business is likely to be affected by similar seasonal influences, money will be easy or tight for each of them at the same time, and the problem then becomes one that can most easily be resolved by the provision of special facilities at the central bank. A money market has, thus, an obvious task to perform : to seek out the funds wherever they may be and to channel them into the hands of the institutions that require them.

For an individual banker money market provides a place where changing liquidity requirements can be accommodated. But it is much more than that. It also serves the liquidity needs of an endless variety of individuals, business, and government units. It is an invaluable agency for promoting the flexibility, mobility, and full utilization of community resources.

The money market serves the business and industry by meeting their working capital requirements through the system of exchange bills, commercial papers, and so on. It constitutes a reservoir from which short-term funds are obtained when special demands arise. It also enables the business community to economize in its use of cash by inducing it to invest its surplus funds for short periods with an assurance that these investments can be quickly liquidated or converted into cash. A smoothly functioning money market channelises the flow of funds to the most important uses throughout the economy. It is the channel for mobilising idle resources and investible funds of the community for productive purposes.

A well-organised money market facilitates the working of the capital market also, because the institutions operating in the capital market often make use of funds obtainable in the money market. Conditions in the money market and the short-term rates of interest prevailing therein influence the conditions in the capital market as well as the long-term rates of interest.

The money market mainly helps the government mainly in two ways. It enables the government to obtain short-term accommodation by means of treasury bills without interfering with the normal functioning of the banking and credit system in the economy. Moreover, the money market gives stability to the government security market and strengthens the credit of the government.

A well-developed money market is the basis for an effective performance of a central bank's policy of monetary management. "It is in the money market that the central bank comes into contact with the financial sectors of the economy as a whole and it is through varying the liquidity in the market and thereby influencing the cost and availability of credit that the Bank achieves its economic objectives." The money market and short-term rates of interest which prevail there serve as a good barometer of monetary and banking conditions in the country and thus provide a valuable guide to the determination of central banking policy. Prof. Sen observes that the short-term money market is "the place where the strain on the banking system is first felt in periods of pressure, and it is the place where ease in the banking system is first felt in periods of monetary superfluity." Central banks, therefore, regulate the short-term money market so as to influence cost and availability, demand and supply of money.

Prof. Sen has shown how different sub-markets of a developed money market help in the proper functioning of the cental bank. The developed money market, being a highly integrated structure, enables the central bank to deal with the most sensitive of the sub-markets so that the influence of its operation may spread to other sub-markets also. "The more organised a money market, the greater is the smoothness with which the central bank can exercise control over the banking system."

CAPITAL MARKET

Normally, a business enterprise needs finance for two purposes : (i) for buying capital equipment and fixed assets, such as machinery, tools and implements, power plant, construction of factory building and workshops etc., which are referred to as long-term capital requirements, and (ii) for buying raw materials, holding the stock of finished goods, for payment of wages, etc., which are referred to as short-term capital requirements. Thus, an industrial house has to borrow short-term funds as well as long-term funds. The money market caters to the short-term needs only. The long-term capital needs are satisfied by the capital market.

Concept of Capital Market

The term "capital market" is used to describe the institutional arrangements for facilitating the borrowing and lending of long-term funds. Usually, stress is laid on the markets for long-term debt and equity claims, government securities, bonds, mortgages, and other instruments of long-term debts. Thus, the capital market embraces the system through which the public takes up long-term securities, either directly or through intermediaries. It consists of a series of channels through which the savings of the community are mobilised and made available to the entrepreneurs for undertaking investment activities. The capital market consists of two sub-markets – the one dealing in new issues, and the other concerned with the transfer of the ownership of existing issues. The new issue market attracts new capital investment, while the other only determines the ownership of the existing securities, i.e., by whom the existing securities shall be held. Those who supply long-term finance and those who require them are linked together by a chain of specialist intermediaries. A firm,in need of capital, obtains it through the sale of shares usually through an issuing house which not only advises on the terms of the issue but gets it underwritten by a firm of underwriters who, for a small commission, will take at an agreed price the entire issue or the unsold part of it, which it will sell afterwards at a premium, depending on the performance and the image of the firm. Apart from the issuing house and the underwriting firm is the Stock Exchange where both Government and private securities or stocks are traded.

Conventionally, short-term credit contracts are usually classified as money market instruments, while long-term debt contracts and equities are regarded as capital market instruments. In practice, however, there is a thin line of demarcation between the money market and the capital market, because, quite often, the same institutions participate in the activities of both the markets, and there is flow of funds between the two markets.

Distinction between Money Market and Capital Market. As noted in the beginning of this chapter, money market is sometimes defined as a centre, where dealings in borrowed funds take place. The duration of such funds may vary from a very short period e.g. a few hours to a long-term e.g., a few years. Thus, some writers include both long-period capital market and short period money market proper in the term "money market." However, a distinction between the two i.e., short-period market and long-period market for borrowed funds, is often made. Money market refers to the market, where dealings only in the short period take place. The term 'capital market' is employed to denote the market, where funds for long period are lent and borrowed. Capital market may be further divided into two types – one market for new capital and another for old capital. It must be noted that money market and capital market thus defined are not independent. On the other hand, one is closely related to and dependent on the other. Same dealers deal both in the capital market and in the money market. Commercial banks, for example, lend funds for the short period as well as for long period. Thus, a close relationship exists between the two.

Institutional Structure of Capital Market

The structure of any capital market is composed of the sources of demand for, and supply of long-term capital (money capital).

The Demand for Capital. The demand for capital comes from various categories of borrowers such as the Central and State Governments, local authorities and private industrial and manufacturing groups (joint stock companies). In India, the government constitutes an important source for demand of capital funds on account of widespread expansion of the public sector initiated by the Planning era. There has been an increasing trend of public borrowing in the country during the last three decades. The Central and State Governments as well as local public bodies have floated loans by issuing securities, bonds, etc., the duration of which range from five to fifteen years.

In the private sector, a large demand for long-term capital comes from the joint stock companies. These companies raise funds by the following methods:

(i) issuing shares, (ii) selling debentures, (iii) borrowing from specialised financial institutions – called "Development Banks," and (iv) inviting fixed deposits from the general public.

The local public bodies such as Municipal Corporations, etc., have also been borrowing on a large scale in recent years. They borrow in order to finance lumpy capital expenditure. In particular to cover a planned expenditure, local bodies resort to the capital market. According to the study by the Reserve Bank of India on this issue, the outstanding debt of local bodies has increased much.

The Supply of Capital. There are many channels for the supply of funds to the capital market. A large part of the funds is obtained directly from individual investors through equity capital. There are specialised financial institutions such as the development banks, which also supply capital funds to industries. In India, there are term financing institutions like the Industrial Finance Corporation of India, State Financial Corporations, Industrial Reconstruction Corporation of India, and the Unit Trust of India, which are playing a very significant role in industrial finance since Independence.

In the capital market of India, there are other sources known as financial intermediaries. These are the Life Insurance Corporation and other insurance companies.

It may be observed that within two years (1986–88), the loan sanctioned by the term financing institutions has increased by more than 70 per cent.

Again, in recent years, Provident Funds are becoming a very significant medium of savings for the working classes, but their money does not flow into the private sector investment market, because it is mostly invested in Government securities, Government small savings, and other trustee securities.

In recent years, commercial banks are also indirectly coming into the picture of the capital market. Though, basically, commercial banks are confined to short-term lending, they are, at present, showing an interest in catering to the medium and long-term credit needs of industry and agriculture, through subscribing to the share capital and debentures of special financial institutions like IFC, SFCs and ICICI. Again, banks underwrite shares and debentures issued by joint stock companies.

Importance of Capital Market

The capital market plays an important role in mobilising savings and channelising them into productive investments for the development of commerce and industry. As such, the capital market helps in capital formation and economic growth of the country. We discuss below the importance of capital market.

1. Link between Savers and Investors. The capital market acts as an important link between savers and investors. Funds flow into the capital market from individuals and financial intermediaries which are absorbed by commerce, industry and government. It thus facilitates the movement of stream of capital to be used more productively and profitably to increase the national income.

2. Incentive to Savers. The capital market provides incentives to savers in the form of interest or dividend and transfers funds to investors. Thus it leads to capital formation. In fact, the capital market provides a market mechanism for those who have savings and to those who need funds for productive investments. It diverts resources from wasteful and unproductive channels such as gold, jewellery, real estate, conspicuous consumption, etc., to productive investments.

3. Stability in the values of Stocks and Securities. A well-developed capital market comprising expert banking and non-banking intermediaries brings stability in the values of stocks and securities. It does so by providing capital to the needy at reasonable interest rates and helps in minimising speculative activities.

4. Encourages Economic Growth. The capital market encourages economic growth. The various institutions which operate in the capital market give quantitative and qualitative direction to the flow of funds and bring rational allocation of resources. They do so by converting financial assets into productive physical assets. This leads to the development of commerce and industry through the private and public sectors, thereby inducing economic growth.

In an underdeveloped country where capital is scarce, the absence of a developed capital market is a great hindrance to capital formation and economic growth. Even though the people are poor, yet they do not have any inducements to save. Others who save, they invest their savings in wasteful and unproductive channels, such as gold, jewellery, real estate, conspicuous consumption, etc. Such countries can induce people to save more by establishing banking and non-banking financial institutions for the existence of a developed capital market. Such a market can go a long way in providing a link between savers and investors, thereby leading to capital formation and economic growth.

Questions

1. What is money market ? Describe the important constituents of the money market.
2. Discuss the role of different institutions in the money market. How far is it essential that the central bank should have an effective control over all these institutions ?
3. Distinguish between a developed and an undeveloped money market. What is the significance of a developed money market in an economy ?
4. What do you understand by capital market ? Explain its importance.
5. Distinguish between :
   (a) Money and Capital markets,
   (b) Developed and Underdeveloped money markets.

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3
THE SUPPLY OF MONEY

The supply of money is a stock at a particular point of time, though it conveys the idea of a flow over time. The term 'the supply of money' is synonymous with such terms as 'money stock', 'stock of money', 'money supply' and 'quantity of money'. In this chapter, we shall study the definitions of money supply and its determinants.

Money is used as a means of payment and is freely accepted for the settlement of debts. The term "supply of money" thus means the aggregate stock of domestic money owned by the public in a country. The term 'public' refers here to private individuals and business firms, operating in the economy, but it excludes from itself the Central Government, the Central Bank and the commercial banks. The cash balances held by the Central Government and the State Governments with the Central Bank or with the Treasury or the cash reserves owned by the commercial banks are not included in "money supply" on the ground that they are not in actual circulation in the country. The "money supply" in a country means the *"total stock of money in circulation."* We have, thus, to take into account only that stock of money which is held by the public in a spendable form. Cash balances held by the Central Government, Central Bank or the commercial banks are no part of money supply since they are not in actual circulation in the country. *Thus, the supply of money at any particular moment of time means the total amount of money in circulation at that moment of time.*

The total money supply in a country comprises, (i) currency money, i.e., metallic coins and paper notes issued by the Central Bank or the Central Government and circulating in the country, and (ii) demand deposits (subject to withdrawal by cheques) held by the public with the commercial banks.

Since currency money is issued by the Central Bank or the Central Government, it enjoys a legal status. It is legal tender money. It cannot be refused in the settlement of debts. Its general acceptability as a means of payment has made it an important constituent of money supply. Likewise, demand deposits (subject to withdrawal by cheques) held by the public in commercial banks are also money because they can be withdrawn without prior notice to the banks. But the time-deposits held by the public in commercial banks cannot be regarded as full-fledged money, because they can be withdrawn only at the expiry of the fixed period, except in certain special circumstances. Time-deposits, at best, can be regarded only as quasi-money, not full-fledged money. They, no doubt, function as a store of value, but, by no means, do they constitute a means of payment. Similarly, the over-draft granted by commercial

banks on current account to their customers cannot be regarded as money unless they are actually made use of by the customers in question.

The monetary gold stock held in reserve as a backing to paper currency cannot be included in money supply for the simple reason that it is not permitted to circulate within the country. Likewise, we must exclude from money supply the cash balances held by the Treasury, the Central Bank and commercial banks as reserves to support publicly-owned demand deposits.

To sum up : *the money supply in a country comprises* of two items, (i) currency item (i.e., coins and paper notes), and (ii) deposit item (i.e., demand deposits withdrawable by cheques). The total money supply bears a certain variable proportion to the national income of a country. For example, the money supply in India, roughly speaking, constitutes 26 per cent of its national income (at current prices). This means that the *income velocity of money* in India is 4, or in plain words, every rupee circulates four times to facilitate commercial transactions in the country.

DIFFERENT APPROACHES REGARDING MEASURE OF MONEY SUPPLY

Based on different notions of defining money in a modern economy there are four major approaches regarding the appropriate measure of money stock, namely :

1. The Traditional Approach;
2. The Chicago School or Monetarist Approach;
3. The Gurley-Shaw Approach; and
4. The Radcliffe or Liquidity Approach.

1. Traditional Approach

The traditional approach emphasises the significance of money as a medium of exchange and holds that money supply is constituted by currency money plus chequable demand deposits of banks held by the public. Conventionally, thus, the stock of money at a point of time is measured by aggregating the issue of currency notes and the amount of demand deposits with banks. This approach is functional and function determines nature.

According to this view, money supply is defined as currency with the public and demand deposits with commercial banks. Demand deposits are current accounts of depositors in a commercial bank. They are the liquid form of money because depositors can draw cheques for any amount lying in their accounts and the bank has to make immediate payment on demand. Demand deposits with commercial banks plus currency with the public are together denoted as M_1, the money supply. This is regarded as a narrower view of the money supply because all other liquid assets are excluded from the definition of money supply. Time deposits, post office savings bank deposits and liabilities of non-banking financial institutions cannot by themselves act as a medium of exchange. These liquid assets must first be converted into currency or demand deposits for using the value stored for making payments.

2. The Chicago School or Monetarist Approach

The Chicago school, led by Miton Friedman, however, on empirical consideration, holds that money includes all those things which are perfect substitutes for one another. Thus, according to their view, money supply constitutes currency plus demand deposits plus time deposits. Professor Friedman defines the money supply at any moment of time as "literally the number of dollars people are carrying around in their pockets, the number of dollars they have to their credit at banks in the form of demand deposits, and also commercial bank time deposits." Time deposits are fixed deposits of customers in a commercial bank. Such deposits earn a fixed rate of interest varying with the time period for which the amount is deposited. Money can be withdrawn before the expiry of that period by paying a penal rate of interest to the bank. So time deposits possess liquidity and are included in the money supply by Friedman. Banks in India usually charge a penalty of 2 per cent for premature encashment of time deposits or for their conversion into demand deposits.

Friedman also gives a very strong empirical argument in favour of including time deposits in the definition of money supply. Empirical work done by Friedman and his friends on the U.S.A. data shows that changes in money income are very closely related to and are brought by changes in money supply. The correlation between changes in money supply and changes in money income improves when time deposits are also included in the definition of money supply.

Thus, this definition includes M_1 plus time deposits of commercial banks in the supply of money. This wider definition is characterised as M_2 in America and M_3 in Britain and India. It stresses the store of value function of money or what Friedman says, 'a temporary abode of purchasing power.'

3. The Gurley-Shaw Approach

The Gurley-Shaw approach, on the other hand, visualises that totality of financial liabilities of monetary and non-monetary financial intermediaries as a whole and not the quantity of money alone determines the spending decisions of the public – households and the firms – in a modern economy. Thus, their concept of money supply includes all alternatives, liquid stores of value such as currency, demand deposits, time deposits, equity share, units of Unit Trust, etc.

They include in the supply of money, M_2 plus deposits of savings banks, building societies, loan associations and deposits of other credit and financial institutions.

Gurley and Shaw suggest that money supply be defined as 'a weighted sum of currency and demand deposits and substitutes with weights assigned on the basis of the degree of substitutability ranging from one to zero.' The more imperfect a substitute, the less the weight.

An illustration will make it clear. Let us assume that at a particular time public's total assets are as follows : (i) Rs. 1,000 crores in the form of currency, (ii) Rs. 2,000 crores worth of time deposits, (iii) Rs. 2,000 crores as liabilities of non-bank financial intermediaries. So public's total assets are Rs. 5,000 crores.

Let us assume, for understanding, that degree of substitutability between time deposits and currency is 0.75 and between currency and liabilities of NBFIs is 0.50. Currency, of course needs to be assigned a weight one because it has 100 per cent substitutability.

Now we can calculate the weighted sum of money supply as follows :

Rs. 1000 (currency) x 1 + 2,000 (time deposits) x 0.75 + 2,000 (liabilities of NBFIs) x 0.50 = 1000 + 1500 + 1000 = Rs. 3,500.

Therefore, according to Gurley and Shaw the real money supply will be Rs. 3,500 and not Rs. 5,000.

4. The Radcliffe or Liquidity Approach

The Radcliffe or Liquidity Approach, however, offers a completely new line of thinking and a much wider concept of money supply. The Radcliffe Committee holds that money supply is just a part of the wider structure of liquidity that is relevant to the spending decisions of the community. Prices are affected by the spending decisions of the people which in turn is determined by the general liquidity of the economy. Hence, the concept of money supply should be viewed in terms of general liquidity which includes cash, all kinds of bank deposits, deposits with other institutions, near-money assets, and the borrowing facilities available.

The Radcliffe Committee, however, observes that money supply, in a modern economy, cannot be successfully measured empirically, as the degrees of liquidity of different constituents of money supply (money, near-money and real assets) are varying in nature and are relative in time variation.

The choice between these alternative concepts of the money supply depends on two considerations : *one* "a particular choice of definition may facilitate or blur the analysis of the various motives for holding cash," and *two* from the point of view of monetary policy an appropriate definition should include the area over which the monetary authorities can have direct influence. If these two criteria are applied, none of the four definitions given above is wholly satisfactory.

The first concept of money supply may be analytically better because M_1 is a sure medium of exchange. But M_1 is an inferior store of value because it earns no rate of interest, as is earned by time deposits. Further, the Central Bank can have control over a narrower area if only demand deposits are included in the money supply.

The second concept that included time deposits (M_2) in the supply of money is less satisfactory analytically because "in a highly developed financial structure, it is important to consider separately the motives for holding means of payment and time deposits." Unlike demand deposits, time deposits are not a perfect liquid form of money. This is because the amount lying in them cannot be withdrawn immediately by cheques. Normally, it cannot be withdrawn before the due date of expiry of the deposit. In case a depositor wants his money earlier, he has to give a notice to the bank which allows the withdrawal after charging a penal interest rate from the depositor. Thus time deposits lack perfect liquidity and cannot be included in the money supply. But this concept is more

appropriate from the point of view of monetary policy because the Central Bank can exercise control over a wider area that includes both demand and time deposits held by commercial banks.

The third concept of money supply that includes M_2 plus deposits of non-banking financial institutions is unsatisfactory on both the criteria. First, they do not serve the medium of exchange function of money. Secondly, they almost remain outside the area of control of the central bank. The only advantage they possess is that they are highly liquid store of value. Despite this merit, deposits of non-banking financial institutions are not included in the concept of money supply.

The liquidity approach is also not satisfactory. Needless to point out that liquidity being a state of the mind – a matter of mood and psychology of the people cannot be precisely measured.

CREATION OF MONEY-VARIATIONS IN THE MONEY SUPPLY

After having explained the various approaches of money supply, we shall explain how the money supply increases or decreases. Variations in money supply arise out of the action of money-creating agencies i.e., the treasury, the central bank and the commercial banks. These agencies require assets of various kinds and issue in payment their liabilities.These liabilities are payable on demand. Thus, money supply consists of debts of these money-creating agencies. Commercial banks cause variations in money supply by creating credit through their operations of accepting deposits and advancing loans. The central bank is a government agency that has the virtual monopoly of note-issue. Besides this, the central bank exercises a control over the credit structure of the whole economy. In addition to these two agencies, the Government also issues token coins and notes of smaller denominations. In India, the one rupee notes are issued by the Ministry of Finance of the Govt. of India. As mentioned above, money supply consists of the debts of the various financial institutions (commercial banks, central bank and Treasury). Therefore, any change in the debt composition of these institutions is likely to have its effect on money supply. We may, therefore, explain in what follows as to how these institutions can bring about changes in the money supply through their lending and borrowing activities.

Commercial Banks and Money Supply

The commercial banks' demand deposits are generaly referred to as a constituent of money supply, because they are transferable by cheques in the settlement of debts. It is through payments made by cheques that the volume of demand deposits undergoes constant change.

The creation of bank money or demand deposits depends on the credit creation activities of the banks, which are based on the cash volume – the high-powered money – held by them. The bank money, as such, is regarded as secondary money.

Conventionally, therefore, the total money supply is composed of high-powered money and secondary money.

The relative amounts of the two constituents of money supply, namely, currency, money and demand deposits, depend upon the degree of monetization of the economy and the banking habits and banking development in a country. In economically advanced countries such as the U.S.A., and the U.K., etc., it is demand deposits which form the bulk of the total supply of money. For instance, almost 80 per cent of the money supply of the U.S made up of demand deposit accounts of the public in commercial banks. Thus, most of the payments are cheque payments in that country. It is estimated that over 90 per cent of the total volume of money payments are made with deposit money as the medium of exchange. In underdeveloped countries, such as India, on the other hand, the proportion of currency money to the total money supply with the public is considerably large because a very high percentage of transactions are performed through cash payments, rather than through cheques. On 1st January, 1989, for instance, the total supply of money (M_3) in India amounted to Rs. 1,87,557 crores, of which currency money with the public (M_1) amounted to Rs. 64,836 crores or 34.6% and deposit money was Rs. 1,22,721 crores or 65.4%.

Reserve Bank of India's Measure of Money Stock

Holding a notion of the liquidity approach to money, the Reserve Bank of India sums up the following assets as "aggregate monetary resources" :

(1) Currency (C) ;
(2) Demand Deposits of Banks (DD) ;
(3) Other deposits of the RBI (OD) ;
(4) Post Office savings deposits;
(5) Time deposits of banks; and
(6) Time deposits of post offices.

The report of the second working group on money supply (of the Reserve Bank of India), published in 1977, thus, spelled out a meaningful definition of money supply to facilitate rational policy formulation of the Indian economy.

According to the group, money supply is both an economic and a policy-controlled variable. Money supply, as an economic variable, is influenced by the portfolio behaviour of the public and banks. As a policy-controlled variable, changes in money supply are affected by the monetary authority's view and action regarding the appropriate size of primary as well as secondary money. The group observes that for monetary analysis and policy, a single measure of money stock cannot be used, as it would be not only inadequate but misleading, too. In fact, from the standpoint of money as a medium of exchange, in a modern economy, the monetary aggregates are fundamentally composed of those assets which possess the attribute of "superior liquidity."

In recent years, the, Reserve Bank of India has adopted four measures of money stock in a descending order of liquidity criteria as follows :

(*i*) M_1,
(*ii*) M_2,
(*iii*) M_3, and
(*iv*) M_4

where :

M_1 = (*i*) currency notes and coins with the public (cash on hand of all banks is excluded + (*ii*) Demand deposits with all commercial and co-operative banks (inter-bank deposits are excluded) + (*iii*) Other deposits held with the Reserve Bank of India. (Balance in account No. 1 of the International Monetary Fund, the Reserve Bank of India Employees' Pension, Provident and Guarantee Funds and ad hoc liability items are excluded.)

Conceptually, the category M_1 is the same as traditional sense of money supply with the public.

M_2 = (*i*) M_1 + (*ii*) Savings deposits with Post Office Savings Banks.

The category M_2 is prescribed as a compromise between the need for conceptual neatness and operational feasibility. This is on account of the fact that Post Office Savings Bank deposits are relatively less liquid than demand deposits with banks, but are more liquid than time deposits.

M_3 = (*i*) M_1 + (*ii*) Time deposits of all commercial and co-operative banks (inter-bank time deposits are excluded).

The category M_3 represents aggregate monetary resources (AMR) as has been conceived by the first working group of the R.B.I. on the concept of money.

M_4 = (*i*) M_3 + (*ii*) Total deposits with the Post Office Savings Organisation. (National Savings Certificates are excluded).

The category M_4 is an extension of M_3 as aggregate monetary resources (AMR), aggregated by the second working group.

The second working group claims that these new categories of measuring money stock in the Indian economy have a much wider spectrum of monetary aggregates, in view of their varying degree of liquidity and, thus, may bring the money supply data of the R.B.I. more close to reality.

Table 1 below represents these four measures of money stock by the Reserve Bank of India for some selected years.

TABLE 1

Money Stock in India

(Rs. crores)

Year	M_1	M_2	M_3	M_4
1970-71	7,321	8,311	10,958	12,142
1978-79	21,858	23,634	39,867	44,555
1981-82	24,896	27,237	62,551	69,961
1988-89	64,836		1,87,557	

From the viewpoint of monetary management, M_1 signifies a flow, whereas M_3 refers to stock and both should be tackled with due care.

Questions

1. Explain the concept of 'Money Supply.'
2. Examine the traditional and modern views of the concept of money supply.
3. How is money stock measured by the Reserve Bank of India ?
4. State the different approaches for the measurement of money stock.
5. Discuss the determinants of the money supply. Should time deposits be included under money supply ?

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4
THE DEMAND FOR MONEY

Broadly speaking, the concept of demand for money has been interpreted in two different ways, depending upon the two notions relating to the nature and functions of money. As we know, there are two fundamental functions of money, that of a medium of exchange and that of a store of value. Accordingly, there is the medium of exchange concept of the demand for money, and the store of value concept of the demand for money. The former is the classical view and, the latter, the modern view of demand for money.

CLASSICAL NOTION

To the classical economists, money is demanded by the people only as a medium of exchange and, therefore, implies that it is not demanded for its own sake. Thus, the classicists believed that the demand for money was essentially for money to spend, or for the performance of transactions. This means that the demand for money emerges from the volume of transactions of goods and services to be carried on, during a given period. Thus, they thought that the demand for money was determined by the total quantity of goods and services that had to be paid for, during a given period. Further, the quantity of money people would require to hold (i.e. their demand for money) for these transactions depends on the velocity of circulation of money. This view is also designated as the transactions approach to the demand for money.

Fisher, a classical monetarist, held that the demand for money (M_d) is the product of the voiume of transactions (T), over a period of time, multiplied by the price level (P). Thus,

$Md = PT$.

If, in a year, T is 1,000 units and P is Rs. 10 per unit, then

$Md =$ Rs. $10 \times 1{,}000 = 10{,}000$.

In this approach, the demand for money is defined objectively, in a mechanical sense only, and no attention is paid to the motives behind the demand for money.

MODERN CONCEPT : DEMAND FOR MONEY AS AN ASSET

Modern economists stress the "store of value" function of money and view that the demand for money means demand for money to hold-the demand for cash balances. Money is not just meant for spending. It can be held as a form of wealth or asset which commands other forms of wealth in exchange, all the time.

Thus, money being the most liquid asset, can serve as an efficient store of value; so it is demanded for its own sake. In this sense the demand for money is the inverse of the velocity of circulation; we can say either that the demand for money has increased or that the velocity of circulation, the rate of spending, has diminished, or conversely.

In short, the modern approach to the demand for money stresses the public's need for cash, or money balances, as a store of value at a particular point of time. In this context, it involves evidently, the reason for the people's preference to hold liquid cash, or money, rather than other assets, as a store of value. This desire for money is described by Keynes as liquidity preference. Thus, the demand for money, in the Keynesian sense, is a demand for liquidity or "liquidity preference." Hence the modern approach to the demand for money has been designated as the cash balance or liquidity preference approach.

Now, viewing the demand for money in its modern terminology, the question may be asked : Why should there be demand for money to hold, or why do people prefer to keep idle cash balances ? An obvious answer is provided by the subjective considerations of individuals regarding liquidity motives for the satisfaction of which they desire to hold money balances. Keynes distinguished three such motives which induce people to hold money. These are : (1) the transactions motive; (2) the precautionary motive, and (3) the speculative motive.

To Keynes, the total demand for money implies total cash balances. Analytically, total cash balances may be classified into two parts :

(*i*) Active Cash Balances; and

(*ii*) Idle Cash Balances (The Speculative Demand for Money)

(i) Active Cash Balances

Active cash balances relate to the demand for money held under trasactions and precautionary motives. In other words, transactions demand for money and precautionary demand for money together constitute active cash balances held by the people.

Transaction Demand for Money

Money being a medium of exchange, the primary demand for money balances arises directly out of its use for carrying on ordinary trade and business affairs of the economy. The transactions-prompted demand for money arises on account of the lack of synchronisation between receipts and payments. Individuals, in general, do not receive money income as frequently as they make payments. Thus, when income is received at discrete intervals of time, but is paid out more or less continuously against the exchange of goods and services, it is inevitable that people should need a certain stock of money all the time in order to carry on their transactions.

Briefly, thus, the transactions motive concerns the demand for money balances as a medium of exchange-as a means of bridging the gap between periodic receipts and payments.

In the view of Prof. A.C.L. Day, a major element in the demand to hold money is the transactionary demand. He points out that the transactionary demand to hold money is based on two characteristics of money, namely, convenience and certainty. Money possesses the attribute of convenience, in the sense that, it is universally acceptable, easily portable and storable without any cost. The certainty attribute of money implies the relative stability of the value of money. Thus, money is the most convenient as well as certain form of storing wealth. That is why people prefer to hold money balances in order to bridge the gap between periodic receipts and payments in transactions phenomena.

Keynes, in dealing with the transactions motive, recognised both an income and a business motive.

The Income Motive. This refers to the transaction motive of the households, i.e. consumers' class. From the consumer's point of view, the amount of money which he will hold to satisfy the transactions motive will depend both on the size of his income and on the intervals of time between receipt of the various instalments and then its disbursement. As income increases, there is the income motive for increasing transactionary demand for money. Obviously, a rich man will hold more money to satisfy his income motive than a poor man.

The Business Motive. This refers to the transactions motive of the entrepreneur class or business community. Businessmen require money balances in order to meet business expenses, like payment for new materials and transport, payment of wages and salaries, and allied current expenditure. Money held by producers for these purposes is said to be held to satisfy the business motive. Money balances held under this motive will depend on the turnover of the firm. The larger the turnover, the larger will be the demand for money.

It follows, therefore, that the amount of money balances held under the transactions motive will depend : (i) on the time and size of firm's incomes, and (ii) on the turnover of business. As income rises, and the business becomes more prosperous, the amount of money demanded for the transactions motive will rise.

It is commonly stated that the transactions motive for holding money fluctuates with the level of money income. This is justified by the assumption that transactions and hence, transactionary demand for money, fluctuate in proportion to changes in money income.

It should be noted, then, that the transactions demand for money is income-determined, and is relatively stable because income does not change all of a sudden. Moreover, changes in the rate of interest have no such influence in changing the transactions demand which is determined by the level of income. Thus, the transactions demand for money is interest-inelastic.

There may be seasonal variations in the demand for money held under the transactions motive. For instance, during festive seasons, like Diwali and Christmas, or during vacation period, it may tend to increase at micro-as well as macro-levels. Similarly, in the busy season, after the harvest, the business

community's transactions demand for money tends to increase, while in the slack season, it decreases. Nevertheless, the trend of a community's aggregate demand for money, under the transactions motive, depicts a high degree of correlation of proportionality to the size of money of national income.

In symbolic terms, if L_t represent the transactions demand for money, the money demand function may be stated thus :

$$L_t = f(Y),$$

where, Y stands for the level of national income.

Precautionary Demand for Money

Apart from transactions purposes, people generally desire to hold some additional money balances against unforeseen contingencies. Thus, the second reason for holding money balances is the precautionary motive. The money balances which people hold under the precautionary motive will be devoted to fulfilling the function of a store of value. Out of prudence, people keep some liquid reserves or cash balances to provide for unexpected contingencies-for events such as illness, accidents, unemployment, or some ceremonial occasions. The precautionary demand for money depends largely on the uncertainty of future receipts and expenditures. This demand is very sensitive to the anticipation of the level of income. However, future uncertainty is an important factor determining the precautionary demand for money. Therefore, when uncertainty is present, people tend to hold money balances to act as a buffer against unforeseen contingencies. Naturally the precautionary demand for money varies with the type of emergency envisaged. The increased desire for liquidity, related to the precautionary motive, is described by Keynes as "the desire for security as to the future cash equivalent of a certain proportion of total resources."

Thus, the precautionary demand for money is income-determined and is relatively stable. Obviously, the larger the income of the individual, the larger the cash balance set aside for future contingencies. Moreover, the estimate of future contingencies is normal under normal circumstances, which do not fluctuate suddenly. Thus, the precautionary demand will be relatively stable. Though money is held under the precautionary demand as a store of value, it is not affected by the interest rates. Therefore, the precautionary demand for money is also interest-inelastic, and is income-determined, but, by and large, it changes in response to the changes of uncertainties.

In symbolic terms, by denoting the precautionary demand for money as L_p, we can represent the money-demand function as follows :

$$L_p = f(Y)$$

In practice, however, it is difficult to bifurcate the transactions demand and the precautionary demand for money. And, it must be remembered that both are income-determined. The combined sum of money balances, held under the transactions and precautionary motives, is, however, referred to as "active balances" by Keynes. In symbolic terms, the demand for active balances may be stated as :

$$L_1 = L_t + L_p.$$

Since $L_t = f(Y)$, and $L_p = f(Y)$, it follows that $L_1 = f(Y)$, that is to say, the demand for active balances is a function of income. It has been represented graphically in Fig. 1.

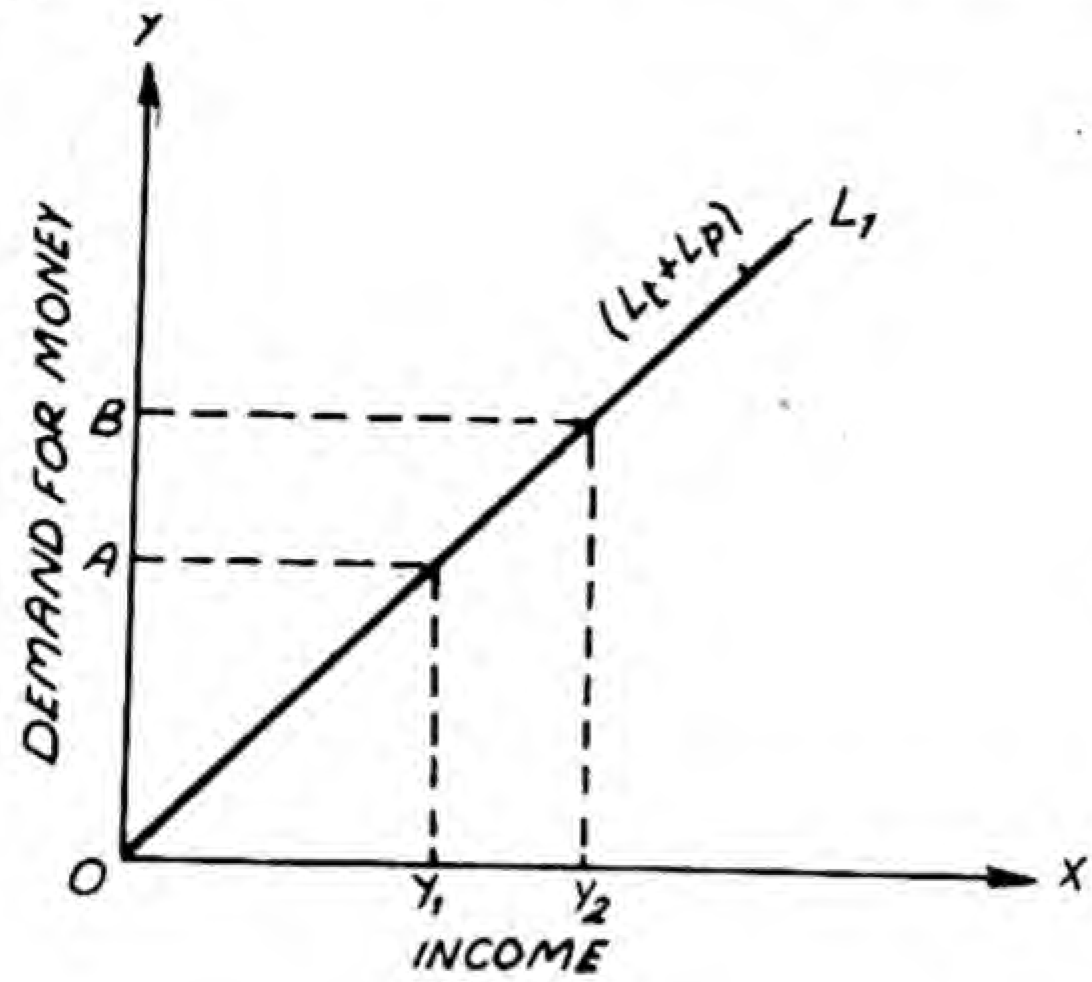

Fig. 1.

In Fig. 1, it can be seen that at income level of OY_1, OA is the demand for money held under the transactions and precautionary motives, i.e. the demand for active balances. At higher income level OY_2, it becomes OB. Thus, there is a proportionate relationshfp between income and the demand for active balances.

(ii) IDLE CASH BALANCES (THE SPECULATIVE DEMAND FOR MONEY)

Keynes pointed out that a section of people in the community hold cash balances for speculative purposes. This demand for money held under the speculative motive is referred to as the demand for "idle balances." The speculative motive for holding cash balances arises from uncertainty about the future rates of interest. More precisely, the speculative demand for money represents the demand for cash for being invested rapidly, as and when attractive opportunities for monetary investments appear.

The speculative motive, in fact, confines itself to the store of value of property of money. Money held under the speculative motive constitutes a store of value, a liquid asset, which the holder intends to use for gambling or to make a speculative gain, e.g. investment in securities at an opportune moment. To Keynes, people make capital gains by speculating in securities or bonds hoping to gain from knowing better than others in the market what the future holds in store for them. Thus, the speculative motive concerns an increase in the demand for money balances as a means to realising a gain, possibly, in anticipation of likely changes in the value of bonds (a form of security asset), but also, most generally, in expected changes in the value of a variety of assets.

It must be remembered that people hold cash balances just to preserve liquidity. But the holding of cash, by itself, does not provide a yield, nor does it

satisfy any want directly. Real assets like jewellery, ornaments, etc. give the pleasure of snob appeal. A car is meant for riding; a house provides shelter or can yield rent, if let out to tenant; shares earn divident; bonds and time deposits receive interest and so on. Whilst money kept idle begets nothing. One has therefore, to pay an opportunity cost for preserving liquidity in terms of the yield forgone. The yield-forgone in keeping cash balances is usually measured in terms of prevailing market rate of interest. The rate of interest is, thus, the cost of being liquid. Thus, at any time, when people have a desire for liquidity, they are supposed to consider the cost element involved. But in holding the active balances, there is the main consideration of convenience and prudence which induces them to have the least consideration for the rate of interest. In the holding of idle balances, however, much attention is paid to the rate of interest, because these balances are held for income-earning purposes of speculative activity. Thus, the amount of money held under the speculative motive depends upon the rate of interest. There is always an inverse relationship between the speculative demands for idle cash balances and the rates of interest. When people expect the prices of fixed income-yielding assets, like bonds, to fall, more balances will be held in cash, than what are just required to satisfy the other two motives (transactions and precautionary). If people expect the rate of interest to fall and prices of bonds to rise, there will be an increased tendency to hold bonds, and other near-money assets, than cash. Thus, the speculative demand for money will be less.

To express it symbolically, thus, the speculative demand for money or the demand for idle balances (L_s or L_2) may be stated as under :

$$L_2 = f(i),$$

where, i stands for the rate of interest. It implies that the demand for idle balances is a decreasing function of the rate of interest. An inverse relationship

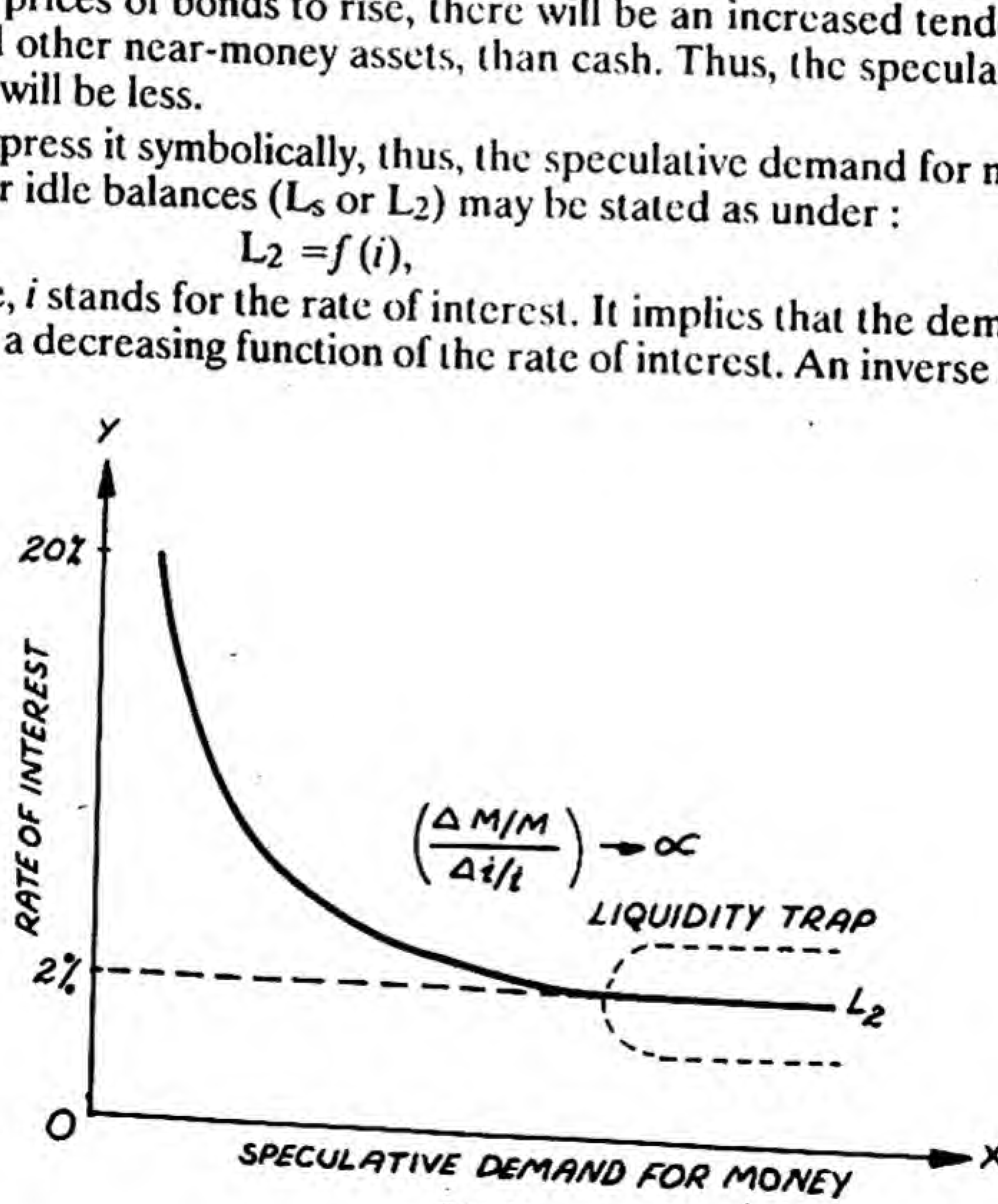

Fig. 2.

between the speculative demand for money and the rate of interest is depicted in Fig. 2.

It can be seen that the demand for idle cash balance is inversely related to the rate of interest. When the rate of interest falls, the demand for speculative balances rises and vice versa. Thus, it is highly interest-elastic. According to Hicks, it is a demand for money act as a liquidity reserve. Therefore, the amount of money held under the speculative motive, as Hicks puts it, or rather, for a liquidity reserve purpose will be a matter of the relative advantage, at the margin, of holding money as against the holding of an interest-yielding asset (near-money asset). It has been observed that at low rates of interest, people prefer to hoard their money rather than use it to buy securities and vice versa. This is because the bond or securities price and interest rates always move in opposite directions. When interest rates rise, bond or security prices fall, when interest rates fall, bond or securities prices rise, so that the capital value of the assets changes accordingly. The reason for this inverse relationship lies in the fact that securities prices (and also of all capital values) actually are the present (capitalised) value of the future flow of income, discounted at the market rate of interest for the type of investment involved.

This may be illustrated with the help of capitalisation formula, as provided by Hamberg :

$$V = \frac{Y}{i}$$

where, V denotes the present value of the future income generated from the security. Y stands for the future income per annum, and i is the market rate of interest.

Now, suppose an investor has invested Rs. 1,000 in government bonds, yielding 6 per cent annual interest and that, at that time, the market rate of interest was 6 per cent. This means that the investor earns Rs.60 as interest. Now, suppose the market rate of interest increases to 10 per cent. Then, if this investor has to sell his bond, no one will pay Rs. 1,000 for it. Because, now Rs. 1,000 in cash can fetch an annual income of Rs. 100. Therefore, in order to sell the bonds he should offer them at less than Rs. 1,000, namely, at a price that would make the annual income of Rs. 60 by the interest on bond exactly 10 per cent of the purchase price. Thus, the capitalised value of the bonds would become 60/0.1 = 600, that is, if the interest rate were to rise from 6 to 10 per cent, bond price would fall from Rs. 1,000 to Rs. 600. Thus, the investment will be a capital loss. Therefore, any individual who expects the rate of interest to rise in the near future will not invest his money in bonds, etc. but will have a strong liquidity preference for money for the present. Similarly the reverse will take place when the rate of interest is expected to fall.

Briefly, therefore, the speculative demand for money is a function of the rate of interest. It is very much interest-elastic. When the interest rate is high, speculative cash balances are minimal, and when the rate of interest is low, the demand for speculative balances may become insatiable.

Moreover, the speculative demand for money, as against transactions and

precautionary demand, is income-determining. The purpose of holding money under the speculative motive is to use it for speculation for earning income. Thus, when the rate of interest is expected to rise, people prefer to hold more money balances at the current rate of interest so that they can take advantage of a rise in the interest rate in future and earn more.

Thus, it is the magnitude of money balances held under speculative motive that determines one's income from it than when it is invested at an opportune moment.

Increased speculative demand for money represents increased preference for liquidity. It is, thus, sometimes called as liquidity proper. It indicates preference for money as the most liquid asset rather than other assets. It is most sensitive because it depends upon speculation or expectations, and is interest-elastic.

THE LIQUIDITY TRAP

The liquidity trap may be defined as the set up of points on the liquidity preference curve where the percentage change in the demand for money, (or$\frac{\Delta M}{M}$), in response to a percentage in the rate of interest, (or $\frac{\Delta i}{i}$), approaches infinity. In other words, there will be a liquidity trap when the demand for money becomes perfectly elastic at a particular low rate of interest.

Refer to Fig. 2 in which the L_2 curve depicts the liquidity preference under the speculative motive, at varying rates of interest.

It is obvious, from the diagram, that at a high interest rate (20%) there is very little demand for money under the speculative motive. As the interest rate moves lower and lower, the demand for speculative balances becomes larger and larger until at 2% it becomes infinitely elastic, i.e., any increase in the money supply will be held as idle cash balances by the people. This situation is called liquidity trap. The trap is depicted in the liquidity preference curve (LP), where the slope of the tangent to the curve becomes horizontal.

The existence of the trap, in real world, would be characterised by a situation, at a moment in time, when large increases in the money stock would simply be absorbed as speculative idle balances, in anticipation of a future rise in the rate of interest.

The liquidity trap is caused by two reasons :

1. At a very low rate of interest on alternative assets in the financial market, the opportunity cost of holding idle balance tends to be the minimum.

2. When interest rates come down to a minimum, the opportunity cost of hoarding idle money is expected to rise in the future rather than decline further.

As such people are induced to hoard as idle balances all the additional money supply made available at this minimum rate of interest. This makes the demand perfectly elastic. As a result, any additional money supply fails to bring down the rate of interest further.

The liquidity trap is also important, partly because it is thought to concern

situations in which the monetary officials lose control over investment in real capital goods via the rate of interest.

According to Bronfenbrenner and Mayer, the phenomenon called "liquidity trap" is mentioned incidentally in terms of elasticity. It makes little difference whether Keynes defines the liquidity trap in terms of an infinite elasticity or a zero slope of a liquidity function.

The liquidity trap suggests that any additional money supply is absorbed by the people as idle balances at a particular minimum rate of interest. Here the demand for money tends to be perfectly elastic which prevents the rate of interest from falling further.

It may be concluded then that there are three motives - transactions, precautionary and speculative - that act as determinants of the demand for money. The transactions and precautionary demand for money is income-determined, more or less interest-inelastic, and relatively a stable phenomenon, whereas the speculative demand for money is interest-elastic, income-determining and very sensitive and fluctuating. In the Keynesian view, it is the speculative demand for money that introduces a dynamic element in the process of general price movements and in the volume of employment and output through a relationship between the current and prospective rates of interest and the profitability of investment.

It has been observed, however, that during inflationary conditions in the economy, the transactions demand for money tends to rise. When prices in general are rising, people find a decreasing value of money, so they may tend to hold more real assets rather than money. Hence their volume of transactions will rise. Again, in view of the rising prices, their routine expenditure will also rise. As such, during inflation, people will be induced to hold more active balances for transactions purposes, i.e. to spend quickly. During deflation, apparently, the transactions demand for money will tend to decrease.

TOTAL DEMAND FOR MONEY

From the above discussion, it follows that the community's total demand for money depends on : (i) the transactions and precautionary motives, and (ii) the speculative motive. In reality, however, the demand for money held under a particular motive is difficult to identify. Hence, for all practical purposes, total cash balances should be regarded as depending on the combined factors relating to these motives. Thus :

$L = L_1 + L_2$

where L stands for an overall demand for money, which is the sum of the demand for active and idle balances.

Since $L_1 = f(Y)$, and $L_2 = f(i)$, it follows that $L = f(Y, i)$. This means that the community's overall demand for money depends upon the level of national income and the rate of interest.

Thus, the liquidity preference schedule of a community can be derived by the super-imposition of the L_1 curves, at each level of income, on the L_2 curve, denoting the relationship between the rate of interest and the idle balances held. The point is made explicit in Fig. 3.

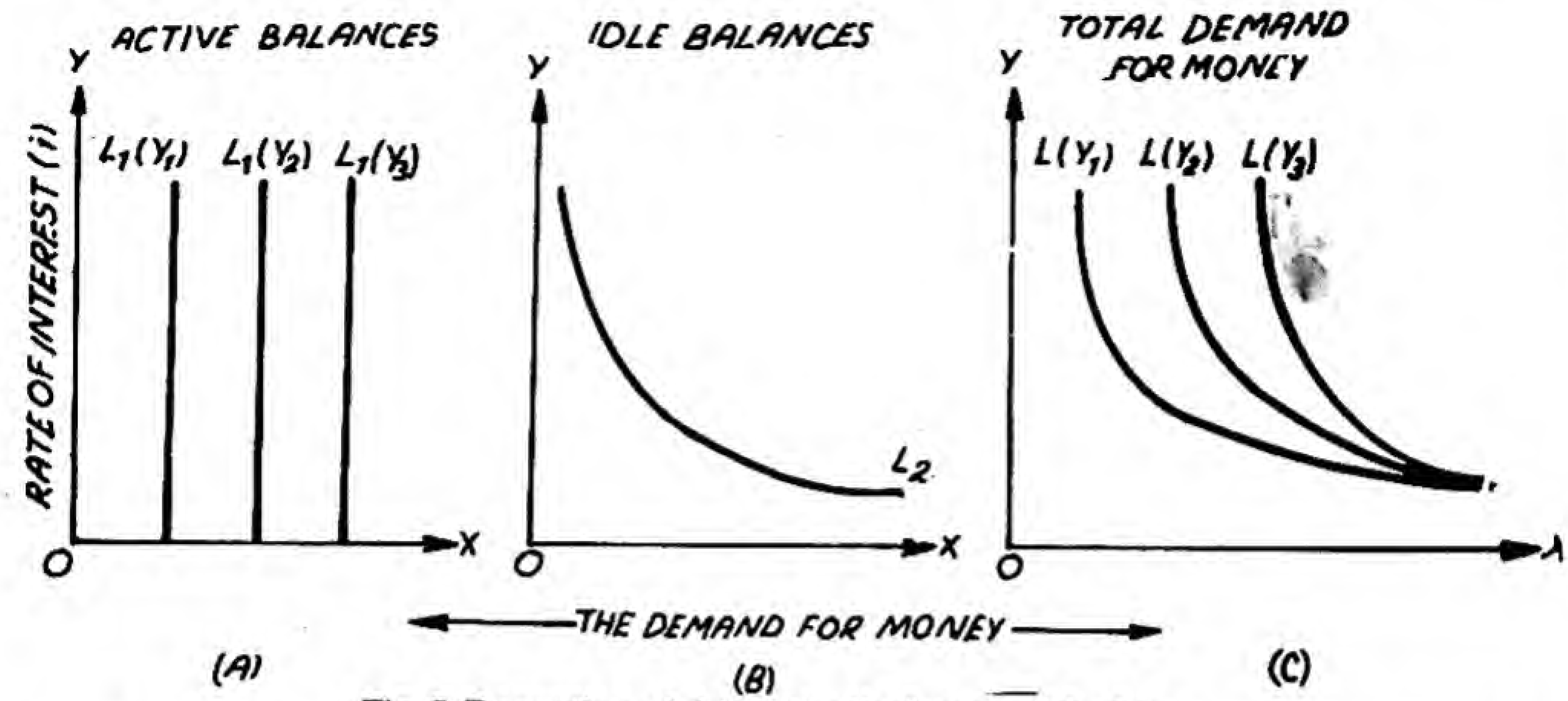

Fig. 3 Derivation of Liquidity Preference Schedule.

In Fig. 3, panel (A) shows the schedule of active balances held at different levels of income. The demand for active balances is perfectly interest-inelastic in the short period. It changes in proportion to the changes in income. Thus, the curve $L_1(Y_1)$ represents the demand for active balances at Y_1 level of income; the curve $L_1(Y_2)$ relates to the demand for active money balances at Y_2 level of income, and $L_1(Y_3)$ at Y_3 level of income. These L_1 curves are vertical straight lines because, on the Y-axis, we have measured the rates of interest, while the demand for active balances are perfectly interest-inelastic. Curve L_2 in panel (B) of the diagram represents the demand for money under the speculative motive (or idle cash balances), which is the inverse function of the rate of interest. Curve L_2, thus, slopes downward but at a minimum rate of interest. It becomes horizontal or parallel to the x-axis. Panel (C) represents the super-imposition of L_1 curves on the L_2 curves. Thus, $L(Y_1)$, $L(Y_2)$ and $L(Y_3)$ curves represent the liquidity-preference schedule of the community, implying that the demand for money varies inversely as the rate of interest, and it increases with the increase in national income. Fig. 3 is a much simplified case. Actually, there may be an infinite number of L curves for all possible levels of income in the figure of the liquidity preference schedule.

To sum up : there are three motives, namely, transactions, precautionary and speculative that influence the demand for money in the community. The transactions and precautionary demands for money are income-determined, interest inelastic and relatively stable in nature. The speculative demand for money is, on the contrary, interest-elastic, income-determining and highly fluctuating in nature.

The classical approach (or the transactions approach), the modern approach (or the cash-balance approach) and the Chicago School Approach will be discussed in detail in the next chapter.

Questions

1. What is demand for money ? Explain the factors which determine the demand for money.
2. The demand for money is a demand for cash balances. Discuss.
3. Elucidate the different liquidity motives influencing the demand for money.

5
THEORY OF MONEY AND PRICES

QUANTITY THEORY APPROACH

Meaning of Value of Money. One peculiar characteristic of money is that it is not wanted for its own sake. In itself money may be quite valueless, particularly in the form of present day paper currency. But, at the same time, money has a great quality. It can be easily converted or changed for useful goods and services. It is precisely this quality of moeny that gives it, its value. Thus money is wanted for the sake of goods and services that it commands. Therefore, when we talk of value of money we have in mind the amount of goods and services that it can get in its exchange. In the words of D.H. Robertson the value of money refers to "the amount of things in general which will be given in exchange for a unit of money." Following Irving Fisher we may further modify this statement, and say that value of money refers to its purchasing power. Purchasing power of money is the reciprocal of price level. Therefore, the study of the purchasing power of money automatically leads us to the study of general price level. In fact, the changes in price level and the value of money are inversely related to each other. When price level rises, value of money falls and vice versa. This inverse relationship between price level and the value of money can be expressed in algebraic terms :

$$V_m = \frac{1}{P}$$

where V_m stands for the value of money and P for the price level.

The value of money, however, does not remain constant over time, it rises and falls. Changes in the value of money affect not only individual owners of the unit of money but also the entire economy. Moreover, variations in the value of money inject an element of instability into the economy as a whole. It is on account of these reasons that the investigation of the factors which govern the value of money becomes of great theoretical and practical importance.

DETERMINATION OF VALUE OF MONEY

Quantity Theory. How is the value of money determined is a question that occupies a central place in the theory of money and has been debated over a long period of time by a number of economists ? The oldest systematic explanation in this regard is, what is called the Quantity Theory of Money ? This theory has a long history behind it and its origin can be traced to those times when the idea of relationship between the supply of goods and their price was accepted by economic theory. It is, therefore, as old as the science of Economics

itself. However, a clear expression of the theory was given by David Hume in his 'Political Discourses' in 1752. Since then, there have been a number of refinements and modifications in the theory, particularly at the hands of classical economists of the nineteenth century. The theory continued to dominate economic thought, right upto 1930's.

In its simplest form, the theory states that value of money varies inversely with its quantity. In other words, price level changes directly with changes in the quantity of money. In fact, it establishes a direct and proportional relationship between the quantity of money and the general price level. Thus, the value of money depends on the transactions it can perform i.e., the number of goods and services it can buy. It can be calculated through the simple process of dividing the quantity of money by the number of goods and services to be exchanged. The theory is, therefore, also called 'Transaction Approach.'

Though the theory is associated with a number of classical economists like J.S. Mill, Ricardo and Taussig, yet it was developed in the U.S.A. by Irving Fisher and Kemmerer. Finally, the credit for bringing final refinement in the theory goes to Fisher. The theory was further modified by Cambridge Economists. In the present century, it has been re-interpreted by Keynes and others.

Before Fisher, J.S. Mill and Taussig have explained the concept of value of money thus : According to Mill, "The value of money, other things being the same, varies inversely as its quantity; every increase of quantity lowers the value and every diminution raising it in an exactly equivalent ratio." Value has been made dependent on the supply of money. Prof. Taussig states, "Double the quantity of money, other things being equal, prices will be twice as high as before and the value of money one half. Half the quantity of money and other things being equal, prices will be one-half of what they were before and the value of money double."

According to Fisher, "Other things remaining unchanged, as the quantity of money in circulation increases, the price-level also increases in direct proportion and value of money decreases and vice versa." Fisher tried to introduce mathematical precision in theory by explaining it with the help of an algebraical equation, in his famous work 'The Purchasing Power of Money' published in 1911. The equation which is popularly known as Fisher's Equation of Exchange *in its final form* runs as under :

$$P = \frac{MV + M'V'}{T}$$

It is based on the simple idea that just as the value of a commodity is determined by the demand for and supply of that commodity, the value of money also is determined by its demand and supply. Now, the question is, what is meant by demand for and supply of money ?

Supply of Money. Supply of money at a particular time refers to the total number of units of money circulating in the country. Fisher indicates it by symbol 'M'. This is supply of money at a point of time. But we are interested to know the supply of money in a period of time – say one year. We know that a unit of

money does not stay in one person's pocket. It continues to circulate and changes hands many times. It thus, performs number of transactions during the course of a year. This is called the velocity or speed of circulation of money and is indicated by symbol 'V'. Thus, the supply of money in a period of time is MV– that is, the quantity of money multiplied by its velocity of circulation. For example, a five-rupee note if changes hands ten times or it passes through ten people in the form of payment, then its value will be equal to fifty rupees and its velocity of circulation will be ten, because it has passed through ten hands.

Another factor to be noted on the supply side is the credit money, which performs the functions of money suitably well, and this is written as M′ and its velocity of circulation, V′. They are indicated as M′V′. Now, the total supply of money in a period of time will be :

$$MV + M'V'$$

Demand for Money. One factor which distinguishes money from other commodities is that it is not wanted for itself. It is demanded to buy certain goods and services, *i.e.,* to perform certain transactions. Larger the amount of goods and services to be purchased, larger will be the demand for money and vice versa. The demand for money is, therefore, indicated by symbol 'T'. Another factor, that influences the demand for money, is the price of goods and services to be purchased. Unless we know their prices, we cannot determine the exact demand for money.

Therefore, the total Demand for money will be equal to the number of goods and services, (T) to be purchased and multiplied with their prices (P), *i.e.*, PT.

To complete the equation, we may say that,

Demand for Money = Supply of Money

or $$PT = MV + M'V'$$

The equation expresses a simple truth that the value of goods received (PT) is equal to the sum of money handed over (MV + M′V′).

We know that the value of money is known through price level. Therefore, to find P, the price level, the final equation will be :

$$P = \frac{MV + M'V'}{T}$$

Through this equation, Fisher makes it sufficiently clear that there is an equi-proportionate relationship between the quantity of money and the general price level, provided V, V′ and T remain constant and there is a fixed ratio between M and M′. It establishes the fact that price level varies directly and proportionately with the total supply of money. To quote Fisher, "One of the normal effects of an increase in the quantity of money is an exactly proportional increase in the general level of prices." There is nothing to interfere with the truth of the quantity theory that variations in money produce normally proportional changes in prices.

It should, however, be remembered that the equation of exchange in itself is not the quantity theory of money. Fisher has used it only as a tool to prove the

validity of the theory. For better understanding, the theory can be expressed in the shape of a diagram given below :—

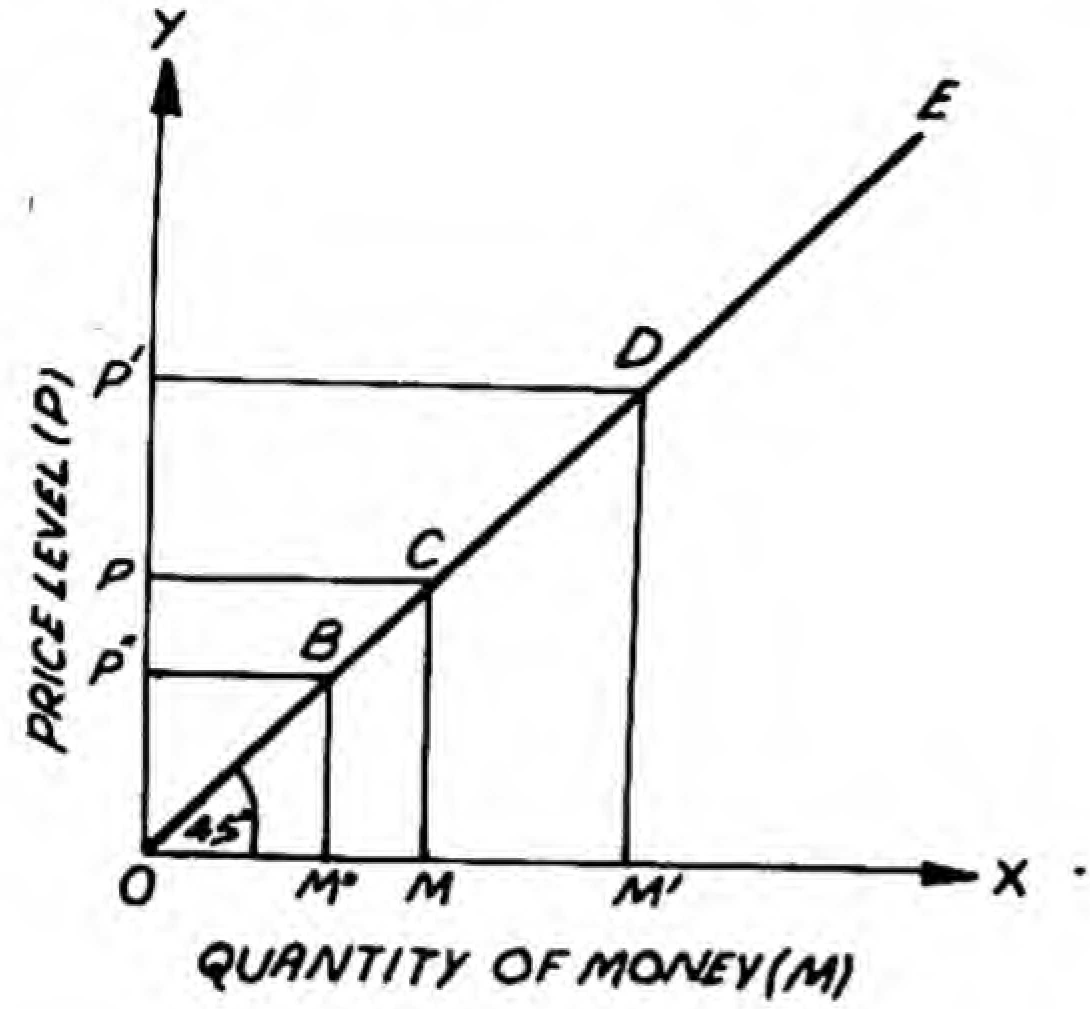

Fig. 1.

In this diagram, quantity of money is represented along the horizontal axis and the general price level along the vertical axis. When the quantity of money is equal to OM, price level is OP. If quantity of money rises to OM', price level also rises to OP'. The proportionate rise in the general price level $\frac{P'-P}{P}$ is equal to the proportionate increase in the quantity of money $\frac{M'-M}{M}$.

Similarly, when the total quantity of money in circulation falls to O M", price level also falls to OP" and the proportionate fall $\frac{P-P''}{P}$ in the general price level is equal to the proportionate fall $\frac{M-M''}{M}$ in the quantity of money. Thus a change in the quantity of money causes an equi-proportionate change in the price level. By joining the points B, C and D, we get the curve BE. This curve forms an angle of 45° with the horizontal axis and represents the equi-proportionate relationship between ΔM (changes in M) and ΔP (changes in P).

Assumptions. Fisher's theory and the equation of exchange used to prove it, are true only if 'other things remain the same'. The theory is, therefore, based on a number of assumptions which are as under :

1. 'P' is influenced or determined by factors like M, M', V, V' and T, but it does not in any way influence these factors. This means that 'P' is a passive factor. Thus, the relation between the price level and other elements is one sided.
2. M and M' bear a fixed proportion, so that where M rises, M' also rises in the same proportion and vice versa.

3. V and V′ are constant and independent of other factors.
4. T, the number of transactions is also assumed to be constant and independent of other factors. This means that changes in natural resources, climatic conditions, techinques of production, productivity of labour etc., do not affect or get affected by the changes in M.
5. Full employment is a normal situation and no idle resources are available to expand the production of goods and services.
6. Absence of barter system.
7. Money is only a medium of exchange.
8. No hoarding. All money is spent for the purchase of goods and services.

9. Assumes long period of time. Fisher does not take note of short-period changes in T or V.

On the basis of above assumptions, we can say that Fisher assumes P as a passive factor and considers other elements such as M, V, V′ and T as constant. He established a direct proportional relationship between the supply of money and price level.

Critical Appraisal of the Theory. The quantity theory, as propounded by Fisher, is by no means the final word regarding the determination of value of money. It has been subjected to a severe criticism and has been denounced as unsuitable, at least, in its rigid form. It is said to be based on unrealistic assumptions, which are far away from the realities of life. The main points of criticism are as under :

1. *No Fixed Relation between M and M′.* The Theory has assumed a constant ratio between the amount of currency (M) and the amount of credit money (M′), so that when M is doubled M′ is also doubled. In actual practice, however, it may not be so. For example, during periods of boom and depression M and M′ do not move in the same proportion, or even in the same direction. During boom period, because of greater demand for credit, M′ will continue to rise even if there is no increase in M. On the other hand, during depression M′ will be low in spite of a rise in M.

2. *V, V′ and T are not Constant.* The assumption that V, V′ and T are constant and independent of changes in M is also not valid. When the quantity of money changes, the velocity of circulation, both of cash money and bank money, is also likely to change. It may move in the direction as changes in M or even in the opposite direction. For example, during inflation, Government may reduce M to check inflation but this may cause V to rise. On the contrary, during depression V may decline in spite of a increase in M.

Similarly, T, the volume of goods and services, is also not free from the influence of M. A rise in M is associated with a period of rising and higher profits. This leads to more investment and increase in the volume of goods and services. Because of all this a change in M may fail to bring about a proportionate change in P. Experience shows that V varies with T; that M may increase without influencing P, if T also increases. Similarly, any increase or decrease in M may not produce proportional changes in P. Germany experienced hyper-inflation

after World War I not so much due to the increase in the supply of money bı due to the increase in V.

3. *Technically Faulty.* Prof. Halm has pointed out a technical flaw in th equation as it is based on inconsistent combinations. He, therefore, warr against over-rating the importance of the theory. He says that "M refers to point of time, whereas V refers to the velocity of money during a period of tim consequently the expression MV would involve the inconsistency of multiplyin non-comparable factors unless the assumption is made that M is the averag amount of money circulating during the period in question or is the sam amount during the whole period. But these assumptions are not compatible wit all possible purpose of the equation."

4. *Over Simplified Generalisation.* The theory implies an oversimplifie generalisation in the sense, that control over money supply would be sufficien to control the price level. This however, is not borne by facts. For example during World Depression of Nineteen thirtees a number of countries tried t check the downward trend of prices by increasing the supply of money, but t no effect. This shows that P is not a function of M alone.

5. *Ignores the study of Relative Prices.* Prof. Hayek in his 'Prices an Production' has pointed out another defect in the theory. The equation o exchange has tried to establish an unrealistic cause relationship between th quantity of money and the general price level, without realising the fact that monetary factors influence the economy only through the process of relative prices. Prices of all commodities may not move to the same extent or they may move in different directions. Thus, the problem of relative prices, which is of greater importance than the general prices, has completely been ignored.

6. *Static Theory.* Another objection against the quantity theory is that it is applicable in a static state, where everything remains the same. But we are living in a dynamic world – a world of continuous change. The theory is, therefore, a misfit in the present day dynamic situation and utterly fails to explain the fluctuations caused by business cycles.

7. *Money not a Medium of Exchange only.* Quantity theorists regard money only as a medium of exchange. They believe that the entire quantity of money is used for purchasing goods and services. It has, however, been pointed out by Keynes and other modern economists that money is used as a store of value also. The amount of money used as a store of value becomes idle and is, therefore, ineffective to influence price level.

8. *Full Employment Not Normal.* The theory is based on the false assumption that there is always 'Full Employment' in the economy. But Keynes has amply proved the hollowness of this assumption. The normal position in capitalist countries is less than full employment or under-employment equilibrium. Under such a situation an increase in the quantity of money leads to an increase in output and employment rather than the price level.

9. *No Proportionality Between 'M' and 'P'.* Keynes also points out that the rigid form of the Quantity Theory establishes a proportional relationship between M and P, which is not true. In actual practice, before the level of full

employment, increase in P will be much slower than the increase in M; while after full employment, rise in P is likely to be more rapid than the increase in M.

10. *Price Level not Infleunced by Money Only.* Quantity Theory creates the impression that money is the only factor influencing price level. No doubt, money is an important factor influencing prices but there are so many other factors like change in demand, availability of substitutes, extent of supply and level of income, which are equally important. It also ignores that the rate of interest is an important link between P and M.

11. *Existence of Barter System.* The theory assumes complete absence of the barter system. But in most of the backward countries barter is still popular with rural population. To that extent the scope of the theory becomes limited.

12. *Not a Correct Explanation of Value of Money.* Another objection against the quantity theorists is that they have completely divorced the theory of money from the general theory of value. In fact, the value of money is determined in the same manner as the value of any other commodity, i.e., through the demand and supply analysis. But the quantity theory through its rigid assumptions, takes into consideration the supply side only.

13. *Ignores Short Period Fluctuations.* The theory considers only long period changes in prices, as a result of changes in the quantity of money. It fails to explain short-period fluctuations which, according to Keynes, are of far greater significance. Even Prof. Fisher admits that V and T are subject to fluctuations in the short run transitional period and they become constant only in the long period.

14. *Does not Explain the Real Cause of Fluctuations in Prices.* According to Crowther the quantity theory can only "explain the 'How it works' of fluctuations in the vlaue of money and in the activity of industry. But it cannot explain the 'Why it works' except in the long period." We notice that sometimes a small increase in the quantity of money may cause a rise in prices, but at other times even a substantial increase in money may have absolutely no effect on prices. This only shows that the real cause of fluctuations in prices lies somewhere else, which the quantity theory is unable to explain.

15. *Does not Explain the Causes of Trade Cycles.* Extending furthe. the argument in the previous paragraph, it is also contended that the quantity theory fails to explain the real causes of trade cycle. Depressions and booms may occur without any shortage or abundance of money respectively. During depression, when income, output, employment and prices are at the lowest level, even a substantial creation of money may not start recovery. Similarly, during boom period a large scale reduction in the quantity of money may fail to arrest the upward trend. Crowther, therefore, regards the quantity theory as, "an imperfect guide to the causes of the trade cycle."

Conclusion. This drastic criticism against theory should not lead us to believe that it is absolutely useless. In fact, in its broad sense the theory conveys a fundamental truth. Quantity of money is, undoubtedly, the most important factor influencing price level. We can, therefore, make a broad generalisation

that other things being equal, an increase in the quantity of money will cause a rise in the general price level, while a reduction in it will cause the prices to decline. Thus, in its rigid sense the theory may not be true. But in its broad sense it certainly has some meaning. It broadly examines the effects of inflation, deflation and increased trade activities. Undoubtedly, in cases of deliberate inflation or deflation, the monetary authorities act upon it. The theory has served as a deterrent to the issuing authorities by relating large issue of money to the shooting prices like hyper-inflation in Germany in 1923 and in 1947-48. In the end, one may agree with Prof. Pigou when he says, "The quantity theory furnished a tool which in the skilled hands of Prof. Fisher has accomplished great things."

CASH BALANCES OR CAMBRIDGE APPROACH

Another version of quantity theory is contained in the cash balances or Cambridge Approach. It was developed and finalised by a group of Cambridge economists, notably Alfred Marshall, A.C. Pigou, D.H. Robertson and J.M. Keynes. The transaction approach lays emphasis on the supply of money. Whereas in the cash balances approach the emphasis is mostly on the demand for money. The transaction approach gives importance to money as a medium of exchange or money on the wing. The Cambridge approach looks at money as a store of value or money 'sitting'. The Cambridge economists have considered only the transactions and precautionary motives for holding cash balances and have ignored the speculative motive to hold money. Under Cambridge approach also, the value of money is determined by the demand and supply analysis but unlike the Transaction Approach, it gives the recognition both to the demand and supply sides. While the supply of money means almost the same thing in the two approaches, but demand for money, in the cash balances approach, refers to that portion of real income which the people want to hold in the form of cash. Thus the demand for money arises from people's desire to keep a certain amount of cash balances with them. Holding of cash represents some security and convenience because it serves as a safeguard against future uncertainties. At the same time, it involves a sacrifice to the person who holds it, because he has to forego the acquisition of useful goods and services. To keep too little cash may mean inconvenience and insecurity; and to have too much of it may require unnecessary suffering. Every individual family and community has to strike a balance between these two extremes and decide the proportion of cash balance to their real income.

This can be explained in the words of Marshall, "In every state of society there is some fraction of their incomes which people find it worth while to keep in the form of currency. It may be a fifth or a tenth or a twentieth. A large command of resources in the form of currency renders their business easy and smooth and puts them at an advantage in bargaining, but on the other hand, it looks up in a barren form resources that might yield an income or gratification if invested say in extra furniture, or a money income, if invested, in extra machinery or cattle."

To quote Crowther, "If the community has decided to keep a store of

oney equal to half its annual income, the actual quantity of money in existence ill have that value, and each unit of money will have its proportionate value."

Let us substantiate it with the help of a simple example. Suppose the real come of the community is 1000 quintals of wheat. If the community decides keep a money stock equal to half its real income, the total quantity of money ill be worth 500 quintals of wheat. Let us suppose further that the total quantity f money consists of 1000 hundred rupee notes. Then one hundred rupee note ill be worth half a quintal of wheat and the price of wheat will be Rs. 200/- per uintal.

If we write R for the real income of the community, K for the proportion f income held in cash, then KR (1000x1/2 = 500 quintals of wheat) will be the alue of total quantity of money. Further, let us write M for the number of units f money.

Therefore, $\frac{KR}{M} = \left(\frac{500}{1000} = \frac{1}{2}\right.$ quintal of wheat) will be the value of a nit of money. Value of money is the opposite of price level. Therefore, to bring ', the price level, into the picture, the final equation will be :

$$P = \frac{M}{KR}$$

The money available to the public is partly in the form of cash with them nd partly in the form of withdrawable current bank deposits. So, Prof. Pigou nodified the equation and expressed it in the following form :

$$P = \frac{KR}{M}\left[c + h\,(1-c)\right]$$

where 'c' denotes the proportion of cash with the public; 1-c denotes the proportion which is kept as withdrawable deposits, 'h' stands for the proportion of cash that banks keep for their liabilities. Thus, $[c + h(1-c)]$ represents the total amount of cash available to the people at any particular time. The working of the above equation can be explained by taking an illustration.

Suppose, in a community, people choose to keep 1/5 the of R – the real resources. Suppose R = 4500 quintals of wheat. This means that the people like to have 900 quintals of wheat in the form of titles to legal tender. Let us suppose that the price per quintal of wheat is Rs.40. It implies that the people will like to have ready command over 36,000 units of titles to the legal tender. Now they may decide to have 36,000 units of currency either in the form of cash or a part of it in the form of current bank deposits. Suppose the community decides to have 1/3 in the form of cash and 2/3 in the form of bank deposits. It means $c = 1/3$ and $1-c = 2/3$. In other words, people keep 12000 units of currency in the form of cash and 24,000 units are kept in the bank. We know that the banks keep only a certain fraction of the total deposits in cash. Suppose banks keep only 10% of their total deposits in cash *i.e.,* they will keep only 10% of 24,000 = 2400 units of currency in cash. The total demand for currency will, therefore, be

12,000 + 2,400 = 14,400

The same can be found out by substituting values for different terms in the equation.

$$M = \frac{KR}{P}\left[c + h(1-c)\right]$$

$$= \frac{\frac{1}{5} \times 4500}{\frac{1}{40}}\left[\frac{1}{3} + \frac{1}{10}\left(\frac{2}{3}\right)\right]$$

$$= 900 \times 40 \left(\frac{1}{3} + \frac{2}{30}\right)$$

$$= 36{,}000 \times \frac{12}{30} = 14{,}400$$

This is how the Cambridge Equation has been derived. This is how Prof. Pigou has expressed the cash balances theory in the form of the above equation. Though different writers have used slightly different versions of the equation but the above equation can safely be regarded as a representative equation.

For the sake of comparison, let us study, the equation used by different economists.

(a) Marshall's Equation

Among the Cambridge economists, Marshall pioneered the cash-balance theory. According to him, people in a country keep a certain part of their annual income plus certain part of their property or assets in the form of 'ready purchasing power,' which may be taken to mean currency plus demand deposits. According to Marshall's equation :

$$M = KPO$$

or $$P = \frac{M}{KO}$$

Here P is the price level; M the quantity of money; O the output or real income and K the proportion of real income held in cash. According to this equation, price level is found out by dividing the money supply by the amount of output over which people like to hold command in the form of money. Similarly, the value of money or the purchasing power of money is found out by dividing the total amount of output which the people want to hold out of total income by the amount of current supply of money with them. In this equation, for the determination of prices it is K, rather than M, which is more significant.

Criticising the equation, one of the modern monetary theorists has observed, "It does not follow from the mere fact that $V = \frac{I}{K}$ as an arithmetic identity, that therefore, the Marshallian analysis is in fact the same thing as the Hume-Fisher analysis. To assert this is to miss entirelv the significance of the K in the Marshallian equation.

(b) Robertson's Equation $M = KPT$

or $$P = \frac{M}{KT}$$

Here P is the price level; M is the supply of money, T stands for physical volume of trade to be affected with money during the year and K is a part of T which people want to keep in the form of money.

It is similar to Marshall's equation except for the point that symbol T has been used to indicate real income instead of Marshall's O. This is Prof. Robertson's most popular equation and is considered better than those of others as it is more comparable with that of Prof. Fisher. For example, if K and T remain constant, the general price level will change in the same proportion as the quantity of money.

(c) Keynes' Equation

Lord Keynes belonged to the Cambridge school, but he was not satisfied with Pigou's equation. He gave his own equation which is known as the 'Real Balance Quantity Equation.' This is explained thus :

$$n = P\,(K + r\,K')$$

or

$$P = \frac{n}{(K + r\,K')}$$

P, represents the price level ;

n, the quantity of money in circulation ;

K, proportion of real balances held in cash ;

K', proportion of real balances held in bank deposits, and

r, proportion of cash reserves of commercial banks to their deposits.

If K, K' and r remain constant then any change in n will cause an equal change in P. This equation also suffers from certain short-comings which were admitted by Lord Keynes himself. The first defect in this equation is that P measures the price level of consumption levels only. It shows that people want to keep balances only to purchase consumption goods. The fact is that balances are held "for a vast multiplicity of business and personal purposes." The second defect lies "in the suggestion that the possible causes of a variation of K' were limited to those which can be properly described as a change of habit on the part of the public." The mistake lay in applying the conception of income deposits to cash deposits. The third shortcoming of this equation is that it applies more or less to the artificial standard.

COMPARISON BETWEEN TRANSACTION AND CASH BALANCES APPROACH

Similarities

It may be borne in mind that differences between the Fisherine type of equation and the Cambridge equation is not as fundamental as is generally supposed. The two types of equations represent different views of the same thing from different stand points. A closer examination of these equations will show that they are more or less similar as both of them show that price level depends upon the quantity of money. Moreover both of them can be easily compared.

$$P=M\frac{V}{T}, \quad P=\frac{M}{KT}, \quad P=M\frac{1}{KR}$$

Fisher (Robertson) (Pigou)

The equations are very much similar. The symbols used by them are also similar. We find that V in Fisher's equation is equal to $\frac{1}{K}$ in Robertson's equation i.e. $V=\frac{PT}{M}$, $K=\frac{M}{PT}$

Besides, these equations show that V and K tend to be the opposite of each other. The higher the fraction of the real income that people hold in money, the lower will be the velocity of money and vice versa. V and K are the reciprocals $\left(V=\frac{1}{K} \text{ Or } K=\frac{1}{V}\right)$. In a period of boom people expect prices to rise and they are, therefore, in a hurry to spend the money and acquire goods. Fisher describes the situation by saying that V is high, Pigou describes the same by saying that the demand for money (K) is low. Thus both versions of the theory can be reconciled by substituting $\frac{1}{V}$ for K and $\frac{1}{K}$ for V into the two equations.

Both the approaches regard price-level or the value of money as directly varying with the quantity of money.

Dissimilarities

There are, however, some notable differences between the two approaches as well. These are as follows :—

(i) **Functions of Money.** The transaction approach emphasises the medium of exchange function of money, whereas Cash Balances approach emphasises the store of value function of money.

(ii) **Demand for Money.** Both approaches give different interpretation to the demand for money. In Fisher's approach, money is demanded for transactions, what Robertson calls as 'money on wings'. In Cambridge equation money is demanded for holding cash balances, what Robertson calls as 'money sitting'.

(iii) **Transaction Velocity and Income Velocity.** Fisherine approach emphasises upon the velocity of money (V) while Cash Balances approach on the income velocity of circulation, velocity of circulation of money stands for the number of times a unit of money changes hands over a certain period of time. Income velocity of circulation refers to the speed with which money on the average completes a full circuit in creating income.

(iv) Flow and Stock. The transaction approach looks upon money as a flow, while Cash Balances Approach as a stock.

(v) Time. Fisher's equation is concerned with the amount of money required by the community during a period of time to finance its transactions, while Cambridge equation focusses attention on the amount of money held in the balances of the individuals at a point of time with a view to finance their future transactions.

(vi) Velocity of Circulation. Fisher's approach emphasises upon the velocity of money (V), while Cash Balances approach on idle balances kept as a part of the national income (K).

(vii) Price Level. P in Fisher's equation is the average of the price level of each good and service at each stage of production and includes the average price-level of all transactions. P in Cambridge equation is the price level of only the goods finally brought to the market.

(viii) Relative Importance of Demand and Supply. In Transaction Approach, there is an emphasis on the supply of money, in Cash Balances approach, there is a shift of emphasis from the supply of money to the demand for money to hold. In this approach, the stage is held by K and the changes in K are regarded as important in affecting the value of money. Thus, there is a marked shift of emphasis.

(ix) Trade Transactions. T in Fisher's theory is the sum total of all transactions, where as R is only the final product which comes to the market.

(x) Nature. The transaction approach is a highly mechanical concept because it depends on the velocity of circulation which is determined by factors like length of pay period, growth of banking, spending habits of people etc. But Cash Balances approach is psychological because it depends on K which is determined by subjective and psychological factors.

Thus there are fundamental dissimilarities in both the theories. In words of Hansen, "It is not true as is often alleged that the cash balance equation is a merely the quantity theory in new algebraic dress."

Superiority of Cash Balances Approach

Emphasising the superiority of the theory, Crowther remarks, "This alternative method of analysis thus takes us one or two steps nearer to reality and as an explanation of what happens in trade cycles, it is much more convincing than merely to ascribe it to change in velocity of circulation." The points of superiority as follows :

(i) More Realistic. The Cambridge approach lays emphasis on the subjective valuations and human motives which are the basis of all

economic activities in contrast to the highly mechanical nature of the concept of velocity of circulation of money in transaction approach.

(ii) **More Comprehensive.** The cash balances version of the quantity theory of money throws light on a new element i.e. the level of income and changes there in. Instead of being concerned with all kinds of happazard transactions, it is concerned with the level of income, which in turn determined the level of output, employment and prices. Truely speaking the problem of price level can not be studied without reference to changes in output, employment and income.

(iii) **Importance to Demand Side.** The emphasis on the demand side is also an improvement over Fisher's preoccupation with the supply side : changes in demand side cause changes in price level even when supply of money does not change.

(iv) **Income Velocity of Money.** Cambridge approach has brought to light another fact that it is not the velocity of money as such which matters but the velocity of circulation of money due to changes in income that matters. Moreover, it is not velocity of circulation of money that is significant, it is change in income velocity of money that is important.

(v) **More Practical.** Cambridge equation (where P is value of money) is more practical and useful than the transaction equation $P = \frac{MV}{T}$ (where P is price-level) to explain the value of money because it is easier to calculate the total cash balances than the total expenditure on transactions.

(vi) **Integration of the Theory of Money with the Theory of Value.** The Cambridge equation makes use of the demand and supply analysis in explaining the value of money and, as such, it goes a long way in integrating the theory of value with the theory of money.

(vii) **Basis of Liquidity Preference Theory.** The cash balances approach has in fact given rise to the famous liquidity preference theory. It has given the precautionary motive for demanding liquidity. People demand money for the sake of security.

(viii) **Significance of K.** Cambridge equation brings to light the significance of K. An analysis of the factors responsible for changes in K offered scope for the study of many important problems like expectations, uncertainty, rate of interest etc. which were not considered by Fisher in transaction version of quantity theory of money. A shift in K reflecting changes in liquidity shows a shift in demand for goods i.e., a movement away from goods to money causing revision in production plans—changes in output, employment and income.

(ix) **Causal Process.** Fisher's approach explains that the changes

in the price-level are caused by changes in supply of money. This does not explain the causal process between money supply and the price level. Cambridge approach explains how price-level changes even if money supply is held constant due to changes in K.

(x) Broader Concept of the Demand. Cambridge equation gives a broader concept of demand because according to his approach money is demanded not only for transaction but also for storing of value.

(xi) Explanation of Trade Cycle. Cambridge equation gives a better explanation of trade cycles. It explains the developments during periods of boom and depression. During inflation, there is a fall in K and the situation known as a 'flight from currency' develops. During depression, changes become self-reinforcing because of a continuous rise in people's desire to hold cash. In this respect it is superior to transaction approach.

Thus, the Cambridge equation is better in certain ways. In words of A.C.L. Dey, the Cambridge version "looks several steps on the way to the modern theory of monetary economics."

Criticism

Despite the superiority of the cash balances approach over transaction approach, it suffers from serious shortcomings as follows :

(i) Ignores Speculative Demand for Money. This approach duly considers the concept of liquidity but does not take into account the complete multiplicity of purposes and complete multiplicity of motives. It ignores the speculative demand for money for holding cash balances. Ignoring speculative demand for money meant that the linkage of theories of the rate of interest and the level of income through the demand for money was not completed.

(ii) Unreal Assumptions. This approach also assumes K and T to be given, thus it becomes subject to all those criticisms which were levelled against Fisher's approach.

(iii) Indeterminate Theory. The theory involves a certain amount of circular reasoning. Desire to hold a part of real income in cash influences the price level but the price level influences the desire to hold money e.g. rise in prices weakens the desire hold cash balances. Thus the cause cannot be separated from the effect. The theory becomes indeterminate.

(iv) Exphasis on Level of Income. Although this theory lays stress on the level of national income, yet it does not take note of other elements like profitability of investment, thriftness—all necessary in a complete theory of the value of money.

(v) Static Theory. Cambridge equation assumes that the elasticity of demand for money is equal to unity. It implies that when value

of money is halved, demand for money is doubled. If the purchasing power of money goes up by 20 percent, the demand for money goes down by 20 percent. Unit elasticity means that stock of money and the volume of goods and services remain constant. It means that the theory can hold only in a stationary state because purchasing power of money is bound to change with the change in money supply.

(vi) Ignores the Effect of Rate of Interest. The cash balances approach fails to assign an explicit role to interest rate, thereby creating an impression that changes in supply of money are directly related to the prices. But a realistic theory of prices in no case can afford to ignore the pivotal role of interest rate.

(vii) Lacks Quantitative Precision. Cambridge approach does not provide an adequate monetary theory which can be made use of to explain and analyse the dynamic behaviour of prices in the economic system, for it does not explain by how much price and output shall change as a result of a given change in supply of money in short period.

(viii) Importance to Consumption Goods. This theory deals with the purchasing power of money in terms of consumption goods or what is called the consumption standard. In Pigou's equation R is the real income of the community and people like to hold a part of it in cash. Keynes also speaks of consumption units and P in his equation is the consumption price level. This means that money is used by people for expenditure on current consumption only which is not correct because balances are held by people for a large variety of business and personal reasons.

(ix) Excessive Emphasis on Value of Money. This theory gives undue importance to value of money. In fact, price level is not the most important aspect of the economy and an explanation of changes in price-level is not the most meaningful part of monetary theory. Changes in production and employment are more significant than fluctuations in the value of money.

(x) Not Comprehensive. Cambridge equation does not explain all the complex forces that affect price-level. The effect of monetary demand for goods and of costs on prices has been ignored. It also does not analyse as to how and why changes in income, saving and investment bring about changes in economic activity.

(xi) Fails to Explain Trade Cycles. It does not help in explaining the trade cycles i.e., why prosperity follows depression and vice versa.

(xii) Lack of Integration. According to Don Patinkin, there is lack of integration of Theory of Value and Theory of Money in Cambridge equation because it ignores the working of real balance effect. Real balance effect means that changes in price-level affect real income of people. This also has an effect on demand and supply of commodities and relative prices of commodities.

But in spite of the criticism levelled against this theory, it may be said that the theory ha. made a valuable contribution by relating the value of money to the changes in the demand for money. It also sheds light on why people hold money balances.

FRIEDMAN'S RESTATEMENT OF THE QUANTITY THEORY OF MONEY

Following the publication of Keynes's *The General Theory of Employment, Interest and Money* in 1936, economists discarded the traditional quantity theory of money. But at the University of Chicago "the quantiy theory continued to be central and vigorous part of the oral tradition throughout the 1930s and 1940s." At Chicago, Milton Friedman, Henry Simons Lloyd Mints, Frank Knight and Jacob Viner taught and developed 'a more subtle and relevant version' of the quantity theory of money in its theoretical form "in which the quantity theory was connected and integrated with general price theory." The foremost exponent of the Chicago version of the quantity theory of money who led to the so called "Monetarist Revolution" is Professor Friedman. He, in his essay "The Quantity Theory of Money – A Restatement" published in 1956, set down a particular model of quantity theory of money. This is discussed below.

FRIEDMAN'S THEORY

In his reformulation of the quantity theory, Friedman asserts that "money does matter." He points out that his quantity theory is a theory of the demand for money. It is not a theory of output, money income, or prices.

According to Friedman, the analysis of the demand for money on the part of the ultimate wealth owning units is formally identical with that of the demand for any durable consumer good. The demand for money depends upon three major sets of factors : "(a) the total wealth to be held in various forms; (b) the price and return on this form of wealth and alternate forms; and the tastes and preferences of the wealth owning units." He, therefore, regards the amount of real cash balances (M/P) as a commodity which is demanded because it yields services to the person who holds it. Thus, money is an asset or capital good. Hence the demand for money forms part of capital or wealth theory.

Broadly, total wealth includes all sources of "income" or consumable services. By income Friedman means "aggregate nominal permanent income" which is the average expected yield on wealth during its life time.

Wealth can be held in five different forms : (*i*) money (M), (*ii*) bonds (B), (*iii*) equities (E), (*iv*) physical non-human goods (G), and (*v*) human capital (H). Wealth holders apportion their wealth among its various forms so as to maximise utility from them. In doing so, they apportion it in such a way that the rate at which they can substitute one form of wealth for another is equal to the rate at which they are just willing to do. Accordingly, the cost of holding various forms of wealth (except human capital) can be measured by the rate of interest that can be earned on such assets as bonds, equities, etc., and the expected rate of change in their prices. People hold more wealth when the rate of interest

decreases or prices of such assets increase, and vice versa. But under normal conditions, the interest elasticity of the demand for money is negligible.

According to Friedman, each form of wealth has a unique characteristic of its own and a different yield. Money is taken in the broadest sense to include currency, demand deposits and time deposits which yields interest on deposits. Thus, money is a luxury good. It also yields real return in the form of convenience, security, etc. to the holder which is measured in terms of the general price level (P). When the price level falls, the rate of return on money is positive because the value of money increases and, when the price level rises, it is negative because of the fall in the value of money. Thus the price level P is an important variable in the demand for money function of Friedman.

The nominal rate of return on bonds, equities and physical assets (non-human goods) consists of (a) any currently paid yield or cost, such as interest and bonds, dividends on equities and cost of storage on physical assets, and (b) changes in their prices.

So far as human wealth is concerned, it is difficult to measure because the conversion of human into non-human wealth or the reverse is subject to institutional constraints. These restrictions cannot be measured in terms of market prices or rates of returns. But there being some possibility of substituting human wealth for non-human wealth, and vice versa, Friedman calls the ratios of non-human to human wealth or the ratio of wealth to income as W. According to Friedman, the level of permanent income (Y) and wealth (W) points towards the income elasticity of demand for money which is greater than unity.

Variables other than income may affect the utility attached to the services of money. These variables are the tastes and prefernces of wealth holders. They also determine the demand function for money along with other forms of wealth. Such variables are denoted as u.

Thus, the demand function for money for an individual wealth holder may be symbolically expressed as :

$$M = f\left(Y, P, r_b, r_e, \frac{1}{P} \cdot \frac{dP}{dt}, w, u\right) \quad ..(1)$$

where M is total stock of money demanded Y *is* total permanent income, P *is* the price level, r_b is the yield on bonds, r_e is the yield on equities, ($I/P.\ dP/dt$) is the expected rate of change of prices of goods w the ratio of non-human to human wealth, and u stands for tastes and preferences and all other relevant variables.

The aggregate demand function for money is the summation of individual demand fuctions. So equation (1) being the demand function of an individual wealth holder also represents the aggregate demand function of all wealth holders in the Community.

This demand function leads to the conclusion that a rise in expected yields on different assets reduces the amount of money demanded by a wealth holder, and that an increase in wealth raises the demand for money.

According to Friedman, this demand equation must be considered

independent in any essential way of the nominal units used to measure money variables. If the units in which prices and money income are expressed is changed, the amount of money demanded would change proportionately. Technically, equation (1) can be regarded as homogenous of the first degree in *P* and *Y*, so that it can be written as :

$$\frac{M}{P} = f\left(\frac{Y}{P}, r_b, r_e, \frac{1}{P} \cdot \frac{dp}{dt}, w, u\right) \qquad ..(2)$$

In this form, the equation expresses the demand for real balances (*M/P*) as a function of real variables, independent of nominal monetary values.

In Friedman's restatement of the quantity theory of money, the supply of money is independent of the demand for money. The supply of money is unstable due to the actions of monetary authorities. On the other hand, the demand for money is stable. It means that money which people want to hold in cash or bank deposits is related in a fixed way to their permanent income. If the central bank increases the supply of money by purchasing securities, people who sell securities will find that their holdings of money have increased in relation to their permanent income. They will, therefore, spend their excess holdings of money partly on assets and partly on consumer goods and services. This spending will reduce their money balances and at the same time raise the national income. On the contrary, a reduction in the money supply selling securities on the part of the central bank will reduce the holdings of money of the buyers of securities in relation to their permanent income. They will, therefore, raise their money holdings partly by selling their assets and partly by reducing their consumption expenditure on goods and services. This will tend to reduce national income. Thus, on both counts, the demand for money remains stable. According to Friedman, a change in the stock of money causes a proportionate change in the price level or income or in both. Given the demand for money, a change in the supply of money will lead to a proportionate change in the price level.

Accordingly, Friedman's equation (1) can be transformed into a theory of money income determination by taking with it a stable velocity function V. Thus, it becomes a reformulated version of the traditional quantity theory of money. Equation (1) can be written as :

$$\frac{M}{Y} = f\left(P, r_b, r_e, \frac{1}{P} \cdot \frac{dP}{dt}, w, u\right) \qquad ..(3)$$

$$M = f\left(P, r_b, r_e, \frac{1}{P} \cdot \frac{dP}{dt}, w, u\right)Y \qquad ..(4)$$

Equation (4) is similar to the Cambridge cash transactions quantity equation $M = kPY$. Since Friedman used the symbol Y to denote money income PY, the Cambridge equation becomes

$$M = kY = \left(\frac{I}{V}\right)Y \qquad ...(5)$$

or $$Y = \left(\frac{1}{k}\right) M = VM \quad \left[\therefore \frac{1}{k} = V\right] \qquad ..(6)$$

Interpreting the Cambridge equation M = ky in terms of Friedman's symbols, M includes time deposits and Y is the permanent income and k is equivalent to Friedman's function

$$f\left(P, r_b, r_e, \frac{I}{P} \cdot \frac{dP}{dt}, w, u\right)$$

of equation (4).

Therfore :

$$\frac{I}{k} = \frac{I}{f\left(P, r_b, r_e, \frac{I}{P} \cdot \frac{dP}{dt}, w, u\right)} = V\left(P, r_b, r_e, \frac{I}{P} \cdot \frac{dP}{dt}, w, u\right)$$

From equations (4) and (6), we have

$$Y = V\left(P, r_b, r_e \frac{I}{P} \cdot \frac{dP}{dt} w, u\right) M \qquad ...(7)$$

In this form, Friedman's equation becomes the usual quantity theory equation, where V is the income velocity. Thus, Friedman's money income determination equation of the quantity theory is a simplified form of the Cambridge equation.

Its Criticism

Friedman's reformulation of the quantity theory of money has evoked much controversy and has led to empirical verification on the part of the Keynesians and the Monetarists. Some of the criticism levelled against the theory is discussed as under :

Friedman has been criticised for using the broad definition of money which not only includes currency and demand deposits (M_1) but also time deposits with commercial banks (M_2). This broad definition leads to the obvious conclusion that the interest elasticity of the demand for money is negligible. If the rate of interest increases on time deposits, the demand for them (M_2) rises. But the demand for currency and demand deposits (M_2) falls. So the overall effect of the rate of interest will be negligible on the demand for money. But Friedman's analysis is weak in that he does not make a choice between long term and short term interest rates. In fact, if demand deposits (M_1) are used a short term rate is preferable, while a long term rate is better with time deposits (M_2). Such an interest rate structure is bound to influence the demand for money.

Friedman regards money as a luxury good because of the inclusion of time deposits in money. This is based on his finding that there is higher trend rate of the money supply than income in the United States. But on such 'luxury effect' has been found in the case of England.

In Friedman's demand for money function, wealth variables are preferable

to income and the operation of wealth and income variables simultaneously does not seem to be justified. As pointed out by Johnson, income is the return on wealth, and wealth is the present value of income. The presence of the rate of interest and one of these variables in the demand for money function would appear to make the other superfluous.

Friedman takes the supply of money to be unstable. The supply of money is varied by the monetary authorities in an exogenous manner in Friedman's system. But the fact is that in the United States the money supply consists of bank deposits created by changes in bank lending. Bank lending, in turn, is based upon bank reserves which expand and contract with (a) deposits and withdrawals of currency by non-banking financial intermediaries; (b) borrowings by commercial banks from the Federal Reserve System; (c) inflows and outflows of money from and to abroad; and (d) purchase and sale of securities by the Federal Reserve System. The first three items definitely impart an endogenous element to the money supply. Thus the money supply is not exclusively exogenous, as assumed by Friedman. It is mostly endogenous.

Money supply and money GNP have been found to be positively correlated in Friedman's findings. But, according to Kaldor, in Britain the best correlation is to be found between the quarterly variations in the amount of cash held in the form of notes and coins by the public, and corresponding variations in personal consumption at market prices, and not between money supply and the GNP.

Despite this criticism, "Friedman's application to monetary theory of the basic principle of capital theory – that is the yield on capital, and capital the present value of income – is probably the most important development in monetary theory since Keynes's *General Theory*. Its theoretical significance lies in the conceptual integration of wealth and income as influences on behaviour."

Questions

1. Critically examine the Quantity Theory of Money.
2. Would you consider cash-balances approach to the value of money as superior to the transaction approach ?
3. What is the salient difference between Fisher's Equation and the Cambridge Equation of Money ? Which one do you prefer and why ?
4. Explain the contribution of Milton Friedman to the quantity theory of money.
5. Explain Friedman's reformulation of the quantity theory of money.

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6

COMMERCIAL BANKING

We have already emphasized upon the importance of credit for the development and phenomenal growth of industries, trade and commerce. In modern times, the system of credit is the base upon which the super-structure of banking has been raised; and the fact is that banking and other financial institutions have become part and parcel of credit mechanism. Modern banking is a broad term which includes several types of specialised banks (foreign exchange, co-operative, land mortgage, central banks etc.). We will discuss in this chapter the development of commercial banking, functions of joint stock banks, structure of banking system and credit creation by the banks.

Origin of the Term 'Bank'. Before the dawn of modern civilization, money-changers facilitated trade by changing different currencies and supplying to the merchants good currency when debased coins were in circulation. Gradually, they began to accept the surplus money of others (particularly in the form of gold) and issue an ordinary receipt. This laid the foundation of confidence which at present pervades the credit organistion. Recorded history reveals that credit transactions were carried on in the countries of the East even before the development of social organistion in the European continent. The element of confidence is very important in the banking business. If people do not have confidence in the bank and fear that it might go out of business; then that bank cannot function for long. So, to attract the depositors banks must win their confidence. Some compare money-lenders with a bank, which is totally wrong. Money-lender lends money and charges a very high rate of interest. People do not deposit savings or valuables with the money-lenders for safety. But banks provide facilities to the customers to this effect. It not only charges interest for the loans advanced but pays interest for money deposited. Before we take up the historical development of modern banking, we should trace the origin of the word 'Bank'.

According to one school of thought the word *bank* has been derived from the German word "Banc" meaning a mound or heap, and from it the Italians adopted banco. Some writers think that the ancient business-houses which were run on old lines, were known as *banchi* from which we have derived the word 'Bank.' Still others believe that this word has been derived from bancus or banc or *banque* which means a bench at which the money changers or medieval bankers used to carry on their banking business in the market.

Definition of Bank. To define a bank is not an easy task. Different writers on the subject of money and banking have defined it differently. In the words of Dr. H.L. Hart, "A banker is one who, in ordinary course of his business

honours cheques drawn upon him by persons from and for whom he receives money on current account." According to Professor Kinly, "A bank is an establishment which makes to individuals such advances of money or other means of payment as may be acquired and safely made; and to which individuals entrust money or means of payment when not required by them for use."

According to Prof. Gautier, "Taken in its general acceptance, a word *bank* expresses the business which consists in effecting, on account of others' receipts and payments, buying and selling either money or gold and silver or letters of exchange, drafts, public securities and shares in industrial enterprises : in a word, all the obligations whose creation has resulted from the use of credit on the part of states, societies and individuals." According to Findlay Shirras, "A banker or a bank is a person, firm or company having a place of business where credits are opened by deposit or collection of money or currency or where money is advanced or loaned."

In the words of Prof. Sayers, "A bank is an institution whose debts are widely accepted in settlement of other people's debts to each other." To use the words of Crowther, "A bank is an institution which collects money from those who have it to spare and lends this money out to those who require it."

The Banking Companies Act 1949 has defined banking as "Accepting for the purpose of lending or investment, of deposits of money from the public, repayable on the demand or otherwise, and withdrawable by cheque, draft, order or otherwise." We have quoted above several definitions of bank, most of which are mere descriptions of the business performed by it excluding agency functions. However. we arrive at the conclusion that a bank is an institution or establishment which deals in money and credit. It is an agency which receives, deposits and makes advances. The definition given by H. White *i.e.*, "a manufacturer of credit and a machine for facilitating exchange" is short but precise.

Evolution of Banking. Banking developed in the different countries in different periods of history. Kautilya-the great economist of ancient India-has recommended public regulation of interest because in his opinion the relationship between the capitalist and productive borrowers was of vital importance to the national economy. The ancient Roman civilization had witnessed the development of banking which went into oblivion with the fall of Rome. The beginning of the 12th century saw the revival of banking and within a short period of time the Jews and Italians developed banking activities. The Bank of Venice was the first commercial bank founded in 1157, followed by the establishment of the Bank of Barcelona in 1401 and Bank of Genava 1407. Although the goldsmiths of Great Britain used to perform one function of money lending, yet the credit for laying the foundation of modern banking in England goes to the Lombards of Italy who had migrated to other European countries including England. Exchange banking developed after the establisment of Bank of Amsterdan in 1609 and the Bank of Hamburg in 1690. The Bank of England established in 1694 undertook central banking. It had nothing to do with joint stock or commercial banking. It was only after the

passage of Banking Act, 1833 that commercial banking began to take colossal strides in that country.

PRINCIPLES OF COMMERCIAL BANKING

Banking is of several types and the success and smooth functioning of every type of banking depends upon the adoption of certain sound principles. In a dynamic world, the banking practice is also dynamic. These principles are meant for the provision of guidance to the authorities of the banking institutions. We will enumerate five principles of commercial banking.

(*i*) **Safety.** Loans granted by a joint stock bank to its customers should be guarded by sound securities. They should not be advanced as far as possible, against landed properties as these transactions may sometimes involve the sale of property and ascertainment as to who is entitled to it. And where there is a question of personal security, the bank manager should study the financial position of the customer and should never lend an amount beyond the paying capacity of the latter. Thus, a careful lending would provide safety to the funds of the bank.

(*ii*) **Liquidity of Advance.** A bank should not provide block capital or should not advance long term loans. It should not advance loans for fixed assets like mills, coal mines etc. from which money is not likely to be realised in time. If advances cannot be realised in time, the bank might not be able to meet the demands of its depositors. In that case the bank would fail. The bank should not be tempted by the rates of interest offered by long-term borrowers. It should advance only short-term self-liquidating productive loans to maintain the liquidity of advance.

(*iii*) **Reserve Management.** The bank reserves are extremely essential for the maintenance of its sound financial position. If its cash reserves are large, they would remain idle and unremunerative; and if these are too small or insufficient, they might lead to its failure. These reserves should, therefore, be reasonable which depend upon the general experience of banking. As far as cash reserves are concerned, they depend upon certain important factors like those of the nature of advances and the amount of bills discounted, legal requirements etc. by the state, size of average deposits, existence of local clearing house, amount of reserves maintained by other local banks, general business conditions, use of cheque system and nature of accounts.

(*iv*) **Diversification of Risk.** Another important consideration is the minimization of risk. The success of the bank depends upon the repayment of loans by customers in time. If large loans have been advanced to a particular group of industries, and if these industries are badly affected by a trade cycle, the bank would be doomed. It is a wise policy to diversify the risk of advances. The bank should give smaller loans to a larger number of industries and businessmen.

(*v*) **Maximum Use of Funds.** The banker before making any investment, must have a clear picture of the financial position of the bank and a telescopic view of business world. The liquidity of funds should never be endangered. The

lower the cost of maintaining liquidity, the more efficient is the bank. As far as possible loans should be advanced against commercial papers and stock and shares of some reputed companies so that they may be realised. Before the approach of a slack season, he must invest his funds in profitable channels; and before the approach of busy season, he should realise all the matured advances.

Banking Principle. The principle on which banks work is the same that underlines the insurance business. All people who are paying premiums on their life policies do not die at one and the same time. If they did so, no insurance company could cover the risks or meet the demands on it. By their experience, Insurance Companies know the approximate death rate among their clients. They make a provision to meet the demands for that percentage of deaths. A certain amount of reserve may be kept to meet any unforeseen demand, but that is not used except in the case of an emergency. The same is the case with a bank. Some people are everyday paying in cash in their deposit accounts across the receipt counter. Others are receiving cash from the bank by drawing cheques on their accounts in the bank. Imagine the bank keeping a small cash safe for every customer. Everyday the bank will have to take out some cash from some safes and pay it to his customers who come to demand it. To other safes it will add the cash received in deposit. The cash in different safes will fluctuate from day to day, but the amount of cash in all the safes together would probably remain the same, for withdrawals on one counter are made good by receipts on the other. The bank authorities would soon realise that the total amount of cash possessed by them is never used fully and they can easily lend out a part without impairing their ability to meet the demand of their customers. Suppose by experience the bank authorities come to the conclusion that only 10% of the deposits is liable to be withdrawn normally on any working day. The bank can easily lend out or otherwise invest 90% of its deposits and keep only 10% in cash. When it is making payment, it is receiving new deposits as well. The cash for payment on next day or even on the same day will be received in the form of deposits on the other counter. The bank would thus be able to carry on its business by keeping only 10% of its deposits in cash. If, however, there were a 'run on the bank' and all the depositors come to demand their money at one and the same time, the bank (like an Insurance Company in the case of the death of all policy holders at one and the same time) will fail to meet the demand and will have to close its doors. Banks thus lend out other people's cash which has been deposited with them on the guarantee that when the depositor needs money,the bank would pay it and this is done in the hope that other people's money would have come in the bank in the form of deposits by that time.

Types of Banks. Although the primary business of a bank is to receive deposits and give loans, the business is capable of a variety of different forms. Deposits are of various kinds and it is possible for banking institutions to specialise only in certain types of deposits. The lending business of banks is done in a variety of ways– by discounting and purchasing bills, purchasing securities, giving advances against tangible security etc. Loans are given for different periods–short periods of upto a year or a little more, medium periods of three years or thereabout, and long periods of five or more years. Moreover,

banks provide finance to business and trade for various purposes. Thus, banking business has a wide range which provides a lot of scope for specialisation. It happens that in some cases a single bank undertakes a number of different types of business, but modern financial institutions are generally specialised institutions, each specialising in a particular type of banking business. We have, therefore, a number of different types of banks.

Commercial Banks. The commercial bank is the usual type of institution which dominates the banking system in every country. The public in general keeps its deposits with these banks. A distinctive feature of the business of this type of banks is that by far the largest part of its deposits consists of current account deposits or deposits withdrawable by cheque. The encashment and transfer of deposits by cheque is the greatest service provided by the commercial bank. The major portion of the loans given by this bank is for short periods, only a small part for medium periods, and generally none for long periods. Thus, large parts of both the bank's debts to its customers and the customers' debts to the bank are for short periods.

Savings Banks. Savings banks attract the savings of small savers and a large part of their deposits are savings account deposits. These deposits are not freely withdrawable by cheque and are so only under certain restrictions. The rules in most cases require the customer to come in person with the pass book to get cash for his deposit. The postal savings banks in India answer to the description of this kind of banks. There are many varieties of savings banks in the U.S.A. and Canada.

Investment Banks. These banks provide permanent capital to business undertakings. Issues of shares and debentures of new joint stock companies are purchased by these banks, and later, if necessary, are sold to the public. Since an investment bank provides long term capital to industry, the deposits it receives are also for long periods.

Mortgage Banks. A mortgage bank is an institution which grants morgtages on real estate and property. Land mortgage banks in India and other countries, which provide long term credit to the agriculturist against the mortgage of agricultural land, are an important example of this kind of banks. Building societies in Great Britain provide finance for the building of houses on the security of that property.

Consumption Banks. Consumption or consumers' banks make loans to individuals to enable them to acquire durable consumer goods. They collect funds by the receipt of deposits, by the sale of their own shares, and by borrowing from commercial banks, and use these funds to provide loans for the purchase of consumption goods. In the U.S.A., this type of banking has made considerable progress, and there are many credit unions and sales finance corporations that specialise in this kind of business.

Exchange Banks. Certain banks in India which have their head offices outside the country have been given this name. They are so called because a large part of the foreign exchange business of the country is in their hands. They are, in fact, banks doing normal commercial banking business. The emphasis on the financing of foreign trade has earned them this title.

Co-operative Banks. Co-operative credit societies of small artisans in towns and agriculturists in rural areas have an important place in the credit system of this country. They provide short term finance to members generally on personal security.

The Central Bank. This is the institution charged with the function of controlling the money market and issuing currency. Usually there is only one central bank in a country and it does little ordinary banking business for the general public. It is a banker to the government and other banks and its main business is to control the operations of the rest of the banking system. Although a central bank may sometimes have shareholders (most of the central banks now are owned by the government), its business is never transacted with the object of making profits. Some profits may come its way as a result of its operations, but it primarily thinks of the effects of its business on the working of the economic system.

Many banks engage themselves in more than one kind of activity, *e.g.*, commercial banks also do savings banking and give loans for consumption, but a certain degree of specialisation has become a notable feature of the banking system of many countries. It is the type of banking business on which particular emphasis is put which gives a banking institution its class.

FUNCTIONS OF COMMERCIAL BANKS

Although the main functions of a modern bank are evident from the very definition of the term, yet there are certain other services too rendered by them. We study them in detail :

1. Borrowing of Money,
2. Lending of Money,
3. Agency Services, and
4. General Utility Services.

1. Borrowing of Money. The first important function of banks is to receive and accept deposits from those who can save and cannot profitably use them themselves. These savings in money can either be kept at home or in a bank. To keep them home is not safe. A person also loses interest. Thus, it is considered more rational that people should deposit their savings in a bank. They get interest and there is no danger of theft. Deposits are received by them in three forms-Fixed deposits, which can be withdrawn after a specified period of time and can fetch comparatively much higher rate of interest; Current deposits, which can be withdrawn any time and in any amount. Generally no interest is allowed on such deposits and if at all it is allowed, it is very low and that also on the condition that the deposits are not allowed to fall below a certain level. Then there are the Savings Bank deposits, which aim at encouraging savings among the people; rate of interest allowed is comparatively higher, but lower than that on fixed deposits. There are certain restrictions on their withdrawal also. Banks collect the savings of the country by means of deposits and thus serve what may be called as the pooling centres for the resources of the country .... tapping the saving stream.

In order to meet the expenses of maintaining these accounts the banks realise incidental charges. Whereas the fixed deposits are known as time liabilities, others are known as demand liabilities of the bank. A new scheme recently introduced is that of the *Home Safe Account*, according to which a safe is supplied to the depositor who drops his little savings in it. This safe is periodically taken to the bank where the amount of the safe is credited to his account. Funds of the bank are thus, collected from these deposits.

2. Lending of Money. Another function of a commercial bank is to make advances, loans, cash credits, overdrafts etc.

What do the banks do with these deposits ? How are they able to earn dividends and from where do they meet their expenditure ? How do they afford to pay interest ? The clue to all these questions is to be found in the fact that what the banks receive in the form of deposits, they advance in the form of loans for which they charge rates of interest higher than what they allow to their depositors. Thus they make profits too. While granting loan and charging the rate of interest, the principle is : the longer the period for which a loan is given, the higher is the rate of interest fixed for it. Loans are also advanced through the discounting of bills, besides making cash credits and thus the banks help in the development of industry and trade in the country.

In case of overdrafts, customers are allowed by the banking authorities to overdraw their current accounts usually against collateral securities. A bank also grants cash to its customers, businessmen, industrialists, manufacturers, etc. The entire amount of cash credit which is given against securities is not given at a time. The borrower draws money out of the credit granted as and when needed. This system of granting cash credit is very popular in Scotland. According to H.D. Macleod, "Cash credit is to Scotland what the river Nile is to Egypt, a fertilizer." Cash credit in our own country is not given against the personal security of the borrower. The function of lending leads us to inevitable conclusion that the growth of industry and commerce directly depends upon the existence of banks.

Dr. Thomas has rightly remarked, "The bank is a great reservoir of loanable capital, into which flow countless small rivulets gathered from every source and direction and from which flow many streams, small and large, destined to drive the wheels of industry and float the vessels of commerce."

Discounting of commercial papers or Hundies is only another form of advancing loans though it is not as direct as a loan against pronote or a tangible security is. When a merchant, whom we may call A, sells some goods to another merchant, called B, he draws a bill on B to the amount of purchases made. The bill is an order on B to pay the amount on the conditions laid down in the bill to A or his order. One of the important conditions is the period of time allowed for payment of the bill. A bill may be a Sight Bill meaning thereby that no time is allowed and the person on whom the bill has been drawn has to pay the amount immediately on presentation of the bill. Or it may be a Time Bill which may allow, 30,60,90 or 180 days for payment. In case the bill is Time Bill, the creditor will have to wait for the payment for the period for which the bill runs. But he may need the money immediately. He can then go to a bank which knows him,

with the bill in hand and present it for payment. If the bank has confidence in his integrity, it will make the payment to him against the security of the bill on his signing the same. After the necessary period has elapsed, the debtor will make the payment to the bank and the bill will thus be discharged. The making of advances against bills of exchange, in this manner is technically known as discounting of bills. Most of the trade, internal as well as external, is financed through the medium of bills.

3. Agency Services. Another important function which the bank perforn is number of direct services which it renders to its clients. These services incluc the following :

(a) Remittance of Funds. It remits funds by mail transfers or drafts o behalf of its clients. Transfers give rise to exchange activities. The banking system "credits one enterprise for a contribution made to the social product, and cancels the credit as it fixes the claim in a definite good or service. It debits another for goods or services produced in excess of past contributions remaining unfixed in definite form, offsetting these debits as the enterprise makes its contribution, and thereby effecting clearance."

Banks help in transferring money from one place to another also. This is done through drafts. All important banks in the country have their branches at all important places in the country and some of them even outside the country. Those which have no branches have agency arrangements with some other bank. A person who wants to send some money to another place can go to a bank and purchase a draft of the same amount from it. He can then send the draft to the person to whom he wants to make the payment and that person would get the payment from the branch or agency of that bank at that place by presenting the draft.

(b) Collection of Dividends, Bills, etc. It collects cheques, promissory notes, dividends, interest on shares and stocks and other types of periodical incomes.

(c) Arrangement for Payment. On behalf of its customers, it pays insurance premia subscriptions to social bodies and societies.

(d) Sale and Purchase of Securities. It also sells and purchases securities at the most favourable rates.

(e) Representation and Correspondence. It also performs the duties of a representative of its clients; and in that capacity it secures passports and tickets for them. It also enters into correspondence with income-tax and other authorities on behalf of its customers.

(f) Trusteeship. As a result of possessing specialised knowledge and employing technically trained staff, it efficiently administers trust properties.

4. General Utility Services. In addition to these three important functions, commercial banks also perform general utility services which considerably increases the facilities to their customers. These include the supply of commercial and statistical information, safe custody of valuables, important documents and securities, supply of information regarding the financial standing and credit-worthiness of their customers (but with a great care), sale

and purchase of foreign exchange, under-writing of the loans raised by trading corporations, public institutions and Governments, issue of letters of credit and traveller's cheques etc.

Banks act as cloak-rooms for the valuables of their clients. Banks appoint their own brokers in important stock exchange markets and undertake to buy and sell securities and shares on behalf of their customers in those markets on commission basis. In the light of the above explained functions and services performed by the commercial banks, we may conclude that bankers are 'public conservators of commercial values.' Thus, a modern bank plays a very vital role in the economic activity of the country. It is on account of this that a well developed banking system provides a firm and durable foundation for the economic development of the country.

ADVANTAGES OF BANKS

The service that the banking system renders to a country can hardly be exaggerated. Banks serve as the fountain of all credit in the country and credit today is the life blood of trade and industry. The drying up of all the credit funds would sound the death-knell of the present day trade and industry. How could any large scale enterprise be carried on without the large amount of capital that banks make available ? Could the volume of international trade be half so large that it is today without the provision of finance for it by the banks that discount bills of exchange ? Banking system is intextricably bound up intc the industrial and commercial organisations in every country and existence and progress of one depends upon the progress of the other. The main services of the bank may, however, be briefly summarised :

1. Collect Funds. Banks collect financial resources of the country by attracting deposits on different terms to suit the convenience of different types of depositors. The savings of a country, if they remain scattered in the hands of the people, will be of little use to industry. When they are pooled, they become a great potential source of wealth in the country.

2. Finance Trade and Industry. They finance trade. They accommodate the buyer and enable him to pay when it is convenient for him to do so. They accommodate the seller and pay him immediately on behalf of the buyer if he so desires. Trade is thus facilitated and turn-over becomes much larger than it would be in the absence of facilities provided by banks.

3. Remittances. Transfer of money from one place to another is affected much more easily and cheaply than it can be done either through the post office or by any other means.

4. Encourage Industries. They provide capital for current requriements to the industrialists and small scale entrepreneurs. This helps productive activity in the country.

5. Stabilize Prices. Banks control credit if they follow a concerted policy under the guidance of the Central Bank. They can effectively check fluctuations in price-level through the control of credit, though it must be admitted that in the past this weapon had been used by banks more to bring about fluctuations than to control them.

6. Economy in the use of Precious Metals. By the introduction of cheque system and other means, they economise the use of precious metals and cash. Today payments by cheques and bills form a far larger portion of total payments in the advanced countries than cash payments.

7. Funds for Profitable use. Banks make available idle funds of the community to those who can profitably use them. Thus they do it by attracting deposits from those who have enough to spare and making advances to those who can profitably use these funds. The banks by combining these two important functions help in co-ordinating the two essential services – "the sporadic potential lending with potential waiting". The availability of these funds helps commerce, trade and industry to grow.

8. Mobility of Goods. Through the credit instruments like the bills of exchange, the banks help in the movements of goods from one place to the other and by making timely advances they help the various processes of production too and thus help to bridge the gap which sometimes occurs between the sale of goods and the payment of the price by the purchaser.

9. Facilitate Economic Growth. The banking system is more important for the economic growth of less developed countries. Banks mobilise savings, channelise them into productive activities and boost up the rate of capital formation in the economy. Moreover, through a network of branches in the rural and backward areas, they can accelerate the process of monetisation in the economy

10. Other Advantages. Other services of banks like collection of bills, making and receiving payments on their customers' behalf and keeping up valuables in safe custody are also of very great importance to the community. Moreover, by creating credit, banks act as a useful agency to help the borrowers.

Thus, commercial banks can play a very useful role in the economy of every modern State. Banks are the pivot of modern commerce and industry. Banking development is, therefore, closely associated with the process of economic development. A country with a developed banking system has a strong foundation of industrial and economic progress. In developing countries like ours, the commercial banks have undertaken what may be described as development banking. They make their fund available to the priority sectors, weaker sections and employment-oriented schemes.

The Liabilities and Assets of a Bank : The Balance Sheet

Banking is a business much like any other business. An indication of the financial position of a business concern may be obtained by examining its statement of liabilities and assets, called the 'Balance sheet.'

Balance sheet is a statement of firm's financial position on a particular day of the year, as on that moment, it provides a complete picture of what the firm owns (its assets) and what it owes (its liabilities). The balance sheet should not be confused with the income statement. The latter summarises, for some stated period of time all the gross income accruing to the business entity and the claims against that gross income. A balance sheet, on the other hand, is a summary of the financial position of a business entity at a specific point of time. It is an

instantaneous snapshot that records the value of everything the business owns and the size of all debts the business owes.

A balance sheet is always prepared in two sections. In one section it is customary to record liabilities, normally on the left side, and assets in the other section on the right side. Liabilities are the debts or amounts of money owed to others. The liabilities of a bank consist mainly of the claims of its shareholders, creditors and the depositors. The assets, on the other hand, include such items as cash, accounts receivable, loans and investments, etc.

A balance sheet always balances in total, but no individual items necessarily matches another. We can easily understand this if we have some elementary knowledge of double entry accounting. Any change in the one side of the balance sheet must be precisely offset by an equal change in some other item on the other side. The equality of assets and liabilities is not peculiar to banking alone. Every balance sheet balances. But a bank's business is, in a very special sense, a balancing of assets and liabilities. A bank acquires assets by increasing its liabilities directly, unlike any other business where liabilities are acquired indirectly as a result of trading. Thus, the first thing we want to know about a bank and its operations is the amount of its debts and credits. Crowther has rightly remarked, "The whole business of a bank is in its balance sheet. The balance sheet also has the merit of demonstrating at a glance the ratios to which the bank is working."

A specimen structure of a bank's balance sheet is given on the next page.

Liabilities

The liabilities side of the balance sheet of a bank is comparatively simple. The liabilities represent others' claims on the bank. The liabilities side of the balance sheet shows how the bank raises funds to function as a dealer in debts and credits. The liabilities of a bank usually consist of the following items :

1. Capital. The bank raises capital from its shareholders by issuing various types of shares, such as ordinary, preference, deferred shares, etc. The balance sheet may show the amounts of authorised capital, issued capital, and subscribed capital. But the actual liability of a bank to its shareholders consists in thecapital originally paid in and any accumulation of undistributed profits.

2. Reserve Fund. It is the amount accumulated over the years out of undistributed profits. The bank may use this fund to offset its unexpected losses in certain years. Sometimes a bank is required by law to transfer a part of its annual profits to the reserve fund so long as the amount in the fund does not become equal to its paid-up capital.

3. Deposits. Deposits from the public constitute the biggest proportion of bank's working funds. The deposits are categorised as the demand deposits and the time deposits. It is the former that represent the bulk of the money supply with public. It is on the basis of their deposit liabilities that the banks make loans and investments after keeping a certain cash reserve ratio.

Demand deposits are distinguished from time deposits. Time deposits are those against which cheques cannot be written. Demand deposits may arise out of the credits created by a bank as a claim against itself. But in simple term, a

Specimen of a Bank's Balance Sheet

Capital and Liabilities	*Amount*	*Property and Assets*	*Amount*
1. Capital account–paid up capital		1. Cash (a) Cash in hand (b) Balances with the Central Bank	
2. Reserve Fund and other Reserves		2. Assets with the Banking System (a) Balances with other banks in current accounts and other accounts (b) Money at call and short notice (c) Advances to banks (d) Other assets.	
3. Deposits and other accounts (*a*) Demand Deposits (*b*) Time Deposits		3. Investments (at cost) in Government securities, including treasury bills and treasury deposit receipts.	
4. Borrowing from other banks.		4. Investments in other approved securities	
5. Borrowing from the Central Bank.		5. Advances, loans, cash credits, overdrafts, etc.	
6. Bills payable		6. Bills discounted and purchased (a) Inland bills (b) Foreign bills	
7. Bills for collection being bills receivable as per contra		7. Bills receivable being bills for collection as per contra	
8. Other liabilities		8. Constituent's liabilities for acceptances, endorsements and other obligations per contra	
9. Acceptances, endorsements, and other obligations per contra		9. Premises	
10. Profit and loss account		10. Furniture and fixtures	
11. Contingent liabilities		11. Other assets including gold and silver	
		12. Non-banking assets acquired in satisfaction of claims	
Grand Total		Grand Total	

customer is said to have "made a deposit" when he gives the bank cash or its equivalent, and the bank gives him a deposit credit. A deposit, therefore, is the promise that the bank gives to the depositor in exchange for the cash he "deposits." The cash is an asset, the deposit is a liability. The depositor is a creditor of the bank having lent its cash, and he can claim repayment at any time.

4. Inter-bank borrowings. Liabilities are created when a bank borrows from another bank on a temporary basis. A large bank, in particular, may have deposit liabilities not only for the account of the general public but also for other banks in the country.

The bank may also borrow from the central bank of the country on the basis of the eligible securities or get financial accommodation in times of need or stringency by rediscounting their bills of exchange.

5. Liabilities relating to bills. The bank may have some bills which are payable by it out of its resources. It may also accept some bills from its customers for collection. The amount when collected is credited to the accounts of the customers. Hence the amount under this head is shown on both the sides of the balance sheet. They become the liabilities of the bank after collection, but they are to be treated as assets before collection. The banks also accept or endorse the bills of exchange on behalf of their customers, which simply means that the bank guarantees the payment of bills at maturity. Thus, when the bank has accepted bills for its customers it is technically liable to meet them on maturity; but since the customers are expected to meet them and have presumably given due security, this liability of the customers to the bank is an offsetting asset against the acceptance.

In addition to the above, the banks make some provision for the contingent or unforeseenable liabilities. The profit earned by the bank is also shown as the liability because it is payable to the shareholders.

It may be emphasised, however, that the assets of a bank are based on its liabilities. Banks, unlike other business organizations, acquire only a very small part of their total assets by issuing capital-account claims. An even more important point is that the volume of capital-account claims or share capital changes only slowly and within narrow limits relative to bank assets. Bank reserves, borrowings from other banks and the central bank are also very small relative to total assets. Most of the assets of banks are acquired by creating and issuing bank credits in the form of deposit claims. The volume of deposits that the banking system can issue depends on its reserve requirements enforced by the central bank and the currency volume of reserves available to banks.

Assets

The assets side of the bank's balance sheet is more complicated but more interesting as well. The soundness of a bank whether it is properly run or not – is clearly reflected in the distribution of the bank's funds on different types of assets. The various items that figure on the asset side of the balance sheet are as follows :

1. Cash. Every bank keeps a certain amount of cash in hand to meet any claims upon it in cash on demand. In addition, the banks are required under

law to maintain some cash reserves with the central bank which are in some proportion to their deposit liabilities. A net inflow of cash to the banking system can result either from new issues of money by the government or the central bank, or from a net decrease of currency outside the banks. Cash holdings of banks are equal to only a small fraction of bank deposit liabilities. They yield no income to banks. However, net flows of cash into the banks have the very important effect of enabling the banks to create more money by purchasing other assets.

2. Assets with the Banking System. A bank may have *balances* with other banks in current accounts and other accounts. *Money at call* or *short notice* mainly comprises funds made available to other banks by way of loans or deposits repayable at call or at short notice of a fortnight or less. *Due from banks* represents loans and advances granted to banks.

3. Investments. Banks make investments in the profit-yielding securities. Investments in government securities represent the book value of central and state government securities including treasury bills and treasury deposit receipts etc. Banks may make investments in other approved securities as well. The different types of investments are shown separately in the balance sheets. Banks regard their short-term investments as their secondary reserves as different from cash which is their primary reserve.

4. Loans and Advances. Loans, cash credits, overdrafts, etc. yield income to the bank in the form of interest earned on them. These assets have low liquidity but high yield. The banks generally advance short term loans and advances. But a good portion of these advances is, in effect, allowed to continue for a fairly long time through the recurrent renewals or rolling over of short-time overdrafts and cash credit arrangements. This implies that the basic character of much of the short-term credit does not impart it sufficient liquidity.

Most of the loans and advances are derivative deposits which constitute a net addition to the money supply, without there being any offsetting decrease in the volume of currency in circulation.

5. Bills Discounted and Purchased. The banks may purchase and discount inland as well as foreign bills. Inland bills cover bills of exchange drawn and payable in the country; including demand drafts and cheques purchased. Foreign bills cover all import and export bills purchased and discounted in the country as also cheques and demand drafts drawn in foreign currencies. The banks like to acquire short-dated bills because these are easily marketable and hence sufficiently liquid and at the same time bring in some income to the bank. Moreover, these bills are eligible for rediscounting with the central bank of the country.

6. Properties. Buildings, furniture and fixtures etc. are the other assets of the banks. These fixed assets are often referred to as 'Dead Stock'. They are generally shown at their depreciated value. These are, in a way, the secret reserves of the banks which can be availed of in case of any crisis or collapse.

The amount of assets that a bank can command depends upon the amount of its liabilities. Many types of assets are available to a bank, profitless and

profitable, liquid and non-liquid. A bank must, therefore, formulate a portfolio policy determining what types and proportions of assets it will acquire and hold.

Thus, the balance sheet is an indicator of the nature of a commercial bank. A thoughtful look at the balance sheet brings to our notice a very interesting fact. Most of the assets of the bank are debts of others. Even deposits at the Central Bank are debts of the Central Bank. Coins and currency notes are the debt of the Government. Liabilities of the bank are also the debts of the banks. It is thus clear that why banks are called as dealers in debt. Banks acquire debts of others on which they receive interest and, in turn they issue their own debt (deposits). Besides this, the signficance of the balance sheet lies in the fact that it throws light on the financial soundness and economic conditions of a particular commercial bank.

CREDIT CREATION

At one time, there was a needless controversy regarding the ability of commercial banks to create credit. Some writers meant by credit, bank loans and investments and, therefore maintained that a bank could never lend more than the amount which it had been entrusted with by the depositors. Suppose, a person deposited Rs.1,000 with a bank, the latter could lend upto Rs. 1,000 and not more. In fact, it could lend far less, since it had to maintain a small margin of cash reserves against the withdrawal of money by the depositors. But the bank could not lend more than Rs.1,000 and, therefore, writers conclude that banks could not create credit.

This argument is only partially true. It is true that a bank cannot lend more than what it has got. But it is also true that what is lent out by bank may come back to the bank by way of deposits, which may again be lent out, and so on-deposit becoming the basis for a loan or investment, which again becoming a deposit to be the basis of new loan etc. Commercial banks, therefore, are able to multiply loans and investments and thus multiply deposits. It is in this sense that banks create credit.

The development of credit has been facilitated by the development of banking. The most important function of bank is to create credit, a function which over-shadows all other banking functions. Credit creation refers to the power of banks to expand or contract demand deposits through the process of more loan advances and investments. Banks are in a position to lend much more than what is actually deposited with them. They can raise a big super-structure of credit on the basis of comparatively small reserves. Banks are, thus, known as factories of credit. They create credit which can be regarded as a form of money for all practical purposes. People deposit their surplus cash with the banks and the banks utilize this idle money by granting loans to those who are in need of it. But generally this money is not granted in the form of cash, but an account is opened in the books of the banks in the name of the borrower, and from this account, the borrower will draw money as and when necessary till the sum allowed as credit is completely exhausted. The banks, in this case, need not keep huge sums in the form of hard cash to meet their obligations, but they keep

a little money for doing so. Thus, banks create credit by advancing loans on cash-credit basis, or over-draft payments. Both these methods are prevalent in India.

Therefore, one of the ways of creating credit is through loans. But how can a bank lend more than its deposits ? It is not difficult to understand this paradox. The explanation lies in the fact that banks do not create money out of thin air, but it is essential that savings are made before credit creation. Imagine, that a businessman applies to the bank for a loan of Rs. 5,000/-. The bank will probe into his credit position and ask him to furnish some security. The security can be in the form of fixed capital or circulating capital and it should be of a value higher than the loan advanced by the bank. Suppose, the businessman convinces the banker and with all assurances of re-payments and interests etc. gets the loan. The banker will not give him hard cash, but will credit the account of the borrower with Rs.5,000/-. He can draw cheques as required to that limit. The point to be noted is that the loan is made by increasing bankers, debits. The bankers have, in return, the borrower's promise to repay the amount alongwith the interest. Therefore, in the banking words, it is said, "Every loan creates a deposit", or "Every loan creates a credit". As a result of this loan, an additional Rs.5,000/- have come into existence. When the loan is repaid, the borrower's account will be debited and there will be cancellation of loan to that extent.

Credit is also created by banks by purchasing securities without paying cash. The banks pay the value by cheques to the sellers of the securities and these cheques are deposited in his bank or the banker credits the seller's account with that amount of securities and thus increases the deposits either of his own bank or some other bank, the result is the same i. ., the deposits of the bank as a whole have increased.

But it must be understood that the money, the banker creates is his liability. In daily transactions, the banker knows that only a few cheques are presented for encashment. He need not keep hundred per cent reserves for the cheques. In addition, cheques are daily drawn on all the banks, and in the evening, all the cheques pass through clearing house, where they are set off against one another. Only difference is paid by the banks. Thus, the banks keep a certain portion of the reserve for meeting the day-to-day requirements of the depositors and for setting the balances of their fellow-banks But these payments form a very small portion of the total cash reserves. So the banks keep the required estimate and some amount extra in order to be doubly sure. The rest they give as loans. The cash-reserve ratio depends on a number of factors which include the banking habit of the people, the growth of the capital market, the use of credit instruments in trade and industry for the discharge of business obligations etc. In advanced countries, where banking has developed, and where people make an extensive use of credit instruments in their daily life for making payments, cash reserve limit is low. But in under-developed countries, where such habits are not very common, the cash-reserve limit of the bank is very high. It also depends upon the reputation of any individual bank.

The banks generally cannot afford to let their reserves fall below a prescribed limit, otherwise people will lose confidence. The whole business

depends on the confidence of the public.

Suppose, the bank keeps 20 per cent cash against its liabilities. Then it means that on cash deposits of Rs. 1,000/-, the bank can lend upto Rs.5,000/-. This can be explained as follows :

Keeping	20% of Rs. 1000/- *i.e.*,	Rs. 200 lends Rs.	800/-
– do –	20% of Rs. 800/- *i.e.*,	Rs.160 lends Rs.	640/-
– do –	20% of Rs. 640/- *i.e.*,	Rs.128 lends Rs.	512/-
– do –	20% of Rs. 512/- *i.e.*,	Rs.102 lends Rs.	410/-
– do –	20% of Rs. 410/- *i.e.*,	Rs. 82 lends Rs.	328/-
– do –	20% of Rs. 328/- *i,e,*	Rs. 65 lends Rs.	263/-
– do –	20% of Rs. 263/- *i.e.*,	Rs. 53 lends Rs.	210/-

and so on......

Ultimately the total amount given as loan will go up to Rs.5,000/-. Thus, it means that by keeping 20% cash reserve, bank can create credit to the extent of 5 times. In America, banks keep 3 to 5% reserve and in England they keep 7 to 8% reserve; and thus, banks in America and England can create much more credit than banks can do in India. The percentage of reserve will be higher in backward countries like India than in advanced countries like America and England. In agricultural countries, high cash-reserve is necessary because transactions are settled in cash and cheque habit is not prevalent. In industrial countries like England, Germany and America etc., people are accustomed to the use of cheques and less cash-reserve is required. In periods of panic, banks have to keep a high cash-reserve to their liabilities. Moreover, if there is no Central Bank in a country, each individual bank must keep a high ratio to meet a possible rush on it.

Thus, from the above discussion, it is clear that the banks create deposits in two ways. Firstly, in a passive manner and secondly in an active manner. In the former, the bank creates passive deposits when it opens a deposit account in the name of a customer who brings cash or cheque to be credited in his account. These deposits are also known as primary deposits, because these form the basis of the loan transactions undertaken by the bank. They provide funds to the bank for advancing loans. The active deposits are created by the bank in a more active manner by opening a deposit account in the name of the borrower. These deposits are created by the bank itself and, therefore, they are known as active or derivative deposits. While primary deposits do not add to the total supply of money, the creation of active deposits does add to the total supply of money in the economy. This is known as creation of credit.

Multiple Credit Creation. So far we have assumed that there is only one bank. In order to explain the process of multiple credit creation, we take the banking system as a whole. We noticed that an individual Commercial Bank, on account of adverse closing balances, can lend only upto the amount of its excess reserves. But the commercial banking system has no such problem as the clearing drain. In this case, clearing drain simply transfers resources from one bank to another. Loss of one bank through an adverse clearing balance will be equally matched by another bank's gains through favourable clearing balance. Therefore, the process of credit creation will go on till the initial excess reserves

become equal to the required reserves through an increase in deposits. The process of multiple credit creration can be explained thus :

When the borrower withdraws the amount of debt from the bank by cheques, he pays to other business men by cheques. These other businessmen deposit those cheques either with the same bank or with some other bank. The bank deposits increase not because people have deposited cash money but on account of the issue of cheques. Thus, the original credit by a bank increases the derivative deposits of other banks. In this way, credit multiplies. Let us take an example. We suppose that bank 'A' has excess reserves to the extent of Rs.1,000. It gives loans to its customers of Rs.1,000 (which become derivative deposits with this bank). The borrowers pay by cheque to other people who are the customers of bank B. When the latter deposits them with bank B, the amount becomes a primary deposit with the next bank. If the bank customarily maintains a reserve of 10 per cent, it means that bank B has an excess reserve; and a derivative deposit of 90 per cent of Rs. 1,000. i.e., Rs. 900. People who borrow money from this bank will also pay by cheques to others who happen to be customers of bank C. In this way, the amount of Rs. 900 becomes primary deposit with bank C which, after keeping a reserve of 10% can create a derivative deposit of Rs. 810. This chain reaction will continue until the original excess reserves of Rs.1,000 are expanded to derivative deposits of approximately Rs.10,000 by the banking system of the country. The process is known as the multiple credit expansion which "increases the circulation of common money, because the resulting increase in the amount of business turnover involves cash payment and correspondingly decreases the available cash reserves."

Thus, the banking system as well can create an amount of active deposits, which is a multiple of the original deposit depending upon the cash reserve ratio to be maintained by the banks. The credit multiplier can operate forward as well as backward. When the bank receives primary deposits, it leads to multiple expansion of credit and when the bank loses a certain amount of cash, it leads to a multiple contraction of credit. The credit multiplier may be defined as the ratio between the ultimate amount of derivative deposits created and the original amount of excess reserves in the banking system. The credit multiplier is arrived at by dividing the total volume of derivative deposits by the original excess reserves. Therefore,

$$\text{Credit multiplier} = \frac{\text{The volume of derivative deposits}}{\text{Original excess reserves}}$$

In the above example, $\frac{\text{Rs. } 10{,}000}{\text{Rs. } 1{,}000} = 10$

The size of multiplier is 10. It should be remembered that the size of the credit multiplier depends upon the size of the cash-reserve ratio. If the cash-reserve ratio is low, the credit multiplier will be high, and vice-versa. Thus, the credit multiplier is the reciprocal of the cash-reserve ratio. In our above

example, we had assumed the cash-reserve ratio to be 10%, i.e. $\frac{10}{100}$ Its reciprocal is $\frac{1}{\frac{10}{100}}$ or $\frac{1}{\frac{1}{10}}$ or $1 \div \frac{1}{10}$ or $1\times\frac{10}{1}$ which is equal to 10. As credit creation depends upon the cash-reserve ratio, the credit multiplier is $K' = \frac{1}{r}$ in which K' is credit multiplier and r is reserve ratio.

In the above example,

$$K' = \frac{1}{0{\cdot}1} = 10.$$

If suppose, cash-reserve ratio is 20%, then

$$K' = \frac{1}{0.2} = 5.$$

From the above analysis, the general formula for credit expansion for the whole banking system can be explained as follows:

$$\Delta D = \Delta a.\frac{1}{r}$$

where ΔD stands for multiple expansion of credit for the whole banking sytem, r for cash-reserve-ratio, and Δa for original excess reserve. In our example,

$$\Delta a = \text{Rs.}1{,}000,\ r = 10\%$$

$$\therefore \quad \Delta D = 1000 \times \frac{1}{0{\cdot}1} = 1{,}000 \times 10 = 10{,}000$$

The credit multiplier has great practical significance for the Central Bank. If the Central Bank knows the exact magnitude of the credit multiplier, it can estimate the ultimate effect on the volume of credit of an initial increase or decrease in the cash reserves of the commercial banks.

The technique of credit creation can also he explained with the help of balance sheets and by assuming : (a) the existence of a number of banks A, B, C, D, etc., each bank with different sets of depositors ;

(b) every bank has to keep 20% of cash reserves;

(c) a new deposit of Rs. 1000 has been made with Bank A to start with.

The balance sheet of the bank will show (taking only the new transaction) :

Bank A

Liabilities		*Assets*	
New deposits ...	Rs. 1000	New cash	...Rs. 1000
	Rs. 1000		Rs. 1000

Under the double entry system, the amount of Rs. 1000 is shown on both sides. The deposit of Rs. 1000 is a liability for the bank since it is obliged to return the amount whenever the depositor demands. At the same time, the amount is an asset to the bank which it may use to earn an interest income. The bank, according to law and convention, has to keep only 20% reserve. In our example, Bank A has to keep only Rs. 200 against its deposit. The bank has a surplus of Rs. 800 which it can profitably employ. Suppose Bank A gives a loan to Mr. X who may use the amount to pay off some of his creditors. After the loan has been made and the amount utilised by Mr. X to pay off his creditors, the balance sheet of Bank A will be as follows :—

Bank A

Liabilities		*Assets*	
Deposits	... Rs. 1000	Cash	... Rs. 200
		Loan to Mr. X	... Rs. 800
Total	... Rs. 1000		Rs. 1000

Now, the creditors of Mr. X, who got Rs. 800 from the latter, may be assumed to deposit the amount with their bank, *viz*. Bank B. The balance sheet of Bank B will be as follows :—

Bank B

Liabilities		*Assets*	
New Deposit	... Rs. 800	New cash	... Rs. 800
Total	... Rs. 800		Rs. 800

Bank B finds that it has excess cash reserves. For according to convention, it has to keep only 20% cash reserve against its deposits. After keeping a cash reserve of Rs. 160, the bank is free to lend the balance of Rs. 640 to any one. Suppose it makes a loan to Mr. Y of this amount. The balance sheet of Bank B will be : —

Bank B

Liabilities		*Assets*	
Deposits	... Rs. 800	Cash	... Rs. 160
		Loan to Mr. Y	... Rs. 640
Total	... Rs. 800		Rs. 800

Mr. Y would have naturally used the amount to pay off his creditors, for no one borrows from a bank to keep it idle in the bank itself. We can assume that the creditors of Mr. Y who received Rs. 640 from Mr. Y would be depositing the amount in their bank, *viz.* Bank C. Bank C's balance sheet will be as follows :—

Bank C

Liabilities		*Assets*	
New deposit	... Rs. 640	New cash	... Rs. 640
Total	... Rs. 640		Rs. 640

Bank C finds that it has excess cash reserves to the extent of Rs. 512 (since under 20% cash reserve, it will have to keep a cash reserve of Rs. 128 only against a deposit of Rs. 640). Suppose Bank C buys some long term Government bonds from Mr. Z. The balance sheet of Bank C will be as follows : —

Bank C

Liabilities		*Assets*	
Deposit	... Rs. 640	Cash	... Rs. 128
		Investments	... Rs. 512
Total	... Rs. 640	Total	... Rs. 640

Now, Mr. Z who sold the long term securities to Bank C for Rs. 512 may be expected to deposit the amount with his bank, Bank D. Which in turn may keep a small percentage as cash reserve and lend the rest. And this process of deposit becoming a loan or an advance or an investment which in turn becomes a new deposit goes on and on till the original deposit of Rs. 1000 is completely exhausted. The original deposit of Rs. 1000 becomes additional deposits of Rs. 800, 640, 512, 409, 328 etc. If we add up all these deposits, the total will be Rs. 5000. The total amount of deposit expansion (or creation of credit) will be the reverse of the cash reserve ratio. If, for instance, the cash reserve ratio is 20% or $\frac{1}{5}$, credit creation will be $\frac{5}{1}$ or 5 times the original deposit ; if the cash reserve ratio is 10% or $\frac{1}{10}$, credit creation will be 10 times; if it is 33% or $\frac{1}{3}$ credit creation will be 3 times and so on.

What are the methods of creating credit or expanding deposits ? In our example, we have shown that new deposits may be made when (a) the bank gives loans and advances for short periods, and (b) when the bank makes investments, *i.e.* when it buys long term credit instruments. But the bank can

also create new deposits through buying real goods such as acquiring land, buildings and equipment. But this is only to a small extent and hence it is not of much significance. Therefore, for the creation of bank credit, an essential condition is the availability of proper securities.

However, it is not really necessary that there should be many banking institutions ; it is just sufficient that there is only one bank. Even then, the process of credit creation will be the same. That is, whenever the bank has excess cash reserves, it will lend or invest the same ; this amount will come back to the bank in the form of a new deposit which will become the basis for yet another loan and so on. The money which comes back by way of new deposits is the same. But the bank is not concerned as to how a depositor gets the cash which was with the bank a while ago. Thus credit creation will take place, whether we consider only one bank in an isolated town or we consider the banking system as a whole.

Just as there is a multiple expansion of bank credit, there is a multiple contraction of bank deposit too, when cash is removed from the banking system.

Limitations of Creation of Credit. Theoretically speaking a bank is described as a factory of credit as it does succeed in creating more purchasing power but this power of bank to create credit is not unlimited and is subject to a large number of limitations and controls. Therefore, a bank is not able to create credit at pleasure. The power of the bank to create credit is enormous in highly industrialised countries where people are in the habit of carrying on transactions through the banks alone and where people do not believe in hoarding. In less developed countries like India, the power of the banks to create credit is much limited. The following are the few limitations on the power of the bank to create credit :

(*i*) The total amount of cash in a country determines the credit that can be created by the banks. The larger the cash, the larger the amount of credit that can be created. But the cash which banks keep depends upon the policy of the Central Bank. The Central Bank can increase or decrease the notes in circulation and thus can increase or decrease the reserve of the banks. Thus the amount of credit that can be created by banks depends upon the policy o the Central Bank.

(*ii*) Another limitation on the power of the banks to create credit depends upon the habit of the people; whether they use cash or cheques. The greater the use of cheques, the greater is the power of banks to create a large amount of credit. This is why in America, England and other advanced countries the banks create more credit than in India.

(*iii*) Creation of credit depends upon confidence of the public in the sound banking system. Bank notes are only promises to pay and the public uses these promises to make its purchase and to pay its debt. To the bank, deposits are a liability. When the bank increases its deposits, it increases its liabilities and if public loses confidence, there will be chaos and disorder in the banking system.

(*iv*) Another limitation arises out of the nature of the process by which the deposits come into existence. Every loan made by a bank is secured upon some form of valuable security. If security is not available creation of credit becomes useless. Thus, banks do not create money out of thin air.

(*v*) Power of banks to create credit is controlled by the Central Bank through bank rate policy and open market opertions. A rise in bank rate is an indication to the banks to reduce credit facilities and vice versa. The banks also move in its-foot steps.

(*vi*) Another important limitation on the power of the bank to create credit is the ratio of cash to its liabilities which the banks must maintain to ensure safety. Loan means liability and an increase in liability of the banks means a fall in cash reserve ratio. Banks cannot function if they do not keep a certain percentage of the reserve to meet the demand of the public. When the minimum limit is reached, the power of banks to create credit comes to an end.

(*vii*) It is said that banks do not create credit until and unless there is demand for credit. Actually the borrowers create credit. Generally credit creating will be high during prosperity and general wave of optimism; during depression banks cannot create credit if there is no demand for money.

(*viii*) Every bank is required to keep certain reserves with the Central Bank of the country. The Central Bank keeps on changing the percentage of these reserves from time to time. In the event of an increase in this percentage, the power of the commercial banks to create credit is reduced to the same proportion and in the event of its reduction, the power of the commercial banks increases in the same proportion.

(*ix*) It has been assumed that all the deposits are in the form of demand deposits only, while actually a fairly large part of total deposits is in the form of time deposits or savings deposits. Since time deposits are not withdrawable by cheques and do not serve as money, the result of an increase in time deposits is to reduce the money supply. This reduction in money supply reduces the power of the bank to create credit.

During war period none of these rules is followed by the banks. Money is required for everything and war always sees great creation of bank money. Government raises money through taxes, loans, and other sources but still sufficient money is not available to finance the war. The other alternative is the creation of money. Even in normal times the Government asks the Central Bank to create credit for financing the development plans. This is not harmful as it leads to economic development of the country, but still the banks keep in view the cash reserves while creating credit.

Economists do not have unanimous opinion about the theory of credit creation. According to Hartley Withers, the initiative of creating credit comes from the bank. When a commercial bank advances a loan, it does not pay the borrower in terms of cash. It simply opens a current account in his name and allows the borrower to draw cheques upto the amount of the loan. In other words, the bank pays the borrower in terms of its I.O.U's. The borrower can make payments to others in terms of cheques and they can pass on these cheques

in payment of their debts. Thus, the creation of a loan gives rise to deposits. When current account is opened in the name of the borrower, the bank's liabilities increase to the same extent. These liabilities are accepted as a means of payment and represent purchasing power. Thus, Hartley Withers holds that loans make deposits.

Dr. Walter Leaf and Prof. Edwin Cannan hold opposite views. According to them, the initiative does not lie with the bank. On the other hand, it lies with the depositors. Accordingly, a bank cannot lend more than its amount of deposits. A banker simply lends out the funds, which it gets by way of deposits. It cannot manufacture money. The banker cannot lend out more money than he has of his own plus what he has of others. Therefore, banks do not create money; they simply lend the money entrusted to them by others.

But the above criticism is not valid. A bank, we have noted, does not pay cash to its borrowers but simply issues a claim against itself. A banking system can create credit many times than its excessive reserves. Thus, the criticism that banks cannot create credit is only apparent and not real. We have explained that banking operations play a vital role in creating deposits.

Questions

1. What do you mean by a Bank ? Discuss the various types of banks.
2. Discuss the functions of a Commercial Bank.
3. Discuss the role that banks play in the economic life of a country.
4. What is a Bank ? State and explain the functions of a Commercial Bank.
5. What are the types of banks ? Discuss the importance of banks.
6. Define Balance Sheet. Draw an imaginary balance sheet of a commercial bank and explain the items mentioned therein.
7. How do banks create credit ? Can the banks create as much credit as they like ? If not, why not ?
8. "Loans create deposits and deposits create loans." Discuss. Explain the limitations to create deposits.

7
CENTRAL BANKING

Central Banking which forms an entirely separate branch of banking has developed in the last few decades. The powers enjoyed by and the importance attached to a Central Bank are very great as it occupies the central position in the banking structure of the country. According to Will Rogers, there have been three great inventions since the beginning of time : fire, the wheel and central banking. It occupies a 'central' or pivotal position in the monetary and banking structure of the country. The art of Central Banking has reached such a stage of fullness and precision as Dr. De Kock considers it legitimate to call it "The Science of Central Banking." Few countries had the Central Bank in the 19th century, but the popularity of the Central Bank as an istitution has greatly increased in the 20th century, Today, there is hardly any country in the world which does not have a Central Bank of its own.

Nature of the Central Bank. Both the Central Bank as well as the commercial banks are basically monetary institutions. Both the institutions create credit and extend short term loans only because this helps them in maintaining liquidity in their resources. At the same time, the central bank differs from commercial bank in the following respects :

1. A commercial bank is run for profits but the central bank is set up for the public service rather than profit. In the words of De Kock, "The guiding principle of a central bank is that it should act only in the public interest and for the welfare of the country as a whole and without regard to profit as primary consideration."

2. The central bank does not compete with commercial banks but on the other hand, it acts as the banker's bank and the lender of the last resort.

3. Unlike commercial banks, the central bank does not directly deal with the public. Normally, it does not accept deposits from the public nor does it pay interest on such deposits.

4. A central bank is normally a State-owned institution, whereas commercial banks are generally private owned.

5. Though central bank is State-owned and there exists close cooperation between the State and the central bank but in no case it is subservient to the government. It is free from all political influences and follows actively and vigorously an independent monetary policy conducive to the best interest of the country as a whole. On the other hand, commercial bank can be under the influence of industry and political parties.

6. Central bank possesses the monopoly of note-issue and this right is no longer held by commercial banks now.

7. The Central Bank maintains the foreign exchange reserves of the country and maintains stability in the exchange rates. The commercial banks only deal in foreign exchange under the directions of the central bank.

Definition. Writers on this subject have based their definitions on one or the other function performed by it. According to Messrs Kisch and Elkin the essential function of a Central Bank is the maintenance of the "stability of the monetary standard." This definition involves full control over the circulation of money.

Kent defines it "as institution which is charged with the responsibility of managing the expansion and contraction of the volume of money in the interest of general public welfare." According to the Bank of International Settlements, "A Central Bank is the bank in any country to which has been entrusted the duty of regulating the volume of currency and credit in that country." Vera Smith defines it, "The primary definition of Central Banking is a banking system in which a single bank has either a complete or a residuary monopoly of note-issue."

In the words of Shaw, "The one true but at the same time all sufficing function of a Central Bank is control of credit." Hawtrey, on the other hand, holds that "The essential function of a Central Bank is the lender of the last resort." According to Mr. Jauncey, "Clearing is the main operation of Central Banking." We will quote two opinions more – those of Prof. R. S. Sayers and Dr. M. H. De Kock – The Governor of the South African Reserve Bank. Writing about the constitutional questions on central banking. Prof. Sayers has observed, "The business of a Central Bank, as distinguished from a Commercial Bank, is to control the commercial banks in such a way as to promote the general monetary policy of the state." At the same time, he has emphasised three fundamental points, namely :

1. The central bank is subordinate to the State.
2. It does not hanker after maximum profits.
3. It must possess means of effectively controlling commercial banks.

A careful study of the definitions given above would show that they emphasise one aspect or the other. The definition of Dr. De Kock however, is more comprehensive, all embracing and more elaborate, though it is more lengthy. According to him, "A Central Bank is a bank which constitutes the apex of the monetary and banking structure of its country and which performs, as best as it can in the national economic interest the following functions :

1. The regulation of currency in accordance with the requirements of business and the general public for which purpose it is granted the sole right of note-issue or at least a partial monopoly for the same.
2. The performance of general banking and agency services for the State.
3. The custody of the cash reserves of the Commercial Banks.
4. The custody and management of the reserves of international currency.
5. The granting of accommodation in the form of rediscounts or collateral advances to commercial banks, bill brokers and dealers or other financial institutions, and the general acceptance of responsibility of lender of the last resort.

6. The settlements of clearance balances between the banks.

7. The control of credit in accordance with the needs of business and with a view to carrying out the broad monetary policy adopted by the State.

The definition of De Kock is self-explanatory but one point more may be added and it is that De Kock has emphasised that the Central Bank should deal with the public directly as sparingly as possible and "only in such forms and to such extent as in the circumstances of the particular country, it considers absolutely necessary for the purpose of carrying out its monetary and banking policy."

Historical Development. Central Banking as a specialised branch of banking is of a recent origin although certain banking functions were performed by certain institutions in olden days. The bank of England although established in 1694 was the first bank to perform central banking functions in 1844. Thus the beginning of Central Banking starts with this important year. Other banks followed and by the close of the 19th century practically every country in the West, had got a Central Bank of its own and then the movement spread to the East. These banks were not governed by any theoretical principles. The Riks Bank of Sweden which had sprung from a private bank established in 1656 and which was recognized as a state bank in 1668 is undoubtedly the oldest Central Bank in the world which acquired the sole-right of note-issue in 1897. The Bank of England enjoyed a partial monopoly and its notes were declared legal tender in 1833. The Bank Act of 1844 limited the issues of all other banks to the amounts then in circulation. The Bank of England had come to be regarded "as the custodian of the cash reserves of the private banks and, thus of the country's gold reserves." It had become a full-fledged Central Bank in 1890. On these very lines, the Bank of France was founded in 1800 by Napolean Bonaparte. The most important event of the beginning of the present century is the establishment of 12 Federal Reserve Banks of the U.S.A. in 1914 which were granted monopoly of note-issue and lenders of last resort in their respective areas.

A large number of central banks were established between 1921 and 195- in compliance with the resolution passed by the International Financia Conference held at Brussels in 1920. The South African Reserve Bank, Th Central Bank of China, the Central Bank of Ceylon, the Reserve Bank of Indi were established in 1921, 1928, 1950 and 1935 respectively. So India has it Central Bank established in 1935 under the name of the Reserve Bank of Indi At present, there is no country in the world which has not set up a central ban of its own.

Need for a Central Bank. The need for a Central Bank arises from the va expansion of monetary, fiscal and trade activities of the government and th people. Commercial banks, by creating credit, provide financial facilities for th expansion of commerce and trade. Indiscriminate expansion or contraction credit leads to fluctuations in the level of economic activity in the country. order to control such activities of commercial banks there is need of son institution and that institution is the Central Bank.

A Central Bank is also needed to issue paper currency. The Central Ba issues paper currency according to the needs and requirements of the count

The task of issuing notes has been assigned to the Central Bank in all the countries of the world.

A Central Bank is also needed to help the commercial banks in times of need or economic crisis. In the absence of the central bank, the commercial banks are likely to fail at the slightest crisis in the economy. That is why, the central bank is known as a lender of the last resort.

Only the Central Bank is in a position to implement successfully the monetary and financial policies of the government because it exercises full control over the banking system of the country. Because of the above mentioned reasons, there is a need of a separate institution like Central Bank to control the economy of the country.

The emergence of Central Bank in every country forms one of the most important developments in monetary theory.

The Central Bank is the final authority for the control of currency and credit in a country. It is the leader of the money market, the lender of the last resort, the fountain of all credit, the financier of the Government as well as other banks are also the issuer of all paper currency. Next to Government, it is the supreme monetary authority in the country.

Constitution. There was good deal of discussion about the constitution of Central Bank. It was believed only three decades back that the Central Bank should be as free from Government control as far as possible. They should be privately owned and should give dividends at controlled rates to their shareholders. They should be entitled to appoint or approve the appointment of Governor of the Bank of their country and his deputies. But all this has changed with the rise of the ideology of planning. if economic life is to be planned, it is very necessary to have an effective control on credit in the country. This cannot be done without making central bank an agency of Government. It is on this principle that Bank of England and Reserve Bank of India have been nationalised. Nationalisation of Central Banks is necessary in those countries also where full employment has been adopted as one of the ends of economic life. For policy of full-employment, effective control of credit is very necessary.

FUNCTIONS

The structure, management and policies of Central Banks differ from country to country but there are certain functions which are commonly performed by all the Central Banks. A summary of the functions of the Central Bank is to be found in the evidence of the then Governor of Bank of England before the Indian Committee on Currency and Exchange in 1926. He said, "It should have the sole right of note-issue, it should be the channel and sole channel for the output and intake of Legal Tender currency. It should be the holder of all Government balances; the holder of all the reserves of the other banks and branches of banks in the country. It should be agent, so to speak, through the financial operations at home and abroad of the Government. It would further be the duty of the Central Bank to affect, so far as it could, suitable contraction and expansion of currency, in addition to aiming generally at

stability of exchange and to maintain that stability within and without. When necessary, it would be the ultimate source from which emergency credit may be obtained in the form of rediscounting of approved bills, or advances on approved short term securities of Government paper."

This bank occupies the key position in the banking system and is supposed to supervise and co-ordinate the activities of the member banks in the country. To understand its role in the banking system, it is essential to describe the functions of a Central Bank. The following are its important functions :

1. Note-issue. This is a very important function of a Central Bank and today, every Central Bank enjoys the sole monopoly of note-issue. In the words of De Kock, "The privilege of note-issue was almost everywhere associated with the origin and development of Central Banks." The main grounds, why this privilege has been made the sole monopoly of Central Bank are :

(a) It introduces uniformity in note-circulation and makes far more effective control of the State.

(b) It is an effective method to put a check on the creation of credit by the Commercial Banks.

(c) Issue of notes by the Central Bank which also enjoys the patronage of the State, lends more grace and prestige to the paper currency and creates more confidence which is very essential particularly during economic crisis.

(d) It will also equip the Central Bank to deal with the various monetary and banking problems in a more effective manner and to help resolve them more effectively.

For this purpose, a Central Bank generally maintains a separate department popularly known as the Issue Department and carries on its policy as the interests of the country demand it. There are economists who have very seriously objected to this concentration of power, but in spite of all this the fact stands that the Central Banks continue to enjoy this privilege. Mr. Lewinski prefers bank note-issue to State note-issue as the latter may be issued "without being preceded by any demand for it."

Principles of Note Issue. In the issue of notes two conflicting aims have to be reconciled. On the one hand, the note-issue must be elastic. The circulation should expand or contract in accordance with the demand of trade. On the other hand, the confidence in the notes must be preserved by maintaining its convertibility. The first is the principle of elasticity and the second is that of security. This requires a proper regulation of note issue.

Currency Principle Vs. Banking Principle. There was a keen controversy as to what should be the right principle of note-issue. One school of thought advocated Currency Principle and the other Banking Principle. The advocates of Currency Principle insisted on full metallic backing (i.e., 100% reserve). For every note-issue there must be kept in the currency chest coins of the same value. The currency note was only a convenient and economical substitute for metallic money. Those who advocated Banking Principle were in favour of leaving the business of note-issue to the discretion of the banks – to vary the currency in response to the legitimate needs of trade. Any excess of note-issue would

automatically come back to the banks by being presented for cash payment. The banks would naturally keep reserves to honour these notes. Thus the Currency Principle provides safety but lacks elasticity whereas the Banking Principle ensures elasticity but is wanting in security. A sound system of note-issue, however, must have both elasticity and safety. Hence the Central Bank has to evolve systems each of which represents a compromise between these two principles. Among these we may mention (a) Fixed Fiduciary Principle or Partial Deposit System; (b) Proportional Reserve System; and (c) Minimum Reserve System, etc.

2. Banker to the Government. A Central Bank also acts as a banker to the government and in that capacity, it carries on all banking business on her behalf. It maintains her accounts. It receives deposits from the government but without any interest. It also makes advances to the government, but they are short term loans. It collects taxes on her behalf and also raises loans from the public and thus manages public debt also. It also acts as her financial agent, often her financial advisor in matters like financial, fiscal, monetary and economic. In the words of Dr. De Kock, "As the manifold financial activities of the state can in certain circumstances exercise a disturbing influence on money market conditions and exchange rates, and counteract the credit policy of the Central Bank, the centralisation of government banking operations of the Central Bank at least gives the latter a better opportunity of judging the general financial situation at any time, giving the appropriate advice to the government and taking the necessary remedial measures."

As an agent of the government, Central Bank undertakes the administration and management of the national debt. It represents the government at the specialised financial institutions in the country as well as at the international institutions and conferences.

As an adviser to the government, the Central Bank gives advice to the government on important matters of economic policy like deficit financing, devaluation, trade policy, foreign exchange policy etc.

3. Custodian of Cash Reserves of Commercial Banks. The Central Bank of the country acts as a bankers' bank and in this capacity has to hold the cash reserves of commercial and other banks and acts as a sort of guardian to them. In the words of Crowther, "The Central Bank stands to the member banks in exactly the same relation as the member banks themselves to the public." To safeguard the interests of the depositors as also to look after the economic interests of the nation these reserves are to be maintained by the Central Bank to use them in times of financial stringencies. The Commercial Banks, thus are under an obligation to keep a certain percentage of their deposits with the Central Bank. The centralisation of cash reserves in the Central Bank is a source of great strength to the banking system of any country. Centralised cash reserves can at least serve as the basis of a larger and more elastic credit structure than if the same amounts were scattered among the individual banks. Although the percentage differs from country to country, every one has got the practice. In India, this percentage used to be 2% and 5% of time and demand liabilities but recently the Reserve Bank has been empowered to raise the

minimum time liabilities to 8% and that of demand upto 20%. It is these reserves which have lent support to the credit and banking system of the country.

4. Custodian of Nation's Reserve of Foreign Exchanges. This function lays down that the Central Bank should also hold the nation's Reserves of International Currency. This is necessary on two grounds : (a) By this the Bank can maintain the external value of the national currency, and (b) It can meet any situation arising out of adverse balance of payments because the reserves that are maintained against the paper curreny cannot be utilised for the purpose, hence the need for this important function. In order to minimize the fluctuation in foreign exchange rates, the central bank has to buy and sell foreign currencies in the market. In case of emergency, the central bank may even impose control on the buying and selling of foreign currencies in the market.

5. Rediscounting of Bills of Exchange. Another important function of a Central Bank is the rediscounting of the bills of exchange. We know that the commercial banks discount bills of exchange of their customers; but in the process, some of these banks get their funds exhausted. They suffer from the shortage of cash. The facility of rediscounting by the Central Banks lends elasticity to the credit structure of the bank and promotes liquidity to credit money. In this way, the commercial banks are allowed to carry on their work even with shortage of funds. It should, however, be emphasised that this policy should be adopted only for the institutions in genuine difficulties and be not allowed in an unrestricted manner because the dangers involved therein are enormous.

6. Lender of the Last Resort. This function of the Central Bank has also been very much emphasised by some people. It aims at accommodating the different banks in a country in their hour of need and difficulty by providing them credit facilities. It should not, however, be confused with the last function of rediscounting of Bills of Exchange. It is quite different function because it implies that even if the bills of certain banks have not been rediscounted, the Central Bank can lend them in order to save them from liquidation or failure. The Central Bank's function as the lender of last resort satisfies an important need of banks viz., the need for liquidity. The commercial banks need larger cash reserves, from time to time, than they normally hold. The Central Bank's services as the ultimate source of cash are useful on such occasions. It is interesting to note that this term 'lender of the last resort' was itself coined by Bagehot in his "Lombard Street." Thus the Bank of England adopted this function for the first time and the other banks that followed automatically and willingly assumed the responsibility of performing this function. Authorities after authorities have harped on this tune. Prof. Burgess described it as "a step in the right direction" while Prof. Sayers considers the function as fundamental to good central banking.

7. Bank of Central Clearance. This function of the Central Bank implies that the Central Bank will be responsible for settling the accounts of the different banks working in the country. It is well known that a bank is prepared to collect the cheques of another bank and this process goes on for ever. It doe not, however, mean that each bank makes only as much of the payments as i

expects to receive. In the process some banks turn to be 'creditors' and others as 'debtors'. This final settlement is made by the Central Bank of the country which is said to work as a clearing house. This function, in fact, becomes an automatic function of a banker's bank. It was first adopted by the Bank of England in 1854 and was later adopted by other banks and it became the normal function. As Prof. Shaw put it "A Central Bank will operate as the clearing house for all its member banks as a mere matter of mechanism or of book-keeping." Kisch and Mrs. Elkin consider it very important and essential for a Central Bank to "set up an expeditious and economical machinery for the clearance of drafts and settlement of internal accounts." This function is performed by the Central Bank either daily or weekly. There are writers like Jauncy and Wills who consider the clearing function of the Central Bank as the most important one.

Broadly speaking, therefore, the Central Bank acts as bankers' bank in three capacities :

(i) as the custodian of the cash reserves of the commercial banks.
(ii) as the lender of the last resort.
(iii) as a bank of central clearance, settlement and transfers.

8. Credit Control. By far the most important function of a Central Bank is credit control. It has been described by some writers as the main function, other functions being merely 'adjuncts' of this one function. In fact, all other functions come to serve this one function and its need and importance has been growing because of the important role which 'credit' has begun to play in the economic life of the country. The growing importance of this function can also be judged from the fact that it has been incorporated in the laws of Central Bank of the country. The Reserve Bank of India Act lays down that it is to "operate the currency and credit system of the country to its advantage." The importance of this function has grown particularly since the twenties of the present century owing to the enormous expansion of credit in the world and it has been found not only desirable but essential to put restrictions on the agencies responsible for the expansion of credit.

It is the special function of Central Bank to determine the credit policy of the country and to direct the market to follow the policy. In the discharge of these duties, Central Bank uses its powers of changing Bank Rate, open money market operations and direct action. Of late, the exercise of several other powers, viz., rationing of credit and changing Reserve Ratio has become more prominent.

9. To Achieve Stability in Price Level. It has to control and regulate the supply of credit in the market so as to achieve stability in price level. The foremost duty and function of the Central Bank is to secure stability in the value of money and in order to achieve this end, it has to keep in hand and manipulate all the strings of credit control.

Apart from the above functions, the Central Banks of most of the countries maintain relations with international financial institutions, such as International Bank for Reconstruction and Development etc. as no country by itself can attain full employment and higher rate of economic growth without close

international monetary co-operation. Above all, the Central Bank everywhere helps in the economic development of a country. For this, they undertake to provide banking facilities to hitherto neglected areas and adopt monetary policy conducive to the requirements of economic growth and development. Besides this, the Central Bank collects and publishes statistics about the various aspects of the functioning of the national economy. This provides valuable information on the basis of which the government can formulate and implement its economic policies.

Now, after considering all the functions of a Central Bank, the question arises : Which function is the most important ? In fact, this question is not easy to answer. Different economists consider different functions as most important. According to Hawtrey, the lender of the last resort is the most important function, whereas Prof. Smith considers note-issue as the most important function and Prof. Shaw thinks that control of credit is the most important function of the Central Bank. As a matter of fact, all the functions are important and the Central Bank is to work in the interest of the country. But the majority of the writers seem to think that the function of regulating and controlling the valume of currency and credit is the most important of all. According to De Kock, "The control and adjustment of credit is accepted by most economists and bankers as the main function of a central bank. It is the function which embraces the most important question of central banking policy and the one through which practically all other functions are united and made to serve a common purpose."

In underdeveloped countries, the Central Bank is required to play a more positive role. This role has to be in the direction of the development of the money and capital markets and the commercial banking system and also to provide banking facilities to the community. It has to promote the development of various financial institutions which can mobilise savings and provide capital for economic development of the country.

Nationalisation of Central Banks. The concept of central banking has undergone radical changes, especially after the World War I, owing to heavy strain on the government finances, the urge for liquidity, to meet emergencies and the increased requirement of foreign exchange and the growing emphasis on economic planning and full employment. Some countries adopted planning in order to set their houses in order, whereas others aimed at the achievement of full employment. And in order to carry out their schemes of economic development, deficit financing was resorted to. The trend of economic thinking also changed. People began to lay emphasis on the development of international co-operation in the sphere of monetary management. In the less developed economies, the government and the Central Bank have to act in complete unison.

Taking all these things into consideration, nationalization of Central Banks became necessary. In some countries, the Central Banks have been nationalized, and in others, they are passing through the evolutionary process of nationalisation. The Bank of England was nationalised in 1946. The Bank of

France was nationalised on December 2, 1945, and Reserve Bank of India was nationalised with effect from January 1, 1949. We find that after nationalization, the operational efficiency of these Central Banks has considerably improved, and they have been able to control monetary affairs more vigorously. On the basis of the successful experiment of these countries, other countries like Canada, Denmark, Australia etc., also nationalised their Central Banks. However, certain economists seem to be in favour of granting greater independence to the Central Banks and protecting them against undue political pressure.

CREDIT CONTROL

The Central Bank in every country is vested with the special powers to control and direct the activities of credit institutions in the country. Credit control is essential for the stability and orderly growth of an economy. Credit control means to control the volume of credit – its expansion and contraction according to the economic needs of the community and thereby controlling the price level and rates of exchange for the promotion of economic welfare. It is to regulate the volume of trade and direction of bank loans. On the volume of credit depends largely the level of employment and the level of prices in the country. Price level increases and decreases according to the expansion and contraction of credit in a country. Central Bank being the leader of the money market exercises in a powerful influence to control this type of situation as determined by the circumstances, prevalent in the country and according to the economic suitability.

The Objectives. A central bank controls credit with the following objects in view : (a) To safeguard its gold reserves against internal and external drains, (b) to maintain stability of internal prices, (c) to achieve stability of foreign exchanges, and (d) to eliminate fluctuations in production and employment.

There is no dispute about the fact that in the interest of the economy there is a great need for credit control in the country. The main objective of the credit control from 1875 to 1931, was *stabilisation of the exchange* rate of the currency and all efforts then were diverted to that end. Gold standard was adopted with this objective in view by some countries and attempts to introduce the same were made by others. After the abandonment of gold standard, emphasis began to be laid on price stability instead of exchange stability because it was argued then that stability of the rate of exchange was not important. Objective of price stability became more prominent as it avoided conflict between the employer and the employee, producer and consumer and creditor and debtor in the society. But modern economists have begun to believe that the cine qua non of maximisation of national economic welfare is neither exchange stability nor price stability but elimination of fluctuations in the employment situation in a country. It is the high level of employment and rise in national incomes that needs the highest priority. Thus, the fundamental objective of monetary policy, today, has been the maintenance of a high level of employment by promoting economic activity. Thus, we have, today, shifted from mere economic stability to economic activity compatible with the policy of full employment.

The necessity of safeguarding gold reserves arises under a gold standard. In a gold standard country, gold can be imported and exported and the currency of the country is convertible by law into gold bullion. In such a country an over-expansion of credit causes inflation. At first high prices at home lead to withdrawal of more cash from the banks and gold from the Central Bank to carry on transactions at a higher level.This is called the "internal drain." Secondly, the home price level being higher than the international price level, imports are encouraged and exports discouraged. An unfavourable balance of trade is created which has to be met by export of gold. This is called the external drain. Gold may also move out because the foreign investors have lost confidence in the future of the currency under question and they begin to withdraw their funds. The Central Bank, therefore, must take steps to contract credit, bring prices down and stop the internal and external drain of gold.

Another object is to maintain stability of internal prices as has been stated above. We have already referred to the various disadvantages of fluctuating prices. Price instability causes disturbances in economic welfare, maladjustment and serious social consequences. The Central Bank, by regulating the supply of purchasing power, according to the needs of the people, can reduce such fluctuations to a large extent.

Instability of foreign exchange (Value of foreign money in terms of home money) disturbs international trade. This tendency has to be checked up. Steadiness in internal price and foreign exchange are desirable and they should be subservient to maintaining the stability of economic life as a whole. Then aim should be to maintain a normal steady growth of business activity and prevent booms and slumps. Credit expansion stimulates business activity and credit contraction retards it, the former may lead to over-investment and set a trade cycle in motion. The desired policy of every Central Bank is to adopt measures of credit control during the periods of propsperity or regulate prices and avoid the occurrence of trade cycles.

Need for Credit Control. The evils resulting from over-expansion of credit necessitates a cautious and planned policy of credit creation and control. The over-expansion of credit has many times resulted in the biggest "Crashes" of the world. The economic fluctuations of the 1920's and the world wide depression of 30's can be easily accounted for the unsound credit policies followed by the different countries of the world. When the policy is carried too far it encourages speculation in the business market and brings in its wake all the evils connected with such a situation. The whole business structure including trade and commerce is upset. The economy of a country receives such a huge 'shake up' that it has to exert a lot to regain the status quo. In fact, the trade cycles and evils associated with the various phases of booms and depressions, falling prices, can all be attributed to the uncontrolled policy of credit in the country.

Difficulties of Credit Control. Even if the bank is able to control the volume of credit, the objectives concerned may not necessarily be achieved. Several difficulties may be noted. First, bank credit is not the only form of credit. There is commercial credit like book credit, bills of exchange and promissory

notes (not discounted by banks). Over there the Central Bank has little control. They have as much purchasing powers as any other form of credit.

Secondly, even as regards bank credit. all banks of the country do not have direct relations with the Central Bank. In the U.S.A. one half of the commercial banks with one-fifth of the resources are outside the Federal Reserve System. In India the indigenous bankers, accounting for 9% of the banking business in the country are still beyond the influence of the Reserve Bank.

Thirdly, even if all banks were member banks, commercial banks may not always co-operate with the Central Bank and may not follow its lead. Such co-operation, as we will see, is essential for the successful control of credit. Fourthly, there are non-banking elements in the financial structure of a country. Among these are various circumstances that affect the temper of the business community. These are beyond the scope of central banking action. Finally, the Central Bank cannot control the ultimate use to which credit may be put. Strictly, commercial loans, for instance, may be used for speculative purposes.

This, however, does not mean that any attempt to control credit on the part of the Central Bank is bound to fail. These are limitations to which the action of the Central Bank is subjected to.

METHODS OF CREDIT CONTROL

Now, we proceed to discuss the methods of credit control, also called the central banking techniques.

There are, broadly speaking, two types of controls used by the Central Banks in modern times for regulating bank advances; (a) Quantitative or General Controls, and (b) Qualitative Controls or the Selective Credit Controls. The aim of the quantitative control is to regulate the amount of bank advances i.e., to make the banks lend more or less. In other words, the general controls are intended to expand or contract the total volume of credit in the banking system without regard to the purpose for which it is used. It is the overall supply that is influenced by the Central Bank. The object of the selective credit controls is to divert bank advance into certain channels or to discourage them from lending for certain purposes. The selective controls have, of late assumed great importance. Let us now discuss these methods of credit control followed by the Central Bank of the country in detail.

(A) Quantitative or General Controls. The important quantitative or general methods of credit control are the following :

1. Bank Rate or Discount Rate Policy. The Bank Rate or Discount Rate is the rate at which the Central Bank is prepared to discount and re-discount first class bills of exchange and by offering the facility to the banks, it provides them with an opportunity to get accommodation at the hands of Central Bank, when their own funds are exhausted. Its importance rises all the more because it becomes the affective rate for lending or borrowing in the market. *Bank Rate* has been defined by M. Spalding as "the minumum rate charged by the Central Bank for discounting approved bills of exchange." According to Keynes, "It is the effective rate for lending or borrowing, which prevails in the market." Since

the Central Bank is the lender of the last resort, the bank rate may be regarded as the wholesale price of credit in the market. A change in the bank rate will naturally lead to a change in the rate of interest on all classes of loans charged by other banks. Thus, if bank rate is raised other banks will revise the rates of interest in the upward direction and vice versa. If the Central Bank wants to correct a general rise of prices or a fall in the rate of foreign exchange, it would raise the bank rates. Rate of interest all over the market would rise. People would reduce their borrowing and increase their deposits in the banks. Loans already contracted would be repaid. The total volume of credit in the market would decrease and the price level would fall. The rate of exchange will similarly rise as a result of this change. Fall in prices would stimulate exports and discourage imports. The demand of bills of that country will rise and supply will fall. Moreover rise in the rate of interest will attract deposits and investment from foreign countries and discourage the flow of gold and capital from that country. This will in its turn create a demand for bills on that country and reduce the supply of bills on that country. The rate of exchange would thus rise. If, on the other hand, Central Bank wants to raise the price level and lower the rate of exchange, it would reduce the bank rate.

The growing importance of this method has been pointed out by De Kock himself. According to him, "It indicates the rate at which the public should be able to obtain accommodation on the specified types of papers from the commercial banks. Secondly, it represents the basis of the rates at which the commercial banks can obtain credit from the Central Bank, thirdly, its psychological value as an instrument of credit control to the Central Bank is very important." If the Central Bank raises the rediscount rate, the credit becomes costlier and the commercial banks hesitate to get their bills re-discounted and if it lowers the same, it encourages the banks to avail of this facility. While credit contracts in the former case, it expands in the latter. The rise or fall in the bank rate has been nicely explained by Mr. Gibson. He says that a rise in bank rate may be regarded as amber-coloured light of warning of a robot system of finance and economics, while a fall may be seen as the green light indicating that the coast is clear and the ship of commerce may proceed on her way with caution. The successful operation depends, among others, on the prestige of the Central Bank and the degree of co-operation which it can obtain from the commercial banks and other credit institutions. It must be remembered here that this cooperation can be had by the establishment of a conventional relationship between the two rates – the discount rate of the Central Bank and the rate of discount charged by the various commercial banks of the country. We would like to refer to three main trends of thought.

(1) According to one trend, the bank rate is used as a means of regulating the quantity of bank money.

(2) Another trend of thought is to conceive of bank rate as protecting a country's gold reserves by regulating the rate of foreign lending, and

(3) The bank rate influences the rate of investment – a rise in bank rate discourages investment relatively to savings, and a decline in it encourages

investment. I . c!. Wicksell thinks that the bank rate affects the relationship between savings and investment.

Keynes' Views on the Bank Rate Policy. According to Keynes, the traditional theories of bank rate concentrated largely on the influence of the bank rate as a means of regulating the quantity of money and of protecting a country's gold reserve. It had not taken account of the influence of bank rate on the rate of investment relatively to savings, and the influence of changes in the relation between savings and investment on prices, production, employment and wages. Keynes criticised Hawtrey who had emphasized investment, but only one particular kind of investment namely, "investment by dealers and middle men in liquid goods" to which adds Keynes, a "degree of sensitiveness to changes in bank rate is attributed which certainly does not exist in fact". According to Keynes, economic situation is affected not through the changes in short term rate of interest and in the stock of working capital goods, but through the long term rates of interest and the volume of fixed capital goods. Changes in the bank rate lead to changes not only in short term rates of interest but also in long term rates affect the investment market. This investment depends upon the prospects of profits and the long term interest. Assuming the prospects of profits to be the same, the higher the long term interest, the less attractive becomes investment for replacement of existing capital. When entrepreneurs spend less on fixed capital goods, employment in the capital goods trades declines. This leads to contraction of total money incomes and a decrease in expenditure on current consumption. This results in the decline of employment in consumption goods sectors. Prices and production fall all around. Conversely, when the rates of interest fall, the opposite is the result. In his General Theory, Keynes emphasizes the importance of equilibrium between savings and investment for general economic stability. He is, however, of the view that apart from regulation of quantity of money through open market operation (to be studied presently) such equilibrium should be attained not by bank rate policy but by the state directly organising investment and starting public work in periods of depression. Keynes regards bank rate policy as out of date method of controlling credit.

The bank rate policy, however, has not yet gone completely out of use, though its relative importance has been much reduced. It is still an instrument for correcting wrong trends and restoring equilibrium through its influence on the supply of, and demand for money. Whether it acts through affecting short term interest rate and investment in liquid goods, as Hawtrey holds, or through long term rate interests affecting investment in capital goods as contended by Keynes is a matter which is difficult of verification. The state of trade and prices is affected by many factors. A change in bank rate may lead to change in holding of stocks as well as in investments in fixed capital goods.

Limitations of Bank Rate Policy. Recently, however, it has been emphasized that the bank rate is not a very potent weapon and that theoretically it is based on interest-saving relationship which has long been exploded. A high rate of interest does not increase savings. On the other hand, it retards investments. Incomes fall and savings decrease. Similarly, a fall in the rate of

interest increases savings instead of reducing it. It must also be remembered that a mere fall in the rate of interest would not induce the investors to borrow more. If they do not expect any profit, even a zero rate of interest will not induce them to borrow more than before. As has been said, you can bring a horse to water but you cannot compel him to drink. The importance of Bank Rate which has been described by the Macmillan Committee as "a most delicate and beautiful instrument of credit control" declined after the termination of the First World War, for which some factors were responsible : (1) radical changes in the condition of the money market, (2) changes in the economic structure of the various countries, (3) increasing use of other methods of credit control, (4) cheap money policy, (5) changes in the methous of financing trade and commerce, and (6) greater importance of fiscal policy, especially after the Great Depression. However, during recent years, a revival in the significance of bank rate policy has been noticed. Inflation is now a world-wide phenomenon and to fight it the countries use the bank rate policy.

For a successful working of credit control through bank rate, a number of conditions have to be satisfied :

(*i*) All the other rates should follow the bank rate in its movement so that credit should expand and contract as the case may be.

(*ii*) The economic structure of the country should be elastic so that change in credit conditions should lead to corresponding changes in wages rents, production, trade, etc.

(*iii*) The short term funds market in the economy should be well organised and developed. In less developed countries like India such a market is highly underdeveloped and disorganised and, therefore, the bank rate policy has been ineffective in controlling credit.

In most of the countries, such conditions are not satisfied to any appreciable degree. Hence, the meagre success of the bank rate policy is there. Moreover, bank rate policy is not always successful and as has already been explained in spite of high discount rate, people may continue to borrow huge amounts only because they expect that the yields will be much more remunerative. Again in certain branches of business interest rate constitutes only a negligible part of the total cost and the cost of interest or discount rate may not be material in such branches. Hence, discount rate can affect only through business expectations.

Again this policy may be much more ineffective during depression than during the boom period. During the period of depression the demand for credit falls very considerably because no body wants to invest owing to the loss of the so-called "business confidence" and no one would like to borrow even if the rates of interest were to be very low. There is virtually no demand for credit. There is no inducement to invest at all.

From the above, it is clear that the bank rate policy like other instruments of credit control, suffers from serious limitations and, therefore, it is not a very dependable weapon. But it could do well when it was joined by other weapons. To use the words of De-Kock, "There is a good reason to believe that the official

discount rate has, nevertheless a useful function to perform in conjunction with other measures of control."

2. Open Money Market Operations. The Bank Rate Policy would be effective only if other credit institutions in the country follow faithfully the lead given by the Central Bank. If other credit agencies do not change their rate of interest, the change in Bank Rate would fail to achieve its objectives. In order to meet such a situation, the Central Bank is given a second power called Open Money Market Operations. It means purchase or sale of first class Government Securities of bullion of the Central Bank in the open market.

When the Central Bank raises the bank rate, in order to bring down the price level and raises the exchange rate but others refuse to raise their rates of interest and continue to allow credit at the old rates, it begins to sell Government Securities and/or bullion at attractive prices. People buy securities and make their payment by drawing cheques on their deposit accounts in the banks. Cash balances in the banks are reduced by these withdrawals, and banks in order to keep themselves liquid are compelled to go to the Central Bank to get accommodation. The Central Bank lends on the new rate which is higher than the earlier rate and the borrowing banks are compelled in their turn to raise their lending rates. This results in contraction of credit, hence fall in prices, and rise in the exchange rate. Conversely, if the bank rate is reduced and other banks do not reduce their rates, the Central Bank begins to buy Government Securities and bullion. Cash which is received by the people in exchange for these, flows into other banks. Cash balances increase and in order to find more borrowers to utilize this cash, banks are compelled to lower their rates of interest. This expands credit and raises price level and lowers exchange rate. This method, therefore, supplements the bank rate policy and is exercised when the latter fails to produce the desired result. In other words, when the Central Bank buys stocks and shares in the open market, it releases funds which go to the sellers of these stocks and shares and thus the reserves of the banks swell up and so do their reserves with the Central Bank and the same are used for the fresh loans. Then takes place expansion of credit, which in turn leads to greater investment, greater employment and higher prices. Similarly, when the Central Bank sells these papers in the open market, payments are made to the Central Bank by the purchasers of these papers whether individuals or banks. This, in turn, means reduction in the reserve of these banks with the Central Bank. Such a reduction necessitates the reduction of loans and advances by the commercial banks. The Central Banks therefore, make use of these methods whenever they want to expand or contract credit.

The Theory of Open Market Operations, in the words of De-Kock, is that "the purchases or sales of securities by the Central Bank tends directly and immediately to increase or decrease the quantity of money in circulation and the cash reserves of the commercial banks : that an increase or decrease in the supply of bank cash and, therefore, in the credit creating capacity of commercial banks tends still further to increase or decrease the quantity of money; and that changes in the quantity of money tend to bring about relative changes in money rates and credit conditions which, in turn, tends to bring about the desired

adjustment in the domestic level of prices, costs, production and trade."

This method of credit control was first adopted during the eighteen thirties, and was generally known as 'borrowing on consols.' The Bank of England used this traditional method of sale and purchase of consols up to the end of 19th century. Though the bank was able to control the supply of money, the application of the device was in many ways unsatisfactory. The operations are adopted during the period of boom or depression. In a broad sense, these operations include "the purchase and sale of banker's acceptances of Government securities and other limited types of transactions." In a narrow sense – the purchase and sale of Government securities.

Limitations of Open Market Operations. This policy has been quite effective and popular because (a) it produces a very direct and immediate effect on the credit conditions in the economy, (b) its scope has increased due to the increased availability of government and other eligible securities in the money market, and (c) the bank rate policy proved ineffective owing to the changes that took place in the economic structure of the various countries. The extent to which these policies would succeed depends upon the way how the money market in the country is organised and the extent to which other banks depend upon Central Bank for funds. The limit to these operations is also set by the availability of the securities in the market and the prevailing conditions in the investment market. The above operations will be valid only if certain conditions are satisfied. These conditions are :

(*i*) The theory is that when the Central Bank purchases securities, the cash reserves of the member banks will be actually increased, and conversely, the cash reserves will be decreased when the Central Bank sells securities. This however, may not happen. The sale of securities may be offset by inflow of gold into the bank or by return of notes from circulation and hoards. The purchase of securities, on the other hand, may be accompanied by an outflow of gold or withdrawal of notes for increased currency requirements or for hoarding. In both the cases, therefore, the cash-reserves of the member-banks may remain unaffected.

(*ii*) But even if the cash-reserves of the member-banks are increased or decreased, the banks may not expand or contract credit accordingly. The percentage of cash to credit is not rigidly fixed and can vary within quite wide limits. The banks will expand and contract credit according to the prevailing economic and political circumstances and not merely with reference to their cash resources.

(*iii*) The third condition is that when the commercial banks' cash resources increase, the demand for loans and advances too should increase and vice versa. This may not happen. Due to economic or political uncertainty, even cheap money rates may not attract borrowers. Conversely, when trade is good and prospects of profits bright, entrepreneurs would borrow even at high rates of interest.

(*iv*) The circulation of bank credit should have a constant velocity. But the velocity of bank deposits is rarely constant. It increases in periods of rising business activity and decreases in periods of depression. Thus, a velocity of

contracting credit may be neutralised by increased velocity of circulation, and vice versa.

(*v*) The expansion and contraction of credit depends, to a large extent, on the business psychology of the people who may continue to demand more or less of credit, irrespective of whether the Central Bank wants to expand or contract the same. Suppose, the Central Bank buys securities and the reserves of the commercial banks increase; but the business community may, due to certain factors, withdraw cash from the banks curtailing their lending capacity; even though in principle, they should be in a position to expand credit. The reverse tendency may also operate. Thus, when the Central Banks want to expand credit, businessmen may not like to borrow and when the Central Banks contract credit, the business community may demand more and more of the same. In this way, the open market policy of the Central Banks may become inoperative.

(*vi*) It is essential that the market for different types of securities should be well organised and developed. In less developed countries, this condition is hardly fulfilled and that is why the open market operations policy has not been as successful as it has been in the countries of the west.

(*vii*) It has been stated that the open market operations are more successful in controlling the expansion of credit rather than in stimulating expansion of credit. During a period of depression, the business prospects not being very bright, the entrepreneurs are hardly encouraged to borrow funds from the commercial banks in spite of the fact that they are available at much lower rates.

Thus, we find that in spite of the theoretical excellence of this policy in practice, the policy has been quite useless. In the words of Prof. Wills, "Open market operations do not constitute an independent kind of influence or type of transactions." Similarly, to quote Sober, "It is very doubtful if they (open market operations) can have any appreciable effect upon cyclical or secular trends in business." Prof. Morris considers them as "a weapon of second rate effectiveness." Keynes however, believes that if undertaken skilfully, open market operations could serve, without the use of bank rate, but for that purpose, an efficient state organisation was essential.

Which Policy is Better – Bank Rate or Open Market Operations ? From the above discussion, it would be seen that neither the bank rate policy, nor the open-market operations are, by themselves successful in achieving the desired objective. Each should be supplemented by the other, in order to be effective. The raising of bank rate alone will not be effective if the banks possess surplus funds, unless the Central Bank withdraws surplus funds by selling securities. Similarly, if the Central Bank sells securities without raising the bank rate, the member banks may replenish their reserves by rediscounting their bills with the Central Bank and thus avoid contraction of credit. That is why, economists are of the opinion that the bank rate policy should be adopted only to correct a permanent disequilibrium, while the open market operations may be undertaken to remove a temporary maladjustment. From the point of view of credit control, open market operations are, therefore, necessarily complementary to the bank rate policy.

Hawtrey has no faith in open market operations alone as instrument of credit regulation, and he considers that the use of discount rate is also essential. Keynes, on the other hand, believes that these operations if undertaken skilfully and extensively, alone could achieve the objective provided they are supplemented by a properly planned investment policy.

3. Variable Cash-Reserve Ratio. Another method of credit control is the change in the reserve ratio of the balances which the commercial banks are obliged to keep with Central Bank and this method combined with the bank rate and open market operations comes to be very helpful. The significance of the method lies in the fact that a rise or reduction in the ratio, reduces or increases the basis of credit creation by the commercial banks. In this way, when it is found that the volume of deposits has increased with the commercial banks and they are capable of expending credit, the Central Bank will raise the rate of the reserve ratio, curtailing the power of the banks considerably and vice versa. As we have already stated above, used in conjunction with other methods, it can become very handy and very effective. It was first used by the U.S.A. in 1936 on the suggestion of Keynes who did much to popularize it as a method of credit control by the Central Bank. Later on, this method was adopted by some other countries, such as Germany, Sweden and France etc. For a long time, this method of variable cash reserve ratio was looked upon as an indispensable method of promoting the overall liquidity and the solvency of the banking system. It inspired greater public confidence in the ability of the commercial banks to meet their obligations to the depositors. But in spite of this, this weapon of credit control is not without certain limitations. The limitations are :

(1) First of all, this reserve ratio cannot be changed except by law.

(2) Secondly, the commercial banks need not bother about the same if their own reserves are so huge that a change in the ratio does materially affect them.

(3) Then there may be difficulty of operating them when there are different ratios for different types of deposits and the banks may manipulate things by shifting deposits from one kind to the other.

Nevertheless in the words of Mr. Burgess, "It is the most useful addition to the system's mechanism for control."

4. Minimum Secondary Reserve Requirements. These reserves are over and above the minimum cash reserves to which reference has been made above. The idea underlying the scheme is to further restrict the power of the commercial banks to expand credit by limiting their capacity to convert government securities and surplus cash assets into business loans. This method has been extensively used by several countries of the world and it has certainly limited the power of the banks to expand credit and to use De-Kock's words. "It can be made to play a valuable part in any positive disinflationary monetary policy under conditions of exceptional inflationary pressures caused by war, rearmament or abnormal circumstances." This method was used in some countries like Belgium, France, Sweden, Norway, Holland and India. This proved to be a very useful measure of credit control. It makes the Bank Rate policy more effective and serves as an anti-inflationary measure.

Co-ordination of General Credit Control Measures. Quantitative or general credit control measures explained earlier are the important weapons with the Central Bank for controlling credit and stabilising the business activities through the control of money.

Bank rate policy has become less effective since 1920 and more particularly since the breakdown of gold standard. Open market operations have become more important since then in the countries which have developed security markets. Changes in reserve ratio are effective in controlling the reserves.

All these measures, when used independently, suffer from various limitations. It is, therefore, desirable that they are used simultaneously. If the Central Bank wants to restrict credit, it should increase bank rate, sell securities in the open market and raise reserve ratios. Looking back over the discussion of credit control through general control measures, we may draw the following main conclusions :

1. Credit control is not possible by any single instrument alone. Judicious and skilful combination of all the methods is essential in order to achieve the objectives of credit control and monetary management.

2. Different methods may be combined in different proportions under different circumstances. A rise in bank rate combined with sale of securities in the open market may be helpful in achieving credit contraction. But more immediate results can be obtained by changing the reserve ratio.

3. The effects of different methods are different on different sectors of the economy. Their effects are not evenly distributed over the whole range of economic activity. They should be used in such a manner that they should not retard the economic growth of vital sectors of the economy.

4. Brakes must be applied at the right time and in the right direction. Applied too soon, they may bring expansion to an end without achieving full employment. Applied too late, it may lead to hyperinflation.

5. The instruments of general credit control may be applied not only quickly but also continuously so as to prevent from developing any recessions or depressions from rising.

(B) Qualitative or Selective Controls. Selective credit control is a recent development in monetary management by the central bank. The objective of qualitative method is to divert the flow of credit into particular uses or channels in the economy. In fact, the objective of selective credit control is to encourage the flow of credit into those uses or channels which help the growth of the economy. Selective credit controls are considered as a useful supplement to general credit regulations. They become more effective when they are used together with general credit controls. These methods have been used in most of the countries of the world. In the U.S.A. these have been used to regulate stock-exchange credit. In India, such controls have been used to prevent speculative hoarding of food grains and raw materials. The following are the main types of selective credit controls exercised by the bank :

1. Regulation of Consumer Credit. One of the methods of selective credit controls is that of regulating the volume of consumer credit for the purpose of

durable consumer goods. Durable consumer goods, such as radios, refrigerators, television sets, motor vehicles etc., are purchased in western countries by the consumers on instalment credit system. Under this system, a certain percentage of the price of the durable consumer goods is paid by the consumers in cash. The balance is financed by the bank credit which is repayable by the consumer in instalments spread over a specified period of time.

This method was first introduced in the U.S.A. in 1941 to regulate the terms and conditions under which the credit repayable in instalments could be extended to the consumers for purchasing the durable goods. Later on, this method was adopted by Britain, Canada, Australia and Newzealand. The objective of this method is to curb the consumption of durable consumer goods which happen to be in short supply in the country. During inflation, the Central Bank can extend this control to a large number of durable consumer goods. This will reduce the demand of such goods. At a time of falling prices, the reverse is done. The Central Bank can raise the minimum cash payment limit and then the consumer will have to pay more cash from his own pocket and it will reduce the volume of credit to be extended by the banks.

This method has proved useful at a time of inflation for controlling the rising prices of consumer durables in developed countries of the West. But it is not applicable to under developed countries, because there is no such system of instalment consumer credit in force in these countries.

2. Regulation of Margin Requirement. This method too was first introduced in the U.S.A. in 1934, when the Federal Reserve System had been empowered to practise selective credit control. Under this scheme the Central Bank of a country is authorised to frame rules and regulations *to prescribe the amount that could be granted by the banks against securities. This method is certainly useful in countries where people are speculative minded.* Speaking of its desirability, Mr. Golden Wilser said, "Margin requirements have served a useful purpose and some light has been thrown upon their possibilities and their limitations as an instrument of policy." But still, it has a limitation in as much as it throws very great responsibility on the Central Bank and which may sometimes be forced to take some decision and action which will directly and immediately influence the profits and even the solvency of a large number of people and groups.

The Reserve Bank of India has been, increasingly, using this method in recent years to control advances against essential commodities like food grains, oil seeds, sugar, etc., with a view to prevent speculative dealings in such commodities.

3. Moral Persuasion. When all other methods of credit control have been used, there still remains one card to be played, and this is in the form of an appeal made by the Central Bank to the other banking institutions to persuade them not to expand credit beyond a certain limit. There is no legality involved in the same. It is only an appeal to the good sense of the banks and is successful to the extent it can exert its influence on the same. There is no binding. In the words of Burgess, *"It is an influence to be exercised with the utmost discretion and would vanish with excessive use."*

The weapon of moral persuasion *implies the issuing of direction and the tendency of friendly advices and suggestions with regard to the policies to be followed and adopted by the commercial banks in connection with credit and investment. As it is where the appeal and influence work,* it is successful and where it fails, nothing can be achieved. In the words of Clark, "Persuasion, as a means of credit control has not been successful (and) the efficacy of warning as an instrument of credit control has been very slight. It is only a moral weapon, which if heeded to, may work wonders because it appeals to the inward than to the outward, but if ignored, it is immaterial." Its success, naturally "depends largely on the prestige and personal authority of the Central Bank, the technical means and the statutory powers at the disposal of the Central Bank, the degree of co-operation between the Central Bank and Commercial Banks as well as other financial institutions and the make up of the country's banking and credit structure." This method has proved quite successful in countries like Britain, France, Holland and Sweden where the commercial banks have accepted the unquestioned monetary leadership of the Central Bank. But this method has not proved very effective in the U.S.A. where the system of unit banking is in operation.

4. Rationing of Credit. It is a method where the Central bank puts a ceiling on the amount of advances that can be made to a bank for purposes of discount and rediscount facilities on any one day. This was first used by Bank of England towards the 18th century, but became a popular weapon with the Central Banks after the First World War. Credit rationing has been extensively used in Germany and Russia. But it does not mean that it is not used elsewhere. According to Wagemann even in more primitive economic conditions, the setting of credit quotas is the only decisive method which the Central Bank has in order to prevent excessive credit demands on the part of business. Countries like Mexico and France have also used the same method.

5. Direct Action. This is another method that has been used sometimes as an alternative to bank rate or open market policy or in conjunction with them. The main purpose of this method is the achievement of the objective of credit control. Direct action may take any form. The Central Bank may refuse to rediscount bills of exchange of the member bank or it may charge penalties over and above the normal rate. But such a policy can be helpful only when the banks actually need accommodation. If their own position is so sound and they have enough resources of their own this policy would not be helpful. Rather undue interference may injure the interest of sound banking in a country.

According to De-Kock, this weapon has serious limitations. There are, however, several limitations to be reckoned with, namely, the difficulty for both central and commercial banks to make clearcut distinction between essential and non-essential industries, productive and unproductive activities, investment and speculation or between the legitimate and excessive speculation or consumption, the further difficulty of controlling the ultimate use of credit by second, third or fourth party; the dangers involved in the division of responsibility between the Central Bank and the commercial banks for the soundness of the lending operations of the latter and the possibility for forfeiting

the whole-hearted and active co-operation of the commercial banks as a result of undue control and intervention.

6. Publicity. Quite a good number of central banks have used publicity as an instrument of credit control. They regularly published statements of their assets and liabilities, reviews of credit, and business conditions and reports of their own operations, general money market and banking conditions. It has been employed quite extensively in the U.S.A., Germany and Sweden. The Reserve Bank of India also periodically publishes returns and statements about the affairs of the commercial bank. Some Central Banks use it as a matter of duty while others attach special importance to it. It may, however be pointed out that this method is very useful for highly industrialised countries, but in less developed countries, where the percentage of illiteracy is very high and where people are ignorant about the significance of banking statistics, this method is hardly of any value.

Appraisal of Credit Control Measures. From the above discussion, we arrive at the conclusion that the two types of credit control measures, quantitative as well as qualitative, are not rivals but on the contrary, they supplement each other. For successful monetary management, the Central Bank should combine the two methods of credit control in appropriate proportions. In fact, a judicious and a skilful combination of general and selective credit control measures is the right policy to follow for the Central Bank of the country.

The methods of selective control are more suitable for under-developed than for developed countries. Firstly, the methods of general credit-control are not very effective in the under-developed countries on account of the absence of well developed money markets. Secondly, it is essential to divert credit to more essential industries in under-developed countries for speedier economic development. The methods of selective credit control can prove helpful in achievement of this objective.

However, it should be noted that the power of the Central Bank to control inflation is greater than that of controlling deflation, and that also when the methods are applied quite in time. Any delay resulting from hesitation etc. is fatal to the economy of the country. The Central Bank should be above politics. It has to take into consideration only economic and financial considerations and nothing else. And when it finds that these considerations demand action that should be immediate and spontaneous only then it can be effective. If the Central Bank does not strike when the iron is hot, it will miss the bus and then it may be too late. Further, the control can be effective only when there is mutual co-operation between the Central Bank and the Commercial Banks on the one hand; and between commercial banks themselves on the other. Again, if the credit is not used by the commercial banks for genuine trade purpose, but for speculative activities, the control cannot be effective. That is why, De-Kock feels the necessity of personal element in regard to credit control.

Then again, it should be clearly understood that the psychology of the people is another important obstacle in the effectiveness of Central Bank's weapons of credit control. Human psychology cannot be changed, but it can

certainly be studied very closely. These feelings of human being – the waves of pessimism and optimism – must be studied and eschewed and action must come on the lines justified thereby. If these things are ignored, results may not be encouraging.

There is accordingly, a need for giving more powers to the Central Bank so that it can help in maintaining economic stability in the country which is its main job.

Questions

1. What is a Central Bank ? Indicate its position in the money market of a country.
2. Describe the functions of a Central Bank in the banking system of a country. How does it influence the price level ?
3. Define a Central Bank. Discuss its main functions.
4. Analyse the various functions of a Central Bank and discuss their significance.
5. Discuss briefly the principal methods adopted by a Central Bank to control credit.
6. How does a Central Bank regulate the volume of the money with the help of bank rate and open market operations ?
7. What do you mean by Selective Credit Control ? In which way are they superior to traditional monetary weapons ?

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8
INFLATION AND DEFLATION

In this chapter we shall apply the analysis of the last two chapters to the problems of inflation and deflation. Inflation and deflation are well-known terms, but there is a considerable confusion regarding their meaning. Even those people who have no idea of the precise meaning of this term, feel that it is something which creates disequilibrium and spells disaster and stresses. With the adoption of paper money towards the end of the 17th century in the USA and France, inflation began to occur at regular intervals. Inflation occurred in the USA in 1775, in Australia and S.Africa in 1848, in Germany in 1920, in Canada in 1920 and 1948, in Brazil in 1950, in India in 1973 and so on. At present almost all the countries of the world are in the grip of inflation. Inflation has been experienced by all countries regardless of their political systems. It is a fact that inflation was experienced by countries at different stages of industrialisation.

MEANING OF INFLATION

The term inflation is used in many senses and it is difficult to give a generally accepted, precise and scientific definition of the term. It is sometimes said that inflation means rise in prices. But all rise in prices may not be inflationary. The rise in prices may occur when the average cost of production increases but it cannot be due to inflation. Let us now study the definitions of inflation as given by certain monetary experts. Crowther has defined inflation as, "A state in which the value of money is falling, *i.e.*, prices are rising." In the opinion of Prof. Kemmerer, "Inflation is too much money and deposit currency, *i.e.*, too much currency in relation to the physical volume of business being done." According to Pigou, inflation takes place, "when money income is expanding relatively to the output of work done by the productive agents for which it is the payment." Hawtrey associates inflation with, "the issue of too much currency." Dr. Gregory calls it a state of "abnormal increase in the quantity of purchasing power." The Economist of London defined inflation as, "excess of demand for everything over the supply of everything." In this definition, the rise in the price level is ascribed to demand, *i.e.*, excess of demand over the supply of goods and services. All these definitions lead us to the inevitable conclusion that inflation is a state of disequilibrium in the economy.

The correct definition of inflation may be *a rise in prices or fall in the value of money when the amount of money in the country is much in excess of the physical volume of goods and services, i.e., too much money purchasing too few*

goods. It simply means that prices rise because of an increase in the volume of money as compared to the supply of goods which money helps to buy. Pigou and Keynes relate inflation to a rise in prices which comes into existence after the stage of full employment. Keynes states that the initial rise in prices upto the stage of full employment is good for the country because there is an increase in output and also in employment. All the unemployed resources are employed and maximum output of goods is made possible. The rise in prices after the stage of full employment is bad for the country because there is no corresponding increase in production or employment. Inflation, therefore, refers to a rise in price level after full employment has been attained. Keynes' description of inflation can be explained with the help of the following digram;

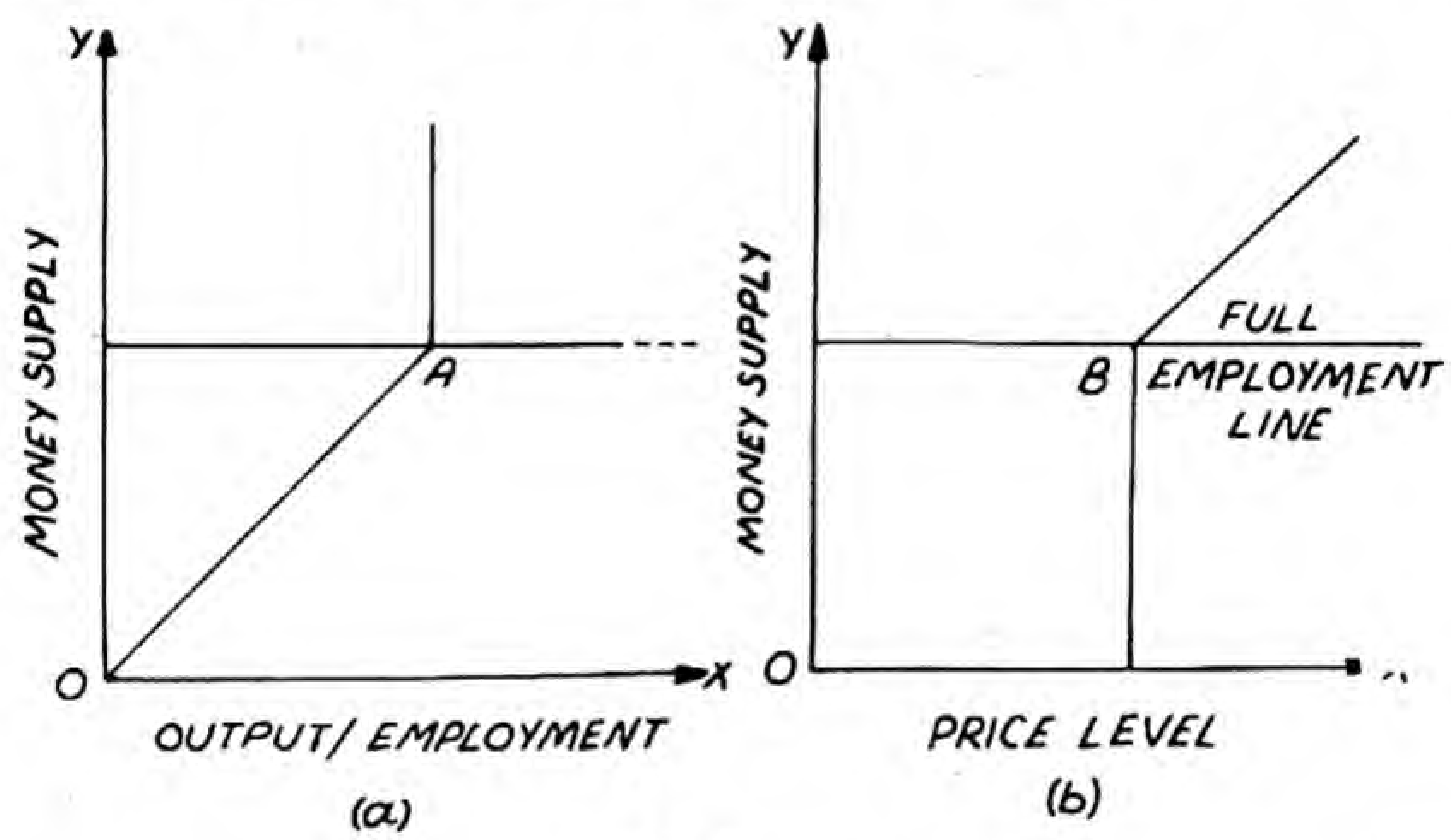


Fig. 1.

In diagram (*a*), an expansion of money supply goes to increase output and employment, not the price level. This position continues upto point A on the full employment line *i.e.,* when more and more money is injected into the economy, it leads to greater increase in output and employment. All the unemployed resources are mobilized and are fully employed. After this point is crossed, whatever additional money is issued, that goes to increase the price level, as explained in diagram (*b*) and not the output and employment. Uptill point A, output and employment increases with the increase in the supply of money and beyond that output and employment become constant. Similarly, upto point B, price level remains constant even though the supply of money increases and beyond that price level starts rising. Thus, according to Keynes, true inflation begins only when the point of full employment has been reached.

When there are higher money incomes and the consequent high demand for goods there would take place a sharp rise in prices. Unless this sharp rise in prices is checked, there would be open inflation or hyper-inflation. This is the stage when prices rise rapidly due to large scale creation of money by the Govt. and the high wages for the labourers. Such an hyper-inflation took place in Germany, China, Russia and Hungary.

In the Keynesian sense, the term inflation can be applied to an under-developed country like India where unemployment of men and materials can exist side by side with inflationary rise in prices. This is due to the existence of certain difficulties like limited amount of capital, machines, transport and communication and the absence of technical know-how. As a result of these difficulties and shortages, a rise in the price level may not lead to increased output beyond a cetain stage even though the country may not have reached the stage of full employment. Keynes says, "So long as there is unemployment, employment will change in the same proportion as the quantity of money, and when there is full employment, prices will change in the same proportion as quantity of money." He does not deny that prices may rise even before full employment, but such a situation is called inflation or semi-inflation. He has defined full employment as the point at which the true inflation sets in as is shown in the figure below :

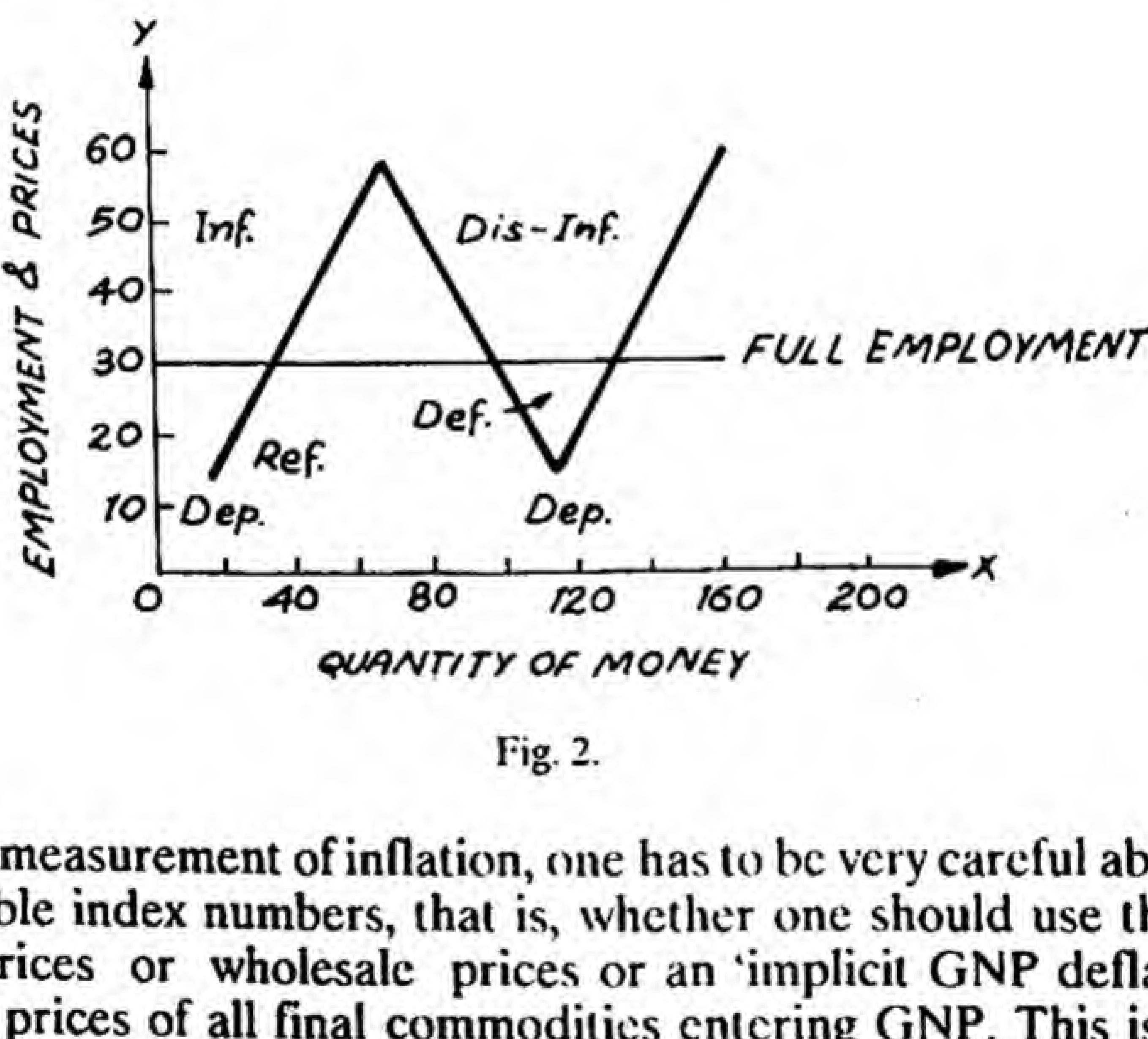


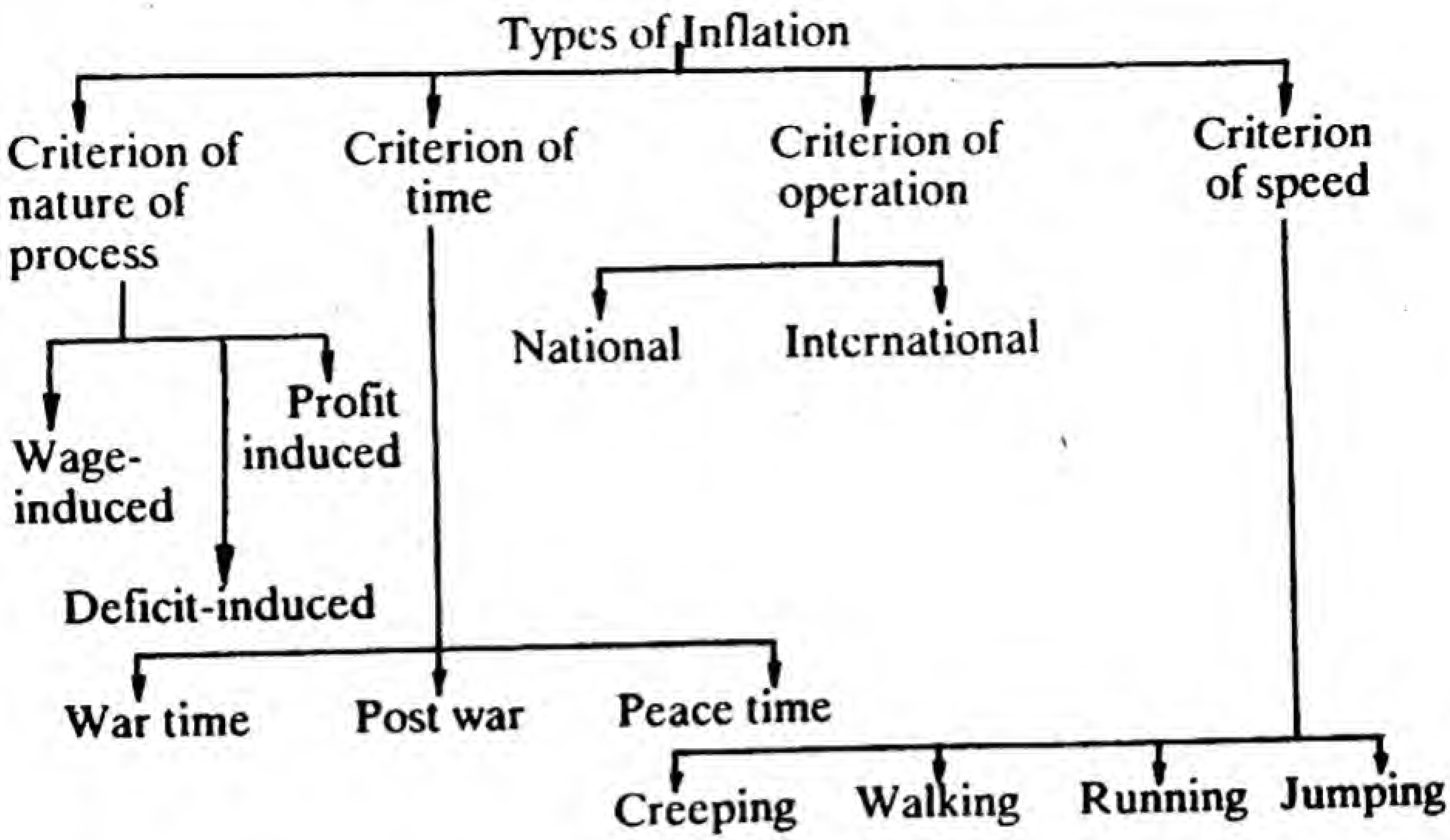
Fig. 2.

For the measurement of inflation, one has to be very careful about the use of the available index numbers, that is, whether one should use the index of consumer prices or wholesale prices or an 'implicit GNP deflator' which includes the prices of all final commodities entering GNP. This is necessary because each index may give a different measure of the rate of inflation and one's diagnosis would depend on the index that one has used. When all price-indices are continuously rising month by month and year by year at an annual rate of 4% or more, there cannot be any doubt that the economy is heading towards inflation.

TYPES OF INFLATION

Inflation can be classified under different headings : –

(*a*) Criterion of nature of Process, (*b*) Criterion of Time, (*c*) Criterion of Sphere of Operation, and (*d*) Criterion of Speed. This classification can be explained with the help of the table given overleaf :



Let us now analyse them one by one :

(*a*) On the basis of the first consideration, inflation has been classified into three kinds : –

(*i*) Wage-induced inflation, which occurs on account of an increase in many wages;

(*ii*) Deficit-induced inflation, which is caused by the adoption of deficit financing or by the government spending excess of revenue receipts; and

(*iii*) Profit-induced inflation, which is caused due to the increase in profit-margin.

(*b*) The second consideration is the time element, according to which inflation may be classified into *war period inflation, post-war inflation and peace-time inflation*. It is generally seen that the demon of inflation raises its ugly head during the period of war and the postwar inflationary trend is the direct legacy of the war-period inflation. During the period of war, currency and credit are inflated for financing the war projects. During peace times, the supply of money is increased in order to avoid the evil consequences of depression; and the post-war inflation (which may rightly be called "Planned inflation") aims at developing the war-torn economy of a country.

(*c*) Based on the criterion of the sphere of operation, inflation may be classified into *national inflation and international inflation*. If inflation is limited within the bounds of a country, it is of national type and when it envelopes the entire world, it becomes international in character.

(*d*) The last classification to be examined is based on the criterion of speed or rapidity with which the price level rises. According to this consideration, inflation has been divided into the following four types :

(*i*) *Creeping inflation,* (*ii*) *Walking inflation,* (*iii*) *Running inflation, and* (*iv*) *Jumping or Galloping or Hyper-inflation.*

Creeping inflation is the mildest form and is conducive to economic progress and growth. In this form the prices rise imperceptibly over a long period. Creeping inflation has been suggested by some economists for a developing economy. It keeps the national economy free from the cankering effects of stagnation and allows constant but slow and secular rise in prices. Some economists feel that this type of inflation is very dangerous for the growth of the economy; and if suitable steps are not taken to regulate its rapidity, it may, in due course of time develop into walking and running types. The inevitable conclusion is that the creeping inflation should not be allowed to persist too long, but must be nipped in the bud. In case of walking inflation, rise in prices becomes more marked as compared with the situation obtaining under creeping inflation and in fact, it is a danger signal of the occurrence of running and jumping inflations under which the rise in prices takes place by fits and starts. It becomes very difficult under these circumstances to measure the magnitude of inflation. Jumping or Hyper-inflation is the last stage which starts after the point of full employment has been reached.

Lord Keynes called it full inflation or true inflation. It has been mentioned earlier that this classification depends upon speed or rapidity with which prices shoot up. Whereas in case of creeping inflation, prices increase by 50 per cent in a country in a period of 25 years, in case of walking, running and jumping inflations, the same increase in price may take place in 10, 5 and 3 years respectively. In fact, these types of inflation represent the stage of the development of an infant. In the beginning, the toddler creeps and after it has learnt creeping, it begins to walk and then with the gradual development in physique it runs and jumps. The diagram ahead is illustrative of these types of

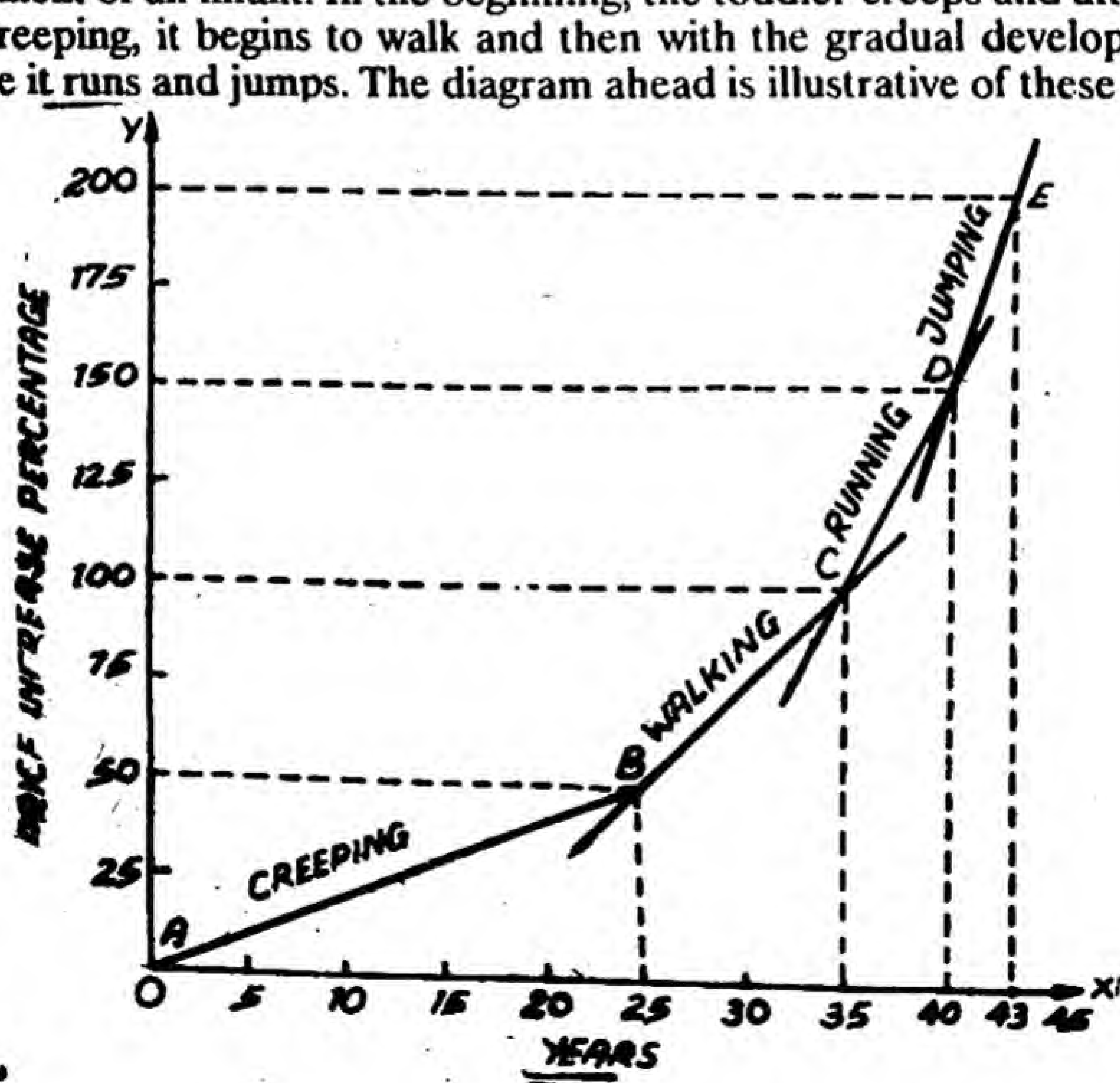


Fig. 3.

inflations. In the first period of 25 years, prices of commodities rise by 50%, the line AB representing creeping inflation. In the second period of 10 years, prices rise by another 50%, the line BC representing walking inflation. In the third period of 5 years, prices rise further by another 50%, the line CD indicating running inflation : and in the fourth brief period of 3 years, prices jump by another 50%, the line DE representing jumping or hyper-inflation. Thus, the entire period covered by these four types of inflation is 43 years (25 + 10 + 5 + 3) of the total increase in prices 200 per cent.

THEORIES OF INFLATION

Inflation is basically the result of disequilibrium between demand and supply forces. There are two schools of thought regarding the possible causes of inflation. One school of thought considers excess demand as an important cause of inflation, another school of thought holds that inflation is mainly caused by the cost push element.

Demand Pull Inflation

According to this theory, inflation is that situation, where at a given price level total demand is more than total supply. Thus inflation is caused by excess of demand. In words of Peterson, "Demand-pull inflation theory holds that inflation is caused by an excess of demand (spending) relative to the available supply of goods and services at existing prices." In words of Shapiro, "According to demand pull inflation, the general price level rises because the demand for goods and services exceeds the supply available at existing prices." There are three approaches to demand pull inflation.

1. Quantity Theory of Money. According to this theory after full employment level, there is direct and proportional relationship between supply of money and the price level. If the money supply is doubled, prices will be doubled, if money supply is halved, prices will be halved. To explain in terms of an equation $MV \equiv PT$, where T is fixed bacause output cannot be increased after the full employment level. If T is constant, P changes as MV (total demand for goods) changes. In 1849 the discovery of gold mines in California and in 1922, a large increase in money supply in Germany led to inflation in these countries.

2. Keynesian Theory. According to Keynes, when aggregate demand is more than aggregate supply, there is increase in prices. $AD = C + I + G$. So whenever there is an increase in consumption, investment or government expenditure, there is a tendency for prices to rise. According to Keynes, consumption is stable in the short period, so inflation is caused by increase in investment or government expenditure.

3. Modern Quantity Theory. This theory is associated with the name of Prof. Milton Friedman. According to this theory, increase in money leads to inflation. This theory does not assume full employment and constancy of velocity. But still it stressed that money supply is an important determinant of inflation. According to this theory, the demand

function of money in real terms $\left(\frac{M}{P}\right)$ is stable. Given the demand function of money, the change in money supply determines the inflation rate.

The traditional theory and modern theory stress that increase in supply of money lead to an excess of demand. Besides supply of money, disposable income, consumer expenditures and business outlays and foreign demand are also inflationary factors on the demand side. This demand pull inflation can be studied with the help of a diagram.

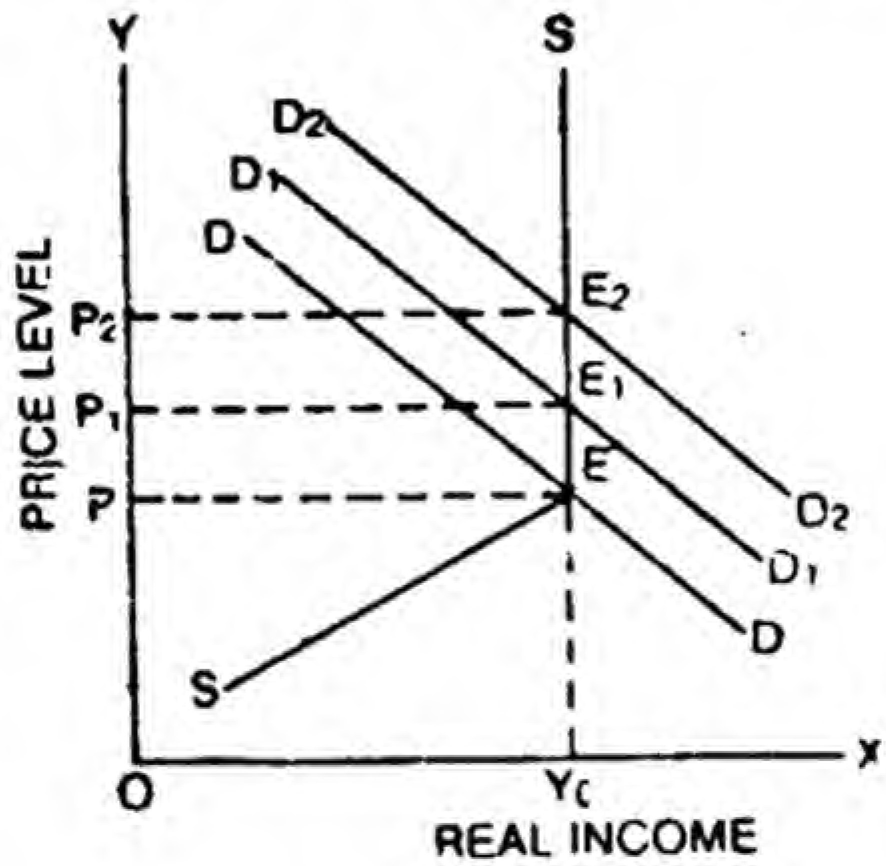


Fig. 2

In Fig. 2, real income has been taken on X-axis and price level on Y-axis. SS is the suppy curve which rises upward from left to right from S to E and after that it become vertical at E which shows full employment DD, D_1D_1 and D_2D_2 are the different demand curves. When DD is the demand curve, it is equal to supply curve at E point at OY_0 level of income. It is also the point of full employment. Given the aggregate demand DD and the full employment output or real income OY_0, the equilibrium point is E, whereas price level is OP. Supply curve being perfectly inelastic after the full employment point E, every shift in demand curve only raises the price level (real income being constant).

Causes of Demand Pull

(i) Increase in Money Supply. There may be in increase in money supply either when more currency is issued or banks create more credit. The increased money supply means increased purchasing power. This raises demand without increasing the supply of goods and services, leading to rise in price-level.

(ii) Increase in Public Expenditure. Today's state is a welfare state. Besides administrative function, state performs many welfare functions. Thus there is increase in public expenditure which increases money income of people. This increases demand, raise prices causing inflationary tendencies.

(iii) **Cheap Money Policy**. If monetary authority adopts cheap money policy, this also leads to excess of demand. On the one hand, lower bank rate makes borrowing cheap and on the other hand, expansion of credit increases money supply. In this way cheap money policy also increases demand.

(iv) **Increase in Disposable Income.** If there is net addition in disposable income, the demand for goods and services will increase. Disposable income might increase because the rate of tax has been lowered or a tax has been withdrawn.

(v) **Black Money**. Black money is the unaccounted money which is earned through transactions. Black money means tax evasion. People spend this money on conspicuous consumption. This raises demand and hence price-level.

(vi) **Increase in Investment**. Another reason of demand pull inflation is increase in investment. When producers have high expectations and they expect high profit, they increase the volume of investment. This increases the demand for factors of production. The factor prices increase, raising the product prices as well.

(vii) **Reduction in Taxes**. Reduction in taxes increases disposable income, thus demand for goods and services increases causing prices to rise.

(viii) **Foreign Demand**. An additional factor in the increased monetary demand is foreign expenditure for domestic goods and services when the demand for exports increases, the income of export goods producing industries increases. This is turn would create more demand for goods and services within the economy.

(ix) **Increase in Population**. The rapid growth of population also raises effective demand by increasing consumption, investment, government expenditure and net foreign expenditure. This also increases inflationary pressure in the economy.

(x) **Repayment of Old Debts**. The government may be repaying old debts. This increases the purchasing power of the people.

(xi) **Dishoarding**. Hoarded money may be brought into the market in the form of greater demand for consumer goods and for investment goods, such an increase in demand may push up prices.

(xii) **Economic Development**. As a country develops there is an expansion of monetised sector. People do more and more transactions with the help of money. If money supply expands faster than the increase in output, prices rise.

COST-PUSH INFLATION

Another theory of inflation relates to cost-push inflation. This type of inflation emerges due to increase in the costs, though for a common man, the explanation of inflation is to be found in rising of prices only. In

words of Bronfenbennex and Holzaman, cost-inflation has been the layman's instinctive explanation of general price increases since the dawn of the monetary system. We do not know of any inflationary movement for which blame has not been put by some people on profiteers, speculators, hoarders, workers and peasants who live beyond their situation. Cost-push inflation is called seller's inflation by A.P. Lerner. Under it a country may experience stagnation *i.e.* rising prices with stagnation of growth and employment, with some unemployment seen in the country. According to Horvitz, "Cost push inflation arises when firms raise prices because of an increase in their cost although there was no excess demand for their product at the original price." According to Wykstrait, "describes the administration of wages and prices by powerful monopoly groups who generates 'cost-push' pressures in resource and product markets". In words of Ranlert, "The basis of cost-push theory of inflation is that organised groups, both business and labour, establish higher prices for their products or service than would prevail in perfectly competitive markets."

In this, inflation results from an increase in the cost of production-like rise in the cost of raw materials and especially of wages. When labour market is imperfect and some strong trade unions are successful in getting higher wages, without increase in productivity, such a situation emerges. Increase in wage rate leads to higher cost of production thus raising price level. There is increase in unemployment. This type of inflation is also called wage-induced inflation. When prices start increasing, trade unions demand still higher wages thus increasing costs and prices all together.

When firms in imperfect competition market want to have a higher profit margin, they increase the prices. This is known as mark-up prices. The production is also reduced. Thus because of fall in production and increase in profits, costs increases, increasing the prices. This inflation is also known as profit-induced inflation.

Thus cost-push inflation is the result of increase in wages and increase in profits which increase cost of production and hence prices. It can be explained with the help of a diagram. (Fig. 3). In Fig. 3, real income is taken on the X-axis and price level on the Y-axis. SS is the original supply curve and DD is original demand curve. The equilibrium is attained at E point at OY level of income and OP price. This is also the point of full employment. If cost of production increases either due to increases in wages or profits, supply curve shifts upwards and the new supply curve is S_1S. The new supply curve intersects demand curve at E_1 point, where the real output is reduced from OY to OY_1 and prices are increased from OP to OP_1. Similarly if supply curve shifts still further, the new point of equilibrium is E_2 at OY_2 level of income and OP_2 level

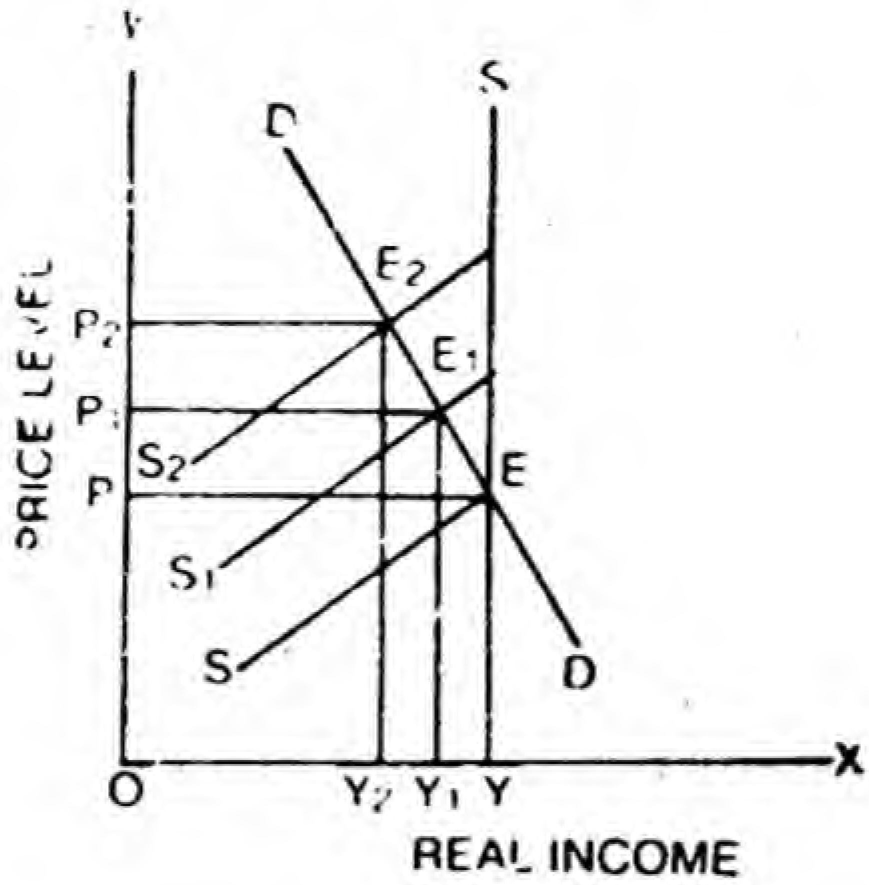


Fig. 3

of prices. Thus we find that on the one hand, price-level rises but on the other hand output and employment level falls.

Factors Affecting Cost-Push Inflation :

(i) Less Production. The major cause of cost-push inflation is shortage of supply of goods and services as compared to their demand. If production is less than the demand, prices are bound to rise.

(ii) Hoarding and Speculation. Sometimes producers and traders may stock commodities to charge higher prices in future. This creates artificial scarcity and prices rise.

(iii) Wage and Cost Spiral. Strong trade unions may be able to increase their wages without an equal increase in productivity. Wage-rise pushes up costs which in turn push up prices, which again push up, wages, costs and prices.

(iv) Taxation Policy. Indirect taxes raise the prices of commodities. If the commodity is subject to the law of increasing returns, prices may rise by more than the amount of tax because less is produced as the demand has gone down due to rise in prices, the cost of production rises.

(v) Industrial Relations. If there is a lack of industrial peace in the country, strikes and lock-outs are very common resulting in fall in production and rise in prices.

(vi) Technical Changes. When some new technique of production is adopted, it takes some time for the production process to be adjusted. In the transitional period, the production may fall but workers and experts have to be paid the same wages. Thus cost of production rises raising the price level.

(vii) Lack of Raw Materials. When there is a shortage of raw materials, cost of production increases, hence the prices.

(viii) War. During war, needs of the army and defence have to be met with first. Thus the supply of goods for civilian consumption is reduced. Thus prices go up.

(ix) **Exports.** If those commodities, which have a strong domestic demand, are exported, it reduces the supply of goods in the country for domestic consumption. This scarcity of goods in the country would raise the price level.

(x) **Sympathetic Price Rise.** Prices rise abroad and domestic prices begin to move up in sympathy the link is provided by international trade. More goods may be exported to earn higher prices in other countries. The reduced supply within the country causes the prices to register a rise.

(xi) **Devaluation.** It means the exchange rate of currency in terms of the currency of another is reduced. It makes imported goods more costly. Exports are encouraged. The supply of exported articles within the country is reduced and price of exported goods go up. Thus the commercial policies of the governments may also lead to inflation.

(xii) **Natural Calamities.** There may be too much rain or too little rain. Agricultural goods decline in supply and their prices rise.

(xiii) **Structure of Production.** Sometimes structure of production is such that producers produce more of luxury items and expensive items which give them higher profits and less of consumption goods. Then there is increase in prices of consumption goods and inflationary pressure sets in.

(xiv) **Industrial Policy.** If industrial policy is such that it discourages the establishment of new enterprises either by imposing heavy taxes or by giving expensive credit, production will be reduced, leading to rise in prices.

INTERACTION OF DEMAND-PULL AND COST-PUSH INFLATION

Sometimes problem arises as to whether inflation is a demand-pull inflation or a cost-push inflation. We know that general price-level is determined by the general demand for goods and services on the one hand and the cost of production on the other, since price in free market economy is fixed by demand and supply forces. The explanation of price level must lie either on the demand side or the supply side or both—the responsibility being divided between the two according to the weightage. If the weightage be in favour of demand, it may be called demand-pull inflation and if it is on the side of supply, it may be called cost-push inflation—when supply fails to keep pace with the increased demand due to mounting cost of production. Mounting cost of production is the sure index of the enhanced scarcity of factors. However, demand and cost are not independent and exclusive entities. What is cost to the entrepreneurs is income to the sellers of the factors and that income is itself responsible for the increase in demand. Costs and incomes are the obverse of the same flow of money—the one glanced upon from the angle of buyers of the factors and the other from that of the sellers of the

factors. All the same, demand-pull inflation and cost-push inflation can be distinguished in the initial stages. When the process starts, it may be rather difficult to make any such distinction between them for both the causes feed each other and merge together to intensify the inflationary pressure. The combined effect of demand pull and cost-push inflation can be studied with the help of following diagram. (Fig. 5)

Real income has been taken on the X-axis and price level on the Y-axis. SS is the original supply curve and DD is the original demand curve SS and DD are equal at E, so this is point of equilibrium at OY level of income and OP level of prices, suppose cost of production increases and the new supply curve is SS demand curve remaining the

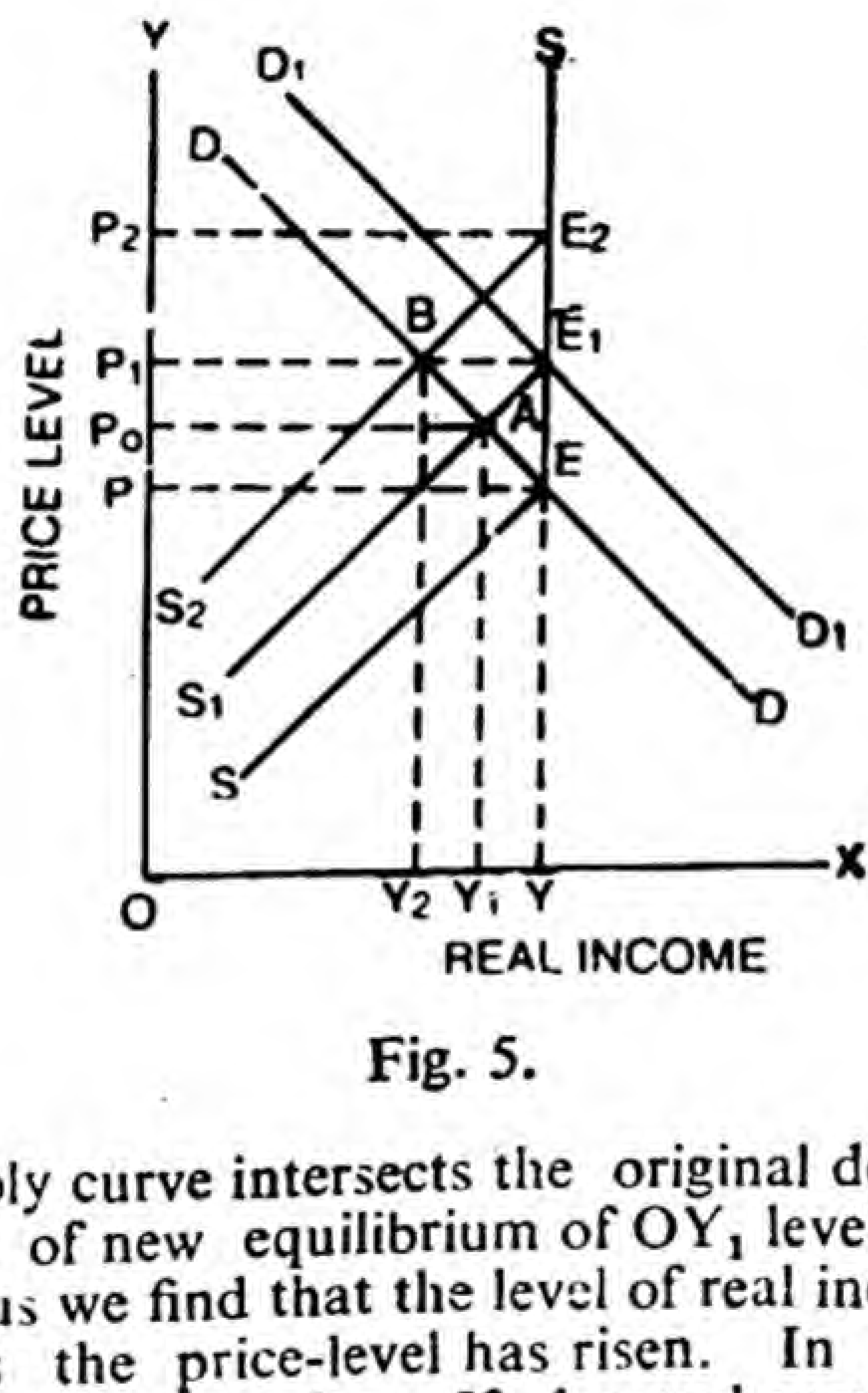


Fig. 5.

same. The new supply curve intersects the original demand curve at point A. This is the point of new equilibrium of OY_1 level of income and OP_0 level of prices. Thus we find that the level of real income or employment has fallen whereas the price-level has risen. In order to achieve full employment, demand has to rise. If demand curve shifts from DD to D_1 D_1, the full employment equilibrium is achieved at point E_1. However, it is possible only at a higher level of prices. The increase in price level would force the trade unions to demand higher wages. It will raise cost of production shifting supply curve to S_2S, which intersects the original demand curve at B at OY_2 level of income showing that employment has fallen. This will further raise demand function. In this way, we find that inflation is an interaction of forces both on the demand side and supply side.

EFFECTS OR CONSEQUENCES OF INFLATION

In words of C.N. Vakin, "Inflation may be compared to a robber. Both deprive the victim of some possession with the difference

that robber is visible, inflation is invisible, the robber's victim may be one or a few at a time, the victim of inflation is the whole nation, the robber may be dragged to a court of law, inflation is legal." However, inflation is not so bad so long as it creates additional employment of factors of production, but becomes so the moment it goes out of control and robs. Let us see what are the effects of inflation on main economic activities.

1. Effects on Production. At the first glance it appears that increasing prices favour production because in a free enterprise economy, production depends on profits which increase during the period of rising prices. The growing profits provide a great stimulus to the producers to undertake large scale investment. Mushroom growth of production units comes into being. Demand for factors will increase leading to rise in income and further rise in demand. This will reinforce inflationary pressure as wage-price spiral will appear *i.e.* cost-push and demand-pull inflations might result in high rate of inflation. Production will be impaired. In that atmosphere people have a feeling of diffidence as regards the value of money and hence they try to get rid of it and prefer to store goods rather than money. "Unless a merchant is paid in some physical unit of some commodity, he will prefer to hoard his stock of goods rather than make the exchange." Eventually there comes a crash which leads to fall in prices, profits, employment, income and demand. In this way inflation has adverse effects on production. The adverse effects are as follows :—

(i) Adverse Effect on Capital Formation. Since savings are adversely affected by inflation, the process of capital formation is also adversely affected.

(ii) Increase in Speculative Tendencies. Since inflation generates uncertainity, businessmen start concentrating on quick income yielding activities. Thus the resources are diverted from productive activities to speculative activites. Thus production is adversely affected.

(iii) Hoarding. Since value of money falls rapidly, every one want to stock goods. This creates artificial scarcity of goods in the market and encourages black marketing.

(iv) Pattern of Production. The pattern of production changes in such a way that resources are diverted from essential goods to luxuries. This adversely affected the economy.

(v) Deterioration in Quality. In order to make a gain out of increased demand and reduced supply, producers lower the quality of the product.

2. Effects on Distribution. If inflation had affected each individual equally, there would not be any grudge and inconvenience. But actually inflation affects the different classes differently. A person is affected by inflation in his capacity as a wage-earner, entrepreneur, stock-holder, bond-holder, debtor, creditor and to payer and in each cate-

gory he is treated differently sometimes favourably and sometimes unfavourably. Those people whose income keeps just pace with rising price will have the same real income. Those people whose income lags behind rising prices will get diminished share of total output. In fact, the effect of inflation on different people depends on the relative movement of the rewards of different factors of production due to inflation.

(i) **Debtors and Creditors**. During inflation the debtors gain and creditors lose because inflation transfers wealth from creditors to debtors. When prices rise, money buys less goods. Thus though the debtors return the same amount of money, they return less in terms of goods. If you lend me Rs. 1000 today and the price level doubles, when I repay, in fact I repay ½ of the real purchasing power I had borrowed.

(ii) **Investors**. Investors are generally of two types—investors in equities (shares) and investors in fixed interest yielding bonds and debentures. Inflation favours the former and hits hard the latter. When prices rise, the returns on equities go up on account of increased profits while bonds and debenture holder gain nothing as their income remains the same.

(iii) **Entrepreneurs.** Entrepreneurs whether they be manufacturers, traders, speculators, merchants or businessmen all gain from inflation. These people gain during rising prices because (a) they are mostly debtors and debtors gain, (b) their profits rise because they add a standard percentage mark upon their costs in setting the prices, (c) their payments for productive factors *i.e.* their costs lag behind rising selling prices *i.e.* profits rise more rapidly than price, (d) they buy raw materials and other goods at old and lower prices and sell when prices rise higher.

(iv) **Wage-earners.** Wage-earners generally lose purchasing power when prices go up and gain it when prices go down, for wages move up and down more slowly than prices. Moreover, there is a lag between change in prices and change in wages. If workers are well organised, they may not suffer much during inflation but if they are unorganised as agricultural workers are in India, they may suffer more, for they may not find it easy to get enhanced wages. However, even though wages lag behind rising prices. the real income of labour as a group often rises in inflation because higher prices come with better business conditions which make more jobs available.

(v) **Fixed Income Group or Middle Class**. The most hard hit section of the society during inflation is the fixed income group usually called the middle class who lives on past savings, fixed interest or rent, pensions, fixed salaries etc. while price rise, their incomes do not, resulting in a decline in the real value of their inelastic incomes. According to Kemmerer, "The middle class, however, by hard work and thrift has built up a fund of savings to educate its children and to provide a livelihood for times of sickness and for old age, finds itself in a desperate situation in a times of serious inflation."

(vi) Farmers. Farmers really gain from rising prices. Price of farm goods usually rise faster in inflation than the prices farmers pay. Thus their real income generally increases in the inflationary periods. Moreover, the farmers generally being debtors pay less in real terms. Land revenues and taxes etc. do not rise much. However, small farmers do not gain as much as big farmers because they do not possess large marketable surplus.

(vii) Tax payers. During inflation, tax payers gain. Though they may pay slightly higher taxes, they hand over less (real) amount in terms of goods than before. The income tax payers gain for they pay in rupees whose value has depreciated as compared with the value of the income when earned.

(viii) Government. State can be put in the category of fixed income group and as such, loses when prices rise. The state has to spend more on goods and services including raw materials for carrying on their projects. But on the other hand the yield from taxes increases even if the rates of taxes remain the same. Moreover, rising prices decrease the real burden of public debts both external and internal.

3. Effect on Foreign Trade. Inflation is incompatible with the promotion of free international trade. A rising prices produce an excess of imports over exports and a drainage of gold. State that cannot curb this price rise may turn to tarrifs and quota systems to avert further drains of foreign exchange. But this restriction on trade will reduce real income and further inhibit the growth potential of the economy.

4. Social Consequences. So far we have not considered social consequences of inflation. But social and ethical consequences are not less serious. Unstable prices generate social unrest and distress. When prices rise, workmen cry for wage increases as their cost of living increases, which usually results in strikes.

5. Moral and Ethical Consequences. Inflation breeds moral degeneration among the people in general and businesses and officials in particular. The gambling spirit dances and corruption, black marketing and bribery are common. Regarding the moral degeneration caused by currency inflation during the French Revolution, Andrew White writes, "In the country the gambling spirit spread more and more......Nor was this reckless and corrupt spirit confined to businessmen, it began to break out in official circles and public men who, a few years before, had been thought above all possibility of taint, became luxurious reckless, cynical and finally corrupt." During post war hyperinflation, "it was a terrible sight to see thousands of men dressed as women and thousands of women dressed as men, dancing in the pervert ball room of Germany in the presence of city police." Dr Murangan quotes in his book 'From Hyperinflation to Devaluation', "Young girls bragged proudly of their perversion, to be sixteen and under the suspicion of virginity would have been considered a disgrace in any school of Berlin at that time, every girl wanted to be able to tell of her adventures, and the more exotic, the better."

6. **Political Effects**. Inflation reduces faith of the people in the government. The dissatisfaction among the masses may result in revolution or toppling of the government. It was the hyper inflation in Germany during 1920's which made Hitler a dictator. According to Dr. D.B. Turroni. "Hitler was the foster child of inflation."

STAGFLATION

Stagflation is a new term which has been added to economic literature in the 1970s. It is a paradoxical phenomenon where the economy experiences stagnation as well as inflation. The word stagflation is the combination of 'stag' plus 'flation', taking 'stag' from stagnation and 'flation' from inflation.

Stagflation is a situation when recession is accompanied by a high rate of inflation. It is, therefore, also called inflationary recession. The principal cause of this phenomenon has been excessive demand in commodity markets thereby causing prices to rise, and at the same time the demand for labour is deficient thereby creating unemployment in the economy.

Three factors have been responsible for the existence of stagflation in the advanced countries since 1972. First, rise in oil prices and other commodity prices along with adverse changes in the terms of trade; second, the steady and substantial growth of the labour force; and third, rigidities in the wage structure due to strong trade unions.

Stagnation is not peculiar to advanced countries. India has also been experiencing it. A number of industries like textiles, textile machinery, steel, tyres, tractors, commercial vehicles, general engineering, etc. have been facing fall in production and rise in labour unemployment. On the other hand, inflation has been continuing.

Causes of Inflation. Inflation always gathers momentum and prices rise in a vicious circle. Whatever maybe the cause, once inflationary rise in prices takes place, it will continue to do so. Causes of inflation may be different in different countries, but broadly speaking, the following are the main causes of inflation :

1. *Deficit Financing.* In time of war or under other abnormal situations requiring a huge increase in government spending, the Government for meeting this expenditure may resort to deficit financing which increases the money supply in the hands of the people, thereby increasing their demand for goods and services. In most of the developing countries, the governments have to obtain funds for executing the development plans through deficit financing and this has led to a situation of inflation.

2. *Increase in the Velocity of Money.* During periods of boom and prosperity, owing to an increase in the MEC and MPC, the velocity of circulation of money increases. Higher velocity of money results in higher prices.

3. *Expansion of Credit.* The expansion of credit may be resorted to either as a matter of policy by the Government or by the commercial banks of the country. The Central Bank can expand credit by lowering the bank rate or by

purchasing government securities. The commercial banks can expand credit by lowering the cash reserves. Expansion of credit is generally resorted to in periods of increasing economic activity and once it starts, it continues for a sufficiently long period of time. With expansion of credit, prices start rising.

4. *Increase in Public Expenditure.* Aggregate demand may also increase as a result of an increase in public expenditure either for meeting the requirements of defence, or of economic development or for boosting the level of economic activity in the economy. With the increase in demands for goods and services, prices increase. Similarly, with the increase in private expenditure, there is an increase in the demand for the services of factors of production. This results in an increase in factor-prices which leads to an increase in expenditure on consumption goods and rise in prices.

5. *Expansion of Exports.* If the export of commodities increases, less goods are available for domestic consumption. This would make the existing demand at home excessive of the available quantity of goods leading to increase in their prices. Increase in exports creates a situation of shortages in the economy giving rise to inflationary pressures.

6. *Increase in Population.* Increase in the population of a country raises the general level of aggregate demand of the people for goods and services. As Prof. Coulborn states, "If population increases rapidly while the aggregate volume of money remains stable, the consequent rise in the velocity of circulation is likely to outweigh the countervailing decrease in the volume of money per head; further, a rapid increase of population may increase output less than proportionately-another factor tending to raise prices."

7. *Trade Union Activities.* These days trade unions are very strong. They continuously agitate for higher wages, shorter hours of work, more holidays with pay and other amenities. In a democractic country, the Government is often compelled to accede to the unreasonable demands of the workers. The increase in wages increases the purchasing power of the workers and hence the aggregate demand which leads to rise in prices.

8. *Reduction in Taxation.* The reduction in taxation offered by the government can also be an important cause for the emergence of excess demand in the economy. When the government reduces taxes, it results in an increase in purchasing power in the hands of the public. With increased purchasing power, the people are in a position to buy more and more of goods and services for private consumption.

9. *Shortage of Supplies of Factors of Production.* Occasionally, the economy of a country may be confronted with shortages of such factors as labour, capital equipment, raw materials, etc. These shortages are bound to reduce the production of goods and services for consumption purposes. In fact, the shortage of productive factors is a serious obstacle to any effort to increase production in the country. Trade union activities also lead to decline in production and rise in prices.

10. *Hoarding by Traders and Consumers.* At a time of shortages and rising prices, there is a tendency on the part of traders and merchants to hoard

essential commodities for profiteering purposes. The stocks of essential goods often go underground during a period of inflation and rising prices, causing further scarcity of these goods in the market. It is not only the traders and the merchants who resort to hoarding at a time of inflation, the individual consumers also hoard essential commodities to avoid payment of higher prices in future. They also hoard essential commodities to ensure their uninterrupted availability for private consumption. This leads to still greater rise in prices.

11. *Natural Calamities Reduce Production.* Natural catastrophies may also bring about inflationary conditions by decreasing the volume of production. Floods, earthquakes and other natural calamities of the kind will decrease the production and will bring inflationary effects.

Thus, due to all the above mentioned factors prices continue to rise and gather speed till a crisis is created.

Effects of Inflation. If prices of all groups such as agricultural, industrial, wages, rents, etc. rise in the same direction and to the same extent, there will be no net effect on any section of people in the community. In practice, however, all prices do not move in the same direction and to the same percentage. Hence some classes of people may be affected more favourably than other sections in the community.

All producers, traders and speculators gain during inflation because of emergence of the wind-fall profits. Prices of goods rise at a far greater rate than costs of production because wages, interests, etc. are more or less fixed. There is also time lag (gap) between the rise in prices and the rise in cost of production. Thus, the producing and trading classes gain abnormally during the period of inflation. Debtors also gain because they had borrowed when money was dear and are returning it when it is cheap. Agriculturists gain because they pay fixed rent but get higher price for their commodities.

No doubt, inflation has certain good effects but evil effects outweigh them. Whereas the vast majority of consumers suffer, only a few persons gain. People have to feel innumerable hardships because money is not properly controlled. Prof. C.N.Vakil has compared inflation to robbery. He observed that "Both deprive the victims of some possession with the difference that the robber is visible, inflation is invisible; the robber's victim may be one or a few at a time, the victims of inflation are the whole nation; the robber may be dragged to a court of law, inflation is legal."

Inflation is very severe on those persons living on past savings, fixed interests and rents, on pensions and other fixed income groups – generally called the middle class. Those persons who are working in Government and private concerns find their money income more or less fixed while the prices of the goods and services which they buy are rising very rapidly. Those with absolute fixed incomes derived from interests and rents – known as rentier class-find that their money income is worthless and their past savings have no value. In fact, the worst sufferers of inflation are the middle classes which are considered as the backbone of any stable society The working classes also suffer because wages do not rise as much as rise in the prices, and also because of the time-lag between the price level and rise in the wages.

Continuous inflation shakes the foundations of the political and economic stability of the system. The German inflation after the First World War was an important factor in the fall of Dr. Cuno's liberal government and in Hitler's coming to power. People wanted anyone who could bring back stability in the system.

Inflation brings about shifts in the distribution of income between different sections of people in the country. The producing classes gain at the cost of salaried and working classes. The rich become richer and the poor, poorer. Inflation, therefore, is considered unjust because it distorts the distribution of income between people. Besides, those who are hard hit by inflation are the young, the old, the sick, the widows and the small savers i.e., all those who are unable to protect themselves. The community as a whole suffers a great loss because of unprecedented decline in capital accumulation resulting from a decline in the willingness and capacity of the investors to save.

Inflation breeds moral degeneration amongst the people, particularly businessmen and officials. The gambling spirit spreads and corruption becomes common. Black-marketing and bribery benefit the profiteers and executives and ruin the consumers. People are robbed of their savings accumulated through hard work as they lose all confidence in the government. This happened during the days of inflation in France during the famous French Revolution.

Thus, inflation will endanger the very foundations of the existing social and economic system. It will create a sense of frustration and distrust, of injustice and discontent and lead people to a state of revolt. It is, therefore, economically unsound, politically dangerous, socially disastrous and morally indefensible. It should be controlled at the earliest.

CONTROL OF INFLATION

Inflation should be controlled in the initial stages otherwise it will take the shape of hyper-inflation which will completely ruin the economy. Inflation is a hydra-headed monster and should be fought with many weapons. Anti-inflationary measures are generally of three types :

(a) Monetary policy,

(b) Fiscal policy, and

(c) Other measures (Miscellaneous).

(a) Monetary Policy. This policy relates to the currency and credit structure of a country. Monetary policy which consists in controlling the supply and cost of money by the Central Bank is enforced by using different monetary instruments which while aiming at reducing the supply of money for speculative activities also raises the cost of obtaining funds from the banking system for hoarding and stock piling of essential goods which are in short supply. The Central Bank may use such methods as the Bank Rate Policy, Open Market Operations, the Reserve Ratios and Selective Control in order to control the credit creation operations of commercial banks and thus restricts the amount of bank deposits in the country. The bank rate is raised to limit investment and the Central Bank starts selling Government securities.

Another measure connected with the monetary policy of a country is related to the blockade of currency and bank accounts. It amounts to the withdrawal of purchasing power from the hands of the people. It is a very drastic action and is not generally followed by the Government. Japan, Russia, Poland etc. adopted these measures to check inflation.

Monetary measures for checking inflation have certain limitations. It is difficult for Central Bank to know exactly when to adopt a particular measure. Sometimes higher rate of interest depresses economic activity. The burden of public debt also increases. Moreover, large sales of government securities by the Central Bank weaken the money market. In many countries of the world including India dear money policy and Central Bank's restrictive credit policy have failed to prevent prices from rising.

(b) Fiscal Policy. It is the policy of the Government with regard to taxation, expenditure and public borrowings. It has a very important influence on business and economic activities. The proper tax policy to control inflation will be to avoid tax cuts, introduce new taxes and raise the rates of existing taxes. The purpose being to reduce the purchasing-power in the hands of the public and thus reduce the demand. But taxes should not go beyond reasonable limits otherwise capital formation will be discouraged. Therefore, a suitable tax policy directed towards restricting demand without discouraging production is a good fiscal measure to control inflation.

Similar effect will be achieved if voluntary or compulsory savings are resorted to. Savings will reduce current demand for goods and thus reduce the inflationary rise in prices. A higher rate of interest and prize-bond schemes etc., are great inducement for savings deposits.

As an anti-inflationary measure, Government expenditure may be reduced. It is, however, difficult to reduce public expenditure especially during war and during a period of economic development. Again Government should go in for public borrowings in order to reduce the spending capacity of the people. The amount thus collected should be spent on productive purposes.

Fiscal methods, however, do not cure inflation. They only suppress the inflationary pressure and postpone the evil days. Keynes wants the adoption of both fiscal and monetary policies together to check inflation. A judicious combination of monetary and fiscal policies requiring close cooperation between the Central Bank and the treasury can prove effective in controlling inflation in the economy if applied at the proper time.

(c) Other Measures (Miscellaneous). Among other anti-inflationary measures adopted by various countries, price control and rationing are generally useful to overcome the difficulty. Price control and rationing go together. Prices of important commodities are fixed and those commodities which are in short supply are rationed so that equal quantity of goods be available to all people in the country. However, price control and rationing have not been found to be very effective because of lack of a competent administrative machinery, especially in developing countries and have often led to black marketing and corruption. Keynes did not favour price control because it fails to bring about equilibrium between purchasing power and available output.

Prof. Kurihara and Friedman also do not support price control because of administrative difficulties. Keynes is also not in favour of rationing because it is wasteful.

Efforts should be made to increase the supply of goods either through increased production or increased imports. Maximum facilities should be provided to the manufacturers to raise production. More capital should be invested in productive industries. Equally important is the imposition of control over foreign trade of the country concerned, *i.e,* exports of essential goods should be stopped, imports of luxuries should either be curtailed or totally stopped for the time being. This step would ensure regular supplies of rationed commodities at controlled rates. Control of wages, sometimes becomes necessary to stop a wage-price spiral. This will necessitate the control of profits as well. At the same time, anti-hoarding and anti-profiteering measures should be undertaken by the government. There should also be publicity and propaganda to educate the people so that they may reduce consumption.

It must be remembered that no measure can ever succeed without the fullest possible co-operation among the different classes of people and without co-ordinte efforts of Government agencies. It is now generally believed by economists that inflation can be controlled in the initial stages. But when it passes beyond a certain stage, it is practically impossible to control it. Hyper-inflation can be checked only through introducing a new currency as against the existing one. This was done in Russia, Hungary, China etc. when there was hyper-inflation in those countries.

Inflation is a domestic problem in each country and it can be checked by means of responsible government economic monetary and fiscal policies, as well as through special efforts aimed at expanding the productive capacity of the economy.

Concept of Inflationary Gap. This is Keynes' contribution to economics of inflation. He tried to explain the phenomenon of inflation in terms of inflationary gap. The inflationary gap for the whole economy may be defined in the words of Prof. Kurihara, "As an excess of anticipated expenditures over available output at base prices." Anticipated expenditures are determined by the consumption-saving pattern plus the taxation structure while the available output is determined by the conditions of employment plus the technological structure of the economy. In other words, "inflationary gap is a situation where the anticipated expenditures (demand for output) exceed the available output at pre-inflation prices. It is measured thus :

Disposable income-available output for consumption. Disposable income mplies the money income of community minus the taxes to be paid to the State. So long as goods are available in plenty at the base price, no inflationary gap will exist; it arises only when the total money income that people want to spend on consumption exceeds the total output available at pre-inflation prices. Such a situation arises during war time as well as during a period of development planning. During war period expenditures are high and consequently incomes received will also be high. Government expenditure on armament, production and on the army increases. Along with it, there is an increase ir civilian

consumption expenditure as well. While money income received by the people is high, the supply of goods available for civilian consumption is very much limited, since most of the production is diverted for war purposes. Thus, a gap between demand for goods and the available supply of goods is created. A similar inflationary gap arises during planning period. The amount of money expenditure will be high because of expenditure on many projects, but the volume of consumption goods produced will not increase correspondingly, because of the time gap between investment on capital projects and the production of goods from them. Thus, inflationary gap comes into existence.

Let us examine how an inflationary gap arises in a war economy :

Money income of the community	= Rs. 1000 crores
Taxes	= Rs. 100 crores
Disposable income	= Rs. 900 crores
Total output at pre-inflation prices	= Rs. 1100 crores
War expenditure	= Rs. 300 crores
Available output for civilian consumption	= Rs. 800 crores
Inflationary gap : (Rs. 900-Rs. 800)	= Rs. 100 crores

Sometimes it is possible that the entire disposable income of the community is not spent on consumption. A part of it is saved for future use. In the above example, if, out of the total disposable income of Rs. 900 crores, say Rs. 50 crores are saved, then the income available for consumption purposes shall be Rs. 900 crores – Rs. 50 crores = Rs. 850 crores. The inflationary gap shall now be reduced to Rs. 850 crores – Rs. 800 crores = Rs. 50 crores. If people voluntarily saved Rs.50 crores, then the inflationary gap shall be wiped out altogether and there shall be no inflation at all.

During inflation, the best policy is to reduce this gap. It can be reduced through taxation and induced savings. During war period, however, this gap is widened as demand is higher than supply of goods. Due to increase in national income, disposable income increases and a major portion is spent on consumption goods, thus tending to raise prices still further.

The concept of inflationary gap can be explained with the help of the following diagram :

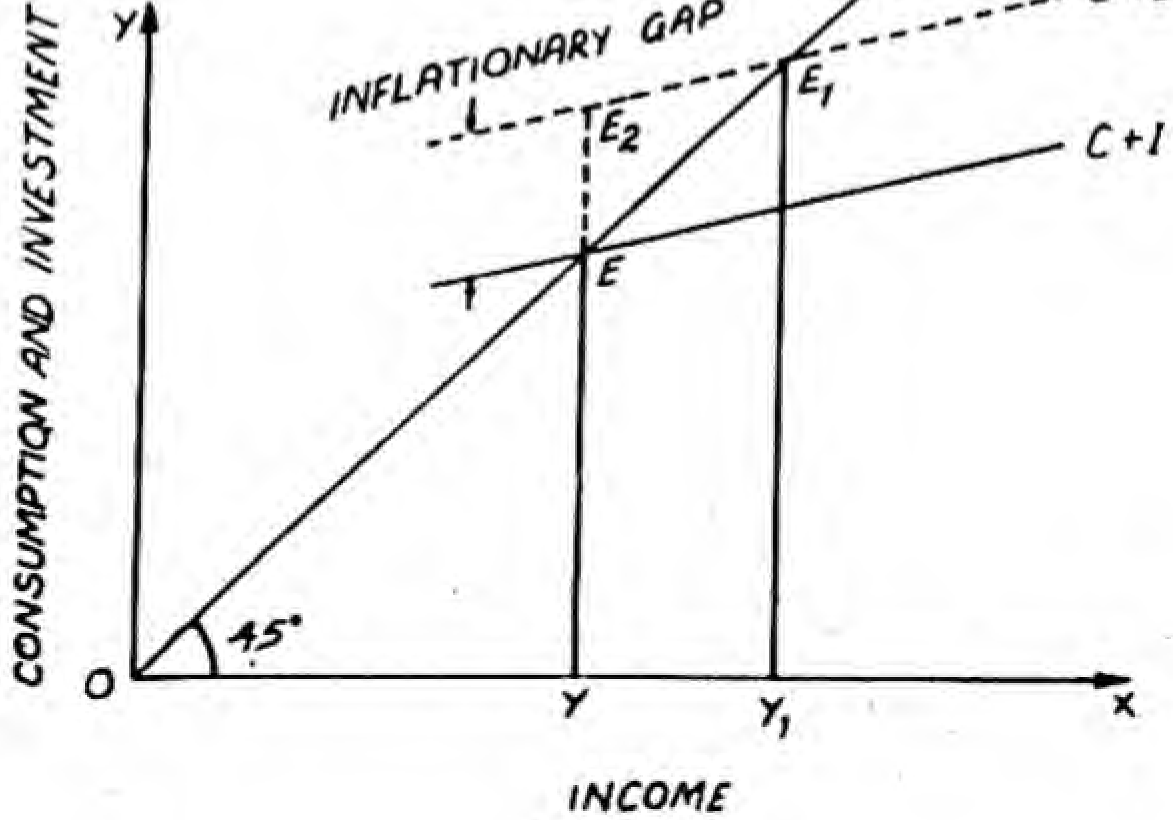


Fig. 6.

In the diagram on the last page, income is measured on the horizontal axis and consumption and investment on the vertical axis. The 45° line known as the consumption is equal to income line (C = Y). It measures the quantitative relation of consumption expenditure to various levels of income and any deviations from the line will show whether consumption will be more or less than the income. This line is zero saving line and all along it income is equal to expenditure. C + 1 line represents the schedule of consumption and investment at different levels of income. The equilibrium between C + 1 line and C = Y line is determined at point E. This indicates that the whole income OY is spent on consumption and investment. There is no excess demand and this is known as full employment income. Now, suppose the government and/or private business units decide to spend more than what they did before, a new C' + 1' line is drawn at a higher level to show this. This line C' + 1' intersects C = Y line at E_1 which shows an excess demand of the order of EE_2. Therefore, EE_2 is regarded as inflationary gap. Unless real income increases from OY to OY_1 this inflationary gap will not be wiped out.

Inflation and Economic Development. Does inflation promote economic development ? This is a very controversial issue in the economics of development. It may be noted that this issue is not as much important theoretically as it is important from the point of view of economic policy. One school of thought including Keynes, believes that inflation does promote economic development. The main arguments in support of their contention are; First, inflation redistributes income in favour of those economic groups who have high marginal propensity to save. Thus, inflation strengthens those groups that can make productive investment. Secondly, a mild inflation is conducive to investment, as it creates an atmosphere in which the business and entrepreneurial class has optimistic expectations. Another argument in support of inflationary financing of economic development is that, in under-developed countries, it is very difficult to bring about capital formation through taxation and borrowings, because of the low level of per capita income in these countries. Thus, capital formation, through created money, constitutes a source of revenue to the Government. Hence it is advocated that deficit financing for capital formation should not be condemned since it would break down the bottlenecks of inflation and bring about more production of consumer goods.

A majority of economists hold the view that inflation is inevitable in the process of economic development, for inflation would not be a very high price for the mobilisation of resoruces. Thus, on the basis of above arguments, it is held that there exists a postiive correlation between economic development and inflation.

Another school of thought opines that inflation does not stimulate development but retards it. Prof. Milton Friedman is highly opposed to the policy of inflationary financing. He is of the firm conviction that inflation is not inevitable in the process of development. To him, there is not much weight in the argument that inflation stimulates investment through its redistributive effects. Prof. Ragnar Nurkse also believes that "the success of inflation as an instrument of capital formation depends largely on the degree to which rise in

prices is unforeseen and unexpected. When a further rise is expected and seems certain, the velocity of circulation of money increases, savings gives place to dis-savings and inflation loses its capital power."

Again, inflation destroys the savings-habit, encourages speculation, demoralises production, breaks down public morale and has adverse social and political consequences. A high degree of inflation drives out the resources and retards capital formation. Inflation also accentuates difficulties with respect to balance of payments position and might seriously affect the availability of foreign exchange for development purposes.

We thus observe that the method of financing development through inflation is too risky and dangerous. If this is pursued by maintaining the stability of money then it may work out nicely. In short, whether inflation promotes or retards development would depend upon whether inflation is steady or intermittent; expected or unexpected; of high or low degree etc. Inflation has various undesirable effects and, as far as possible, it should be tried with care and precaution.

DEFLATION

Fall in price level is good for the community so long as it does not lead to a fall in the level of production or employment. If prices fall from the level of full employment, both income and employment will be affected adversely and this situation will be called as deflation.

While inflation implies excess demand over the available supply, deflation implies deficiency of demand to lift what is supplied. While inflation means rise in money incomes, deflation stands for fall in money incomes. In short, deflation is a state of affairs in which every fall in prices adds to unemploymnent and fall in income of the factors of production in the country.

Causes of Deflation. There are four possible causes of deflation :

(*a*) Definite policy to curb inflation;

(*b*) Sudden increase in total production;

(*c*) Rise in bank rate; and

(*d*) Unfavourable balances of payments.

In order to check the unprecedented rise in prices, sometimes the Government of a country adopts a deliberate policy during or after the period of inflation in accordance with which the volume of currency and credit is contracted either by imposing heavy taxation or by cancelling the inconvertible paper money in circulation. Sometimes surplus budgets which aim at reducing the disposable income of the community and decreasing the effective demand for money are also prepared. Another cause of deflation is an excessive increase in the total volume of commodities and services without a proportionate increase in the supply of money. Sometimes the Central Bank of a country creates deflationary conditions by raising its bank rate and undertaking open market operations under which it sells securities in the market. Deflation may also occur on account of an unfavourable balance of payment position which inevitably results in a reduction of bank deposits and in the exports of huge quantities of gold held by the country concerned.

Effects of Deflation. Deflation is just the reverse of inflation. It is good for some people and bad for others. During deflation consumers gain because with the decline in the prices they are able to purchase more commodities and services with the same amount of money. For the same reason the wage-earners and fixed salary people also gain. The people who may really gain is known as the rentier class who get their income by way of fixed interests and rents.

Deflation is caused by a shrinkage in the total effective demand as a result of fall in the quantity of money. It is characterised by falling prices and increasing unemployment. During this period there are great sufferings and miseries all round and millions of families are thrown in the streets to make their living through begging. During depression period (1929-33) in America 15 million people were unemployed. Millions paraded the streets looking for the work but the work was not available. Similar was the position in England and other countries. Unemployment is next only to war in the degree of human degradation. Unemployment increases the number of devils who could otherwise have served the society as respectable working citizens.

Deflation implies contraction of business activities; men are thrown out of jobs, plants lie idle and factories are closed because effective demand has gone down. Unemployment (due to deflationary policy) reduces productive efficiency, lowers the standard of living and ultimately reduces the volume of production. Millions able-bodied men and women, instead of producing commodities in factories roam about helplessly and stand in queues before poor houses. Deflation thus means a great loss to national income.

The poducers, the merchants and the speculators lose because the prices of their goods fall at a far greater rate than their costs. Due to falling prices they suffer financial losses every year. The farmers also suffer a heavy loss during deflation. The Government of the country finds itself in a very difficult position because it has to borrow money in order to finance relief measures. State revenues decline due to falling money incomes of the people and continuous unemployment.

Methods of Control. Anti-deflationary measures are the opposite of those used to check inflation :

(1) Monetary Policy. Monetary policy that aims at controlling deflation consists of using the bank rate, open market operations and other weapons of control available to the Central Bank of the country to raise the volume of credit of commercial banks. This policy is known as *"cheap money policy."* The idea is to increase the volume of credit and investment. But monetary policy is weak because mere creation of credit does not increase investment. Businessmen due to uncertainty are not prepared to take risks.

(2) Fiscal Policy. Fiscal policy to fight deflation is known as *'deficit financing.'* The Government attempts to reduce the level of taxation so as to leave a larger amount of purchasing power with the public. At the same time the Government increases its expenditure on public works programmes like roads and railway lines etc. Thus, the Government will provide employment to those who are thrown out of employment in the country. The budget deficit

should be financed through borrowings from the banks. Fiscal policy is considered the most important policy for economic stabilization. The successful experiment of the 'New Deal' (policy adopted during depression) tried in America furnishes a glaring example of how a country can pull itself out of depression.

Other methods to control deflation include price support programme and lowering of costs so as to bring about adjustment between price and cost of production. These methods will increase the effective demand, encourage production and investment, mitigate unemployment and free the economy from the clutches of deflation. The best solution of deflation is to have a ready programme of public works to be implemented as and when unemployment makes its appearance.

Choice between Inflation and Deflation. It may be thought that inflation and deflation are exact opposites but it is not so. They are not symmetrical because inflation is a rise in prices without any increase in unemployment, while deflation is a fall in prices accompanied by increasing unemployment.

Both inflation and deflation are necessary evils of the present day economic order and both are indicative of economic disequilibrium. They have some advantages within limits, but they are harmful if go beyond that. Inflation may be considered to be lesser of the two evils. Keynes thus stated, *'Inflation is unjust, deflation is inexpedient; of the two deflation is worse.'* Inflation is unjust and inequitable because it aggravates the inequalities in the distribution of wealth. Deflation is inexpedient because it is unmanageable and does a lot of havoc.

Inflation distorts the distribution of income between different groups of people in the country in such unjust manner that the rich gain at the cost of the poor, but inflation does not reduce the real income of the country. Deflation, on the other hand, reduces national income through contraction of production and increase in unemployment and it also pauperises (ruins) every group in the community. Inflation is unjust and demoralising; it introduces the spirit of gambling. Deflation, on the other hand, does the maximum harm by increasing unemployment. Nothing can be so bad as existence of factories and mills and workers ready to work, but the whole system remaining idle. Inflation at least provides employment to all factors in one way or the other.

Moreover, inflation can be controlled to a large extent, it gets out of control very rarely and that too because of the Government. But deflation, if once started, injects so much pessimism into businessmen and bankers that it is highly difficult to control it.

It will be wrong to conclude that Lord Keynes was in favour of inflation. Both inflation and deflation are evils and there is nothing to choose between the two. Nowhere has Keynes preferred inflation over price stability. He simply considered inflation as a lesser evil because it is worse to provoke unemployment than to disappoint the rentier classes. The proper objectives should be to aim at economic stabilization at the level of full employment.

Questions

1. What is Inflation ? Discuss its causes, effects and remedies.
2. Define the terms Inflation and Deflation. What are their causes ?
3. What are the different kinds of Inflation ? How are they generated ? Indicate briefly the remedies of Inflation.
4. "Inflation is unjust and inequitable and Deflation is inexpedient." (Keynes). Amplify the statement and bring out its implication.
5. Distinguish between Inflation and Deflation. How are the different sections of the community affected by them ?
6. What are the causes of Inflation ? Is it inevitable in the course of economic development ?
7. Critically examine the concept of inflationary gap.
8. Explain the concepts of demand-pull inflation and cost-push inflation.

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PAPER-B

ELEMENTARY STATISTICS

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Meaning, Importance and Limitations

Introduction. The word Statistics seems to have been derived from the Latin word *Status*, or Italian word *Statista* or the German word *Statistik* or the French word *Statistique*, each of which means a political state. In the early years 'statistics' meant a collection of facts about the state or the people in the state for administrative or political purposes. Due to this statistics was regarded as the *science of statecraft* or the *science of kings*. However, with the passage of time, the science of statistics has become very popular. Today statistics is not confined to the activities of the state but it embraces all types of activities. Its use has become so extensive that it is related with nearly every phase of human activity.

Meaning and Definition of Statistics. Statistics has been defined by different writers in different manner at different times. It is because of the rapidly increasing utility of statistics in every field. Broadly speaking, the word statistics has been defined in two senses namely.

(A) Singular Sense
(B) Plural Sense

In a way both the definitions highlight the characteristics of statistics. Let us first **understand** the meaning of statistics in singular sense.

(A) Statistics-Singular Sense. In the singular sense statistics refers to various methods adopted for the collection, classification, presentation, analysis and interpretation of data. Thus, the other meaning of statistics in singular sense is statistical methods. Statistical methods range from very elementary descriptive principles like totals, ratios, percentages etc. (which may be understood by everyone) to those complicated mathematical procedures like interpolation, regression, estimation etc. (which may be understood only by experts). The term statistics in this sense too has been defined differently by different writers. Some of the definitions are given below.

According to *Prof. A.L. Bowley*, "Statistics may be called the science of counting."

The definition given above restricts the scope of statistics to collection of data only. It ignores the methods relating to other important aspects of statistics such as classification, presentation, analysis and interpretation of data. The other definition of statistics

given by *A.L. Bowley* is "Statistics may rightly be called the science of averages."

The definition given above is also not satisfactory because it again restricts the scope of statistics to only averages. Averages are only one method of making analysis of data. The other methods like dispersion, skewness, correlation, regression, interpolation etc. are not covered in this definition. Again *A.L. Bowley* says, "Statistics is the science of the measurement of social organism, regarded as a whole in all its manifestations." This definition is also not comprehensive because it restricts the scope of statistics to sociology *i.e.*, man and his activities. It does not cover physical, biological, astronomical etc. phenomenon within its definition.

In the words of *Boddington*, "Statistics is the science of estimates and probabilities." This definition is also unsatisfactory because estimates and probabilities are only one part of the vast group of statistical methods.

According to *M.R. Spiegel*, "Statistics is concerned with scientific methods for collecting, organising, summarising, presenting and analysing data, as well as drawing valid conclusions and making reasonable decisions on the basis of such analysis." This definition is very comprehensive and covers the statistical methods—from the first stage of collection of data to the final stage of making decisions. Moreover, it depicts the various phases of a statistical investigation.

According of *Seligman*, "Statistics is the science which deals with the methods of collecting, classifying, presenting, comparing and interpreting numerical data collected to throw some light on any sphere of enquiry." This definition by Seligman is also quite comprehensive and includes the various stages in any statistical investigation.

The above two definitions by *Spiegel* and *Seligman* reveal the various types of statistical methods used in any statistical investigation. These are also called the stages of statistical investigation as given below :—

1. Collection of Data
2. Organisation of Data
3. Presentation of Data
4. Analysis of Data
5. Interpretation of Data.

Let us now study, in brief the meaning of each stage of statistical investigation.

1. Collection of Data. The first step in any statistical enquiry is the collection of relevant data. The data must be collected in a systematic manner by keeping in mind the objective of statistical enquiry Utmost care must be taken while collecting data because the collected data form the foundation of the entire statistical analysis. If the collected data is faulty, then the conclusions or findings of the statistical enquiry will also be faulty. The data may be collected either from

primary sources or secondary sources. If the data is collected originally by the investigator himself for the purpose of the enquiry, then it is called Primary data. If the data is obtained from some published sources like newspapers, magazines, reports of government, enquiry commissions etc. it is said to be the secondary data.

2. **Organisation of Data.** After the data has been collected, processing of data is needed. If the data is of secondary nature, it needs to be checked for adequacy, suitability, and reliability. If the data is primary *i.e.*, it has been collected originally by the investigator, then it needs organisation. The first step in organising a group of data is *editing*. The editor examines the data for omissions, irrelevant answers, gaps in coverage and inconsistencies. After editing of data, there comes the stage of *classification* of data. Under classification the data is arranged into certain classes or groups, according to some common characteristics possessed by the items contituting the data. But mere classification will not give us definite results. The last step in organisation is *tabulation* of data. Under tabulation, the classified data are arranged in the form of rows and columns. Thus organisation makes data clear, understandable and highlights its important features.

3. **Presentation of Data.** After the data have been collected and organised, the next step is its proper presentation. The available tabulated data can be presented in the form of Diagrams or Graphs. It should be noted that tabulation is also one of the methods of presentation of data. Diagrams and Graphs attract more attention as compared to data in a tabular form. They are easy to understand and make comparisons between two or more groups of data very easy.

4. **Analysis of Data.** After collection, organisation and presentation of data the next step in any statistical investigation is the analysis of data. The analysis of data brings into light various hidden characteristics of the data. These characteristics relate to average values, extent of variability and skewness in the data, the degree and direction of relationship between two or more sets of data, estimation of values of dependent variable with the given values of the independent variable etc. A large part of this book is devoted to study the various methods of knowing the characteristics of data.

5. **Interpretation of Data.** This is the final step in any statistical investigation. The data is interpreted on the basis of the results of its analysis. The interpretation of data is a very difficult task and requires a high degree of skill, care and judgement. If data is not properly interpreted, then the entire object of investigation may be lost. On the other hand, if results are properly interpreted, then correct decisions can be taken and suitable policies can be framed on the basis of statistical enquiry.

(B) Statistics—Plural Sense. In this sense statistics means numerical data. It is in this sense that the public usually think of statistics. For example, we come across statements like the population of India increased from 36·1 crores in 1951 to 68·38 crores in 1981,

there were 935 females for 1000 males in India in 1981. The production of foodgrains increased from 50·8 million tonnes in 1950-51 to 150 million tonnes in 1985-86. India's share in world exports was 0·45% in 1987-88. The Gross National Product at 1970-71 prices was Rs. 50824 crores in 1980-81 etc. Since the above statements contain figures so they are termed statistics or quantitative statements of facts. From practical point of view such numerical statements of facts are easier to understand than the statements like "Population of India is increasing; number of females is less than males in India; India's share in world exports is low and declining etc." These statements are vague and meaningless as they are not supported by figures.

It should be noted that statistics, in plural sense, are undoubtedly numerical statements of facts but all numerical statements of facts are not termed statistics. For numerical statements to become statistics some conditions must be fulfilled. Different statisticians have explained different conditions for numerical data to become statistics. Some of these are given below :

According to *Bowley*, "Statistics are the numerical statements of facts in any department of enquiry placed in relation to each other."

According to *Yule and Kendall*, "By statistics we mean quantitative data affected to a marked extent by multiplicity of causes."

The above definitions are narrow as they confine the scope of statistics to only those facts and figures which are comparable and are affected by multiplicity of causes. However, the most comprehensive definition of statistics in plural sense has been given by *Horace Secrist*.

According to *Horace Secrist*, "By statistics we mean aggregates of facts, affected to a marked extent by multiplicity of causes, numerically expressed, enumerated or estimated according to reasonable standard of accuracy, collected in a systematic manner, for a predetermined purpose and placed in relation to each other."

The above definition is most comprehensive and exhaustive and highlights certain characteristics which the numerical data should possess so that they may be called statistics. The *characteristics* are as follows :

1. Statistics are Aggregates of Facts. Statistics is a study of group or aggregate of facts. Single or unconnected figures cannot be considered as statistics. A single figure relating to birth, death, import, export etc. does not form statistics but aggregates of figures relating to these variables would be called statistics as these aggregates can be compared with figures relating to other variables. For example, if it is given that annual income of a person is Rs. 10,000 then it will not be called statistics. However, if data regarding his income over a period of say ten years is given, then it will be called statistics. In the latter case we can know whether his income is rising or falling or remains constant. Moreover, we can find average income during the ten year period, the rate of change in income etc. Thus, comparisons, analysis

and conclusions can be drawn only when the facts are stated in aggregates.

2. **Affected by Multiplicity of Causes.** Statistics are those aggregates of facts and figures which are influenced to a very great extent by a large number of causes. For example, statistics or data relating to prices is affected by many causes like demand, supply, money supply, quality of the product, prices of substitutes etc. Similarly, statistics relating to agricultural production are influenced by many causes like quality of seeds, use of fertilisers, fertility of land, weather conditions etc. In physical sciences it is possible to isolate the effect of various causes on any phenomenon. However, it is difficult to do so in social sciences because these causes are not mutually exclusive. Nevertheless, statisticians have developed advanced statistical techniques like multiple correlation, partial correlation etc. to study the role of individual causes on any phenomenon in social sciences also.

3. **Numerically Expressed.** Statistics are only the numerical statements of facts. The qualitative expressions like honesty, friendship, good, bad, poor, old etc. do not constitute statistics. For example, the following statements are simply qualitative in nature and do not come within the perview of statistics.

(*i*) India is a poor country.
(*ii*) Ram and Sham are very good friends.
(*iii*) Mohan is a very honest man.
(*iv*) Sohan's father is very old.
(*v*) The industrial production in India is not sufficient etc.

On the other hand, the aggregates of facts relating to quantitative expressions like marks, age, height, weight of students, profits, sales, income, imports, exports etc. constitute statistics.

4. **Enumerated or Estimated.** There are two ways in which we can collect data relating to any phenomenon.

(*i*) Enumeration or counting
(*ii*) Estimation.

Enumeration is always accurate and precise. This method is generally employed when the scope of enquiry is not large. For example, if the enquiry relates to the average age of a commerce student in a college, then data about age can be actually collected from all the commerce students of that college. On the other hand, if the scope of the enquiry is very large, it is not possible to enumerate all the facts and figures of enquiry. In such a case estimates are made. For example, suppose the enquiry relates to the average age of a commerce student in India. Now scope of enquiry is very large. We make use of sampling techniques and estimate the average age of a commerce student in India. The estimated values will not be as precise and accurate as the actual values.

5. **Reasonable Standard of Accuracy.** The standard of accuracy differs from enquiry to enquiry and it depends on the purpose, nature

and scope of the study. For example, while weighing gold a goldsmith cannot ignore even 1/10th of a gram. On the other hand, while weighing bags of wheat one gram, ten grams or even hundred grams do not have any importance. Similarly, when heights of persons are being measured we have to be accurate even in inches. However, when distance between two cities is to be measured, we can ignore even some furlongs. It is always better to fix in advance the desired standard of accuracy.

6. **Collected in a Systematic Manner.** The collection of data must be preceded by proper planning. It must be collected in a systematic manner. The data collected in a haphazard manner will lead to fallacious conclusions only.

7. **Collected for a Predetermined Purpose.** Before collecting numerical data the purpose or object of the enquiry must be predetermined. The knowledge of purpose helps the investigator to distinguish between relevant and irrelevant data. For example, if the enquiry relates to the prices, then the purpose of this enquiry must be clearly specified. In other words, it must be known whether enquiry relates to the construction of Wholesale Price Index or Retail Price Index. Moreover, it must be known whether a consumer Price Index is to be constructed or a General Index (for all classes of people) is to be constructed. The clear specification of purpose will enable the investigator to choose the commodities for which information about prices is to be gathered.

8. **Placed in Relation to Each Other.** The 'phrase placed in relation to each other' implies that the numerical data must be comparable. Comparison can be on the basis of time or place. For example, we can compare the per capita income in any year with preceding years. This is called comparison on the basis of time. Similarly, we can compare per capita income figures of any country with the corresponding figures in other countries at a point of time. This is called comparison on the basis of space. It should be noted that comparison is possible only if the data are homogeneous. Heterogeneous data like income of a person cannot be compared with his weight.

From the above study we find that the numerical statements of facts will be considered statistics only when they possess various characteristics explained by Horace Secrist. Thus, we can conclude that, "*all statistics are numerical statements of facts but all numerical statements of facts are not statistics.*"

Modern statisticians have joined together the two meanings of statistics *i.e.*, statistical methods and numerical data and accordingly they define the science of statistics as "*a body of principles applied in handling aggregate of facts, observing, estimating, recording, classifying, analysing and interpreting the numerical data affected by multiplicity of causes.*"

Importance and Scope of Statistics. There is no denying the fact that statistics is being increasingly used in almost all the branches of

knowledge. In the absence of statistics and statistical methods most of our problems would have remained unsolved and our knowledge would have remained limited. Looking at the important role of statistics today it appears that *Bowley* was right when he said, "A knowledge of statistics is like a knowledge of foreign language or of algebra ; it may prove of use at any time under any circumstances." The rapidly increasing importance of statistics everywhere is mainly due to the following *causes* or *functions* of statistics.

(*i*) **Statistics presents facts in a definite manner.** The importance of statistics is because of the fact that it presents facts in a clear-cut and definite manner. There is no vagueness in statistics. The statements like population of India is increasing ; there is a lot of unemployment in India etc., have no relevance in statistics. It simplifies mass of figures. Statistics owes its importance to the fact that it condenses a mass of figures. Human mind is such that it can not remember large figures. Statistics solves this problem. Statistical methods like ratios, averages, percentages etc. are deployed for this purpose.

(*ii*) **Statistics facilitates comparison.** Statistics are collected for comparison purposes. Without comparison they have no meaning at all. Comparison is made on the basis of time or on the basis of place. Graphs ; diagrams ; ratios ; averages ; pecentages help in comparing the statistics of same kind collected at different times or for different places.

(*iii*) **Statistics helps in formulating and testing the hypothesis.** In pure as well as in social sciences various hypothesis are formulated and tested. For example, in medicine the effect of a new drug say chloromycin can be tested for typhoid with the help of association of attributes. Likewise in economics and other social sciences various hypothesis are formulated and tested with the help of statistical techniques. The common examples are Law of Demand, Law of Maximum Statisfaction etc.

(*iv*) **Statistics helps in the measurement of uncertainties.** With the help of statistical methods we can study the chance of occurrence of an uncertain event. The theory of probability which is a branch of statistics helps in the measurement of uncertain events. The subjective and objective probabilities are deployed for the purpose.

(*v*) **Statistics helps in the Study of Relationships.** In our day to day life we come across many variables which are related to each other. For example, we generally find that there exists some relationship between :

(*i*) height and weight of a group of persons
(*ii*) ages of husbands and ages of wives
(*iii*) length and breadth of leaves of a tree
(*iv*) amount of rainfall and production
(*v*) price and demand etc.

With the help of statistical methods like 'correlation' we can study the degree and direction of relationship between these variables. If there

are only two variables, then the relationship between them is studied with the help of simple correlation. If the number of variables under consideration is more than two, then the techniques of partial correlation or multiple correlation can be used to study their relationship.

(*vi*) **Statistics helps in forecasting.** With the help of statistical methods we can study the past behaviour of any phenomenon and make predictions about the future events. The techniques of extrapolation, regression are used for making predictions.

(*vii*) **Statistics enlarges human experience and knowledge.** Statistics owes its importance to the fact that it helps in enlarging human knowledge. Without statistics, many fields of knowledge would have even remained closed to mankind. The greatest achievement of statistics in this regard in the 20th century is the landing of space crafts and man on moon.

Let us now study the role or importance of statistics in different fields.

1. Statistics and Economics. The relationship of economics with statistics dates back to 1690 when Sir William Petty published his book '*Political Arithmetic*' and Gregory King tried to demonstrate statistically the relationship between *Supply and Prices*. A lot of statistical material relating to population, occupation, taxes, agriculture, industry, trade, shipping etc. was available, but there was no liason between *Data* and *Economic Theory*. Classical Economists believed in the deductive logic and abstract method of reasoning.

J.S. Mill (1806—1873), a classical economist, admitted the advantage of statistical verification of deductive laws in *some cases*. W.S. Jevons (1835—1882), in his Theory of *Political Economy* in 1871 also advocated verification of the deductive science of economics by the inductive science of statistics and opined that political economy could be developed into an exact science only if commercial statistics were more complete and precise. Because of his work on analysis of time series and pioneering work in the field of price studies and Index number, Jevons has been called the "*Father of Index Numbers*."

Pareto (1848—1923) wrote in 1907, "The progress of political economy in the future will depend in great part upon the investigations of empirical laws derived from statistics which will then be compared with known theoretical laws or will suggest derivation from them of new laws." Keynes in his *Scope and Method of Political Economy* points out that the function of statistics is "first, to suggest empirical laws which may or may not be capable of subsequent deductive explanation and secondly to supplement deductive reasoning by checking its results and submitting them to the test of experience." Now there are no two opinions that induction and deduction are both necessary for the development of economic science. In fact economics and statistics are so intermingled with each other that one is not complete without the

other. The use of statistics can be briefly classified into the following heads :

(*i*) **In the formulation of economic laws.** Statistics are most important tools in the making of economic laws. For example, *Engels Law of Consumption* was based on a detailed and systematic study of the family budget data. *Pareto's* famous *Law of Distribution of Income* was formulated by making an empirical study of income data of various countries at different times. Similarly, *Samuelson's Revealed Preference Theory* was based on data relating to the actual behaviour of consumers in the market.

(*ii*) **In the computation or estimation of national income.** National income is an important indicator of the economic development of any country. It is computed with the help of statistical techniques. All the methods for the measurement of national income like Product method, Income method, Expenditure method rely heavily on statistics. The per capita income which is an index of economic growth is also computed with the help of statistical techniques.

(*iii*) **It helps in the study of economic problems.** What to produce, how to produce, and for whom to produce are the basic problems of an economy. The solution of these problems is based on statistical data about the requirements and available resources in the economy. Moreover, various economic problems like unemployment, rising prices, balance of payments deficit, economic inequalities, population growth etc. can be studied with the help of statistics relating to them.

(*iv*) **It helps in the formulation of economic policies.** All the economic policies are made with the aid of statistics. For example, monetary policies are made on the basis of data relating to prices, interest rates, supply of money, credit conditions etc. A fiscal policy is based on data relating to revenues and expenditure, borrowings, deficit financing etc. Foreign trade policies are based on information about imports, exports, exchange rate, terms of trade, exchange reserves etc.

(*v*) **It helps in economic planning.** When plans are made, various targets are fixed for different activities. These targets are based on data relating to material and human resources. For this the planning authorities need statistics relating to population, agricultural and industrial production, imports, exports etc., performance of past programmes and policies. Just as a house cannot be constructed without bricks, in the same way a plan cannot be made without statistics.

2. **Statistics and the State.** State has to perform a lot of functions these days. Proper execution of these require the collection of data relating to social, economic and other aspects of state activities. The state has to study the past, think of the present and look into the future to make welfare schemes useful for its people. Government collect data for various purposes. Complete data coverage of every activity within and out of the state is useful to the state in carrying out

its programmes most effectively. It is, of course, not possible to get all information on all activities of the state. But basic data relating to population, defence, national income, industrial and agricultural production, exports and imports, finance, commerce, prices, labour, communications, education etc. must be available adequately. The analysis of such data over a period of time depicts certain basic growth or decline. These changes are of utmost utility to government in planning its targets. For example, to check price rise, government has to collect the data for all those factors which have a direct bearing on price line. Although data is used by various agencies, government in almost all countries is the biggest user and collector of statistical data.

3. **Statistics and Business**. In the past, business organisations were small in size and it was localised. A businessman could succeed because of superior energy or sheer luck. With the increase in competition, it has become relatively more essential that business decisions are based upon accurate analysis of facts rather than upon rules of thumb. Over the past couple of decades in the field of business and industrial management there has been an expanding development of a body of quantitative techniques and procedures whose purpose is to aid and improve managerial decision making. Ya-Lun-Chou describes the use of statistics in business as, "In business, statistics has already made radical changes in maintaining and improving output quality, in selecting and promoting personnel, in efficient use of material, in projecting long term capital requirements and forecasting sales, in estimating consumers' preferences and in various other phases of business research and management." It is not an exaggeration to say that today nearly every decision in business is made with the aid of statistical data and statistical methods. One aspect of business which uses statistics is discussed below.

Suppose we want to set up a plant, we have to make a decision on its location and size. A decision on the location and size from amongst various alternatives will depend on the collection and analysis of data on the availability and cost of land, labour, raw materials and manpower etc. Then we need data on local taxes and wage rates. Cost is also connected with scale of the plant. There has to be estimates of cost connected with different scales of plants. Statistics is widely used in production planning, marketing, inventory control, quality control, personnel administration and all other aspects of business operation.

4. **Statistics and Society**. Statistical methods and data are commonly used to analyse various problems which society faces. The problems of drunkenness, crime, poverty, prostitution, theft, and unemployment etc. are all studied by the social scientist with the help of statistical tools. After the analysis, he can suggest the remedies for these social evils.

Education has some targets for the society. To achieve the targets, problems relating to programmes, uses of funds, the service conditions of teachers, libraries, books, discipline and quality of education and

research have to be studied. Statistical data relating to all these problems has to be collected and studied in order to reach rational decisions and different measures have to be taken in the light of study done with the data.

For the proper selection of candidates for different posts, aptitude tests are to be formed and conducted. Most of these tests are purely statistical.

In political science, statistical methods are used to ascertain public opinion on important social, political and economic issues. Statistical methods are widely used in Physics, Chemistry, Agriculture, Meteorology and Medical Science.

5. **Statistics and Investors.** Statistical analysis helps the investors in selecting securities which are safe and which have the best prospects of yielding a good income and increase in price. Such analysis helps in buying and selling of securities. Capital investment in plant and equipment require long term forecasts about sales, prices etc. Among other things, the growth of population, income are anticipated for the long term planning and investment.

6. **Statistics and Banks.** Banks have found it necessary to establish research departments for the purpose of gathering and analysing information on its operations and general economic conditions. The operations of banks are dependent most on the general condition of the economy. Its reserves and funds are influenced by the business condition. While appraising the credit worthiness of the borrowers, various considerations are examined by the bank. The liquidity and profitability of the firm has to be taken into consideration for fixing the repayment schedule. For all this, we need quantitative techniques which are all statistical.

7. **Statistics and Insurance.** Statistics are extensively used in the field of insurance. Actual statistics are needed for the insurance business. For mortality experience mortality tables are made and rates of the premium are fixed. All these tables are made with the help of statistical theory of probability.

8. **Statistics and Accounting, Auditing.** Statistical methods are used also in accountancy and auditing. Statistical data on income, expenditure, investment, profits, production and saving etc. are used for the compilation of National Income Accounts which provide information on the value added by different sectors of the economy and are used for the formation of economic policies. The relationship between profits and dividend helps in the prediction of dividend for the future. The data on assets and liabilities, and income and expenditure are helpful to know the financial results of various operation. In accountancy a number of financial ratios and other ratios are based on statistical methods which

help them in averaging them over a period of time. Most of the accounting and financial calculations are based on statistical formulae.

Another use of statistics is in the area of inflation accounting which consists of reevaluating the accounting records based on historical costs of assests after adjusting for the changes in the purchasing power of money. For this we use price index numbers and price deflators.

In Auditing sampling techniques of statistics are used widely for test checking. It is not possible to check huge volume of business transactions because of the constraint of time and cost involved in it. By sampling audit a sample of transactions or items is checked and inference is drawn about the whole lot.

9. Statistics and Medical Sciences. In medical sciences statistics relating to incidence of disease, body temperature, pulse rate, blood pressure etc. are needed from time to time. Moreover, the relative effectiveness of different drugs needs to be known. For this purpose data relating to the performance of each drug on different patients is collected.

10 Statistics in Defence and War. Statistics occupy a great importance in defence and war. As a matter of fact the statistics has its origin due to the defence requirements of kings in ancient times. Even today a war cannot be fought without statistics relating to personnels in the Army, Navy, Airforce, the strength of the opponent with respect to each force, the amount of weapons etc.

We can conclude our discussion by saying that there is no field of human activity where statistics is not used today. As *Tippett* has maintained that "*Statistics affects everybody and touches life at every point*."

Limitations of Statistics. There is no doubt that statistics is growing in popularity because of its usefulness in every field. But it has its own limitations. One should be aware of these so that the expectations about its results are not over emphasised. According to *Newsholme*, "statistics must be regarded as an instrument of research of great value, but having severe limitations which are not possible to overcome and as such they need our careful attention." Some of the major limitations are discussed below :

1. *Statistics does not study qualitative phenomenon.* We know that statistics is a science which deals with numerical statements of facts only. It means that statistical methods do not apply to the qualitative statements of facts or qualitative characteristics like honesty, integrity, friendship, love, greatness, old, culture etc. Thus, qualitative statements like "Population of India is increasing ; Mahatma Gandhi was a great social reformer ; Ram and Sham are very good friends" do not find any place in statistics. However, various qualitative characteristics like intelligence, beauty etc. are indirectly expressed in numerical terms and given statistical treatment.

2. *Statistics does not study individuals.* We know that statistics is a science which deals with the aggregates of facts. It is, therefore, suited only to those problems where group characteristics are desired to be studied. The statistical techniques do not prove useful where knowledge about individual cases is required. For example, with the help of statistics we can know about the average marks of a group of students in a class. Now if it is required to know the marks of any student say x, then statistics is not helpful. Moreover, statistics can not help in making a study of changes in individual cases. For example, national income statistics would not show how the income of different persons has been affected by its rise or fall, who have become rich and who have become poorer. The statistics will only show whether the nation, as a whole, has grown richer or poorer.

3. *Statistical results are true only on an average.* Statistical laws are not exact laws like laws of mathematics and physics. They are derived by taking a majority of cases and are not true for every individual. They are true only on an average. According to *Connor*, "An exact law, when correctly formulated, is held to be true of every individual case coming under its jurisdiction, whereas a statistical law is only held to be true on the average, or in the long run." For example, when we say that average score of a group of students is 60 it does not mean that all the students get 60 marks only. There may be many students getting more than 60 marks and many students may be getting less than 60 marks. It is quite possible that no student gets exact 60 marks but even then average score is 60. To take another example, we know that the probability of getting a head when a coin is tossed is 0·5. It does not mean that if we throw a coin 10 times, we shall get head exactly 5 times. We may get any number of heads from 0 to 10. We only expect (average value) to get 50% heads and 50% tails.

4. *Statistics does not reveal the entire story.* With the help of statistical methods we can study the characteristics of available data only. For example, if we have data relating to marks of students then we can find average marks, variability in marks, skewness etc. Moreover, we can interpolate and extrapolate about marks by using statistical techniques. But statistical techniques can not answer the questions like "Why a student is getting very less marks, why all the students do not get distinction, why marks of students are different in different subjects. The answers to such questions require information about the likings of students, method of teaching, number of hours devoted to study, intelligence of the students etc.

5. *Statistics are based on sample study.* There are two methods in which any problem can be studied. These are (*i*) Census study (*ii*) Sample study. In the census study each and every unit of the population is studied. It is not possible if scope of the enquiry is large. So we make use of sample study. Under it a few units of population are selected and the results of their analysis are used to describe the

features of entire population. Now if the sample is not properly taken or if investigator is biased then results of sample study will be wrong. It may lead to wrong conclusions also.

6. *Statistics can be misused.* The greatest limitation of statistics is that they are liable to be mishandled and misused. Statistical methods are the most dangerous tools in the hands of inexperts. Misuse of statistics is due to the fact that the figures are innocent and do not bear on their face the mark of their quality. They can be distorted, manipulated or moulded by persons with selfish motives. So with the help of statistics they can either prove or disprove anything. The reasons and examples of misuse of statistics are discussed in detail under the heading *Distrust of Statistics.*

Distrust or Misuse of Statistics. By distrust of statistics we mean the lack of confidence in statistical statements and statistical methods. Despite the fact that statistics is being used in almost all the disciplines and in almost all the activities, still there is lack of confidence in it. This has given rise to a number of expressions which tend to impair the reputation of statistics. The following statements are quite commonly used :

(*i*) An ounce of truth can produce tons of statistics.

(*ii*) Figures do not lie, liars figure.

(*iii*) Statistics can prove anything.

(*iv*) Statistics are lies of the first order.

(*v*) Statistics are like clay of which one can make either a God or a devil as one likes.

(*vi*) Statistics cannot prove anything but can be made to prove everything.

(*vii*) There are three kinds of lies—lies, damned lies and statistics, wicked in the order of their naming.

From the above statements we find that there are two different opinions about statistics (*i*) it cannot prove anything (*ii*) it can prove anything. Those who defend statistics argue that statistics cannot be blamed for its misuse. The blame lies with the persons who misuse it to serve their own interests.

There is no denying the fact that statistical methods are only tools which can be either used properly or misused. If tools are misused by inexperienced and untrained persons, then tools cannot be blamed for wrong results. Statistical tools are just like the tools of an artisan which are capable of producing nothing by themselves. Of course, with their use an experienced artisan can make very useful things. But when these tools fall in the hands of an inexperienced person, then the consequences will not be good. In the words of *Bowley*, "*Statistics only furnishes a tool, necessary though imperfect, which is dangerous in the hands of those who do not know its uses or deficiencies.*"

A few interesting examples can be given to illustrate the point. We all know that medicines are meant for curing people. But if a wrong medicine is taken or a wrong dose of the medicine is taken, then the person may die. Now medicine cannot be blamed for such a result. Similarly, we know that a knife is used for cutting vegetables. Now if a child cuts his finger with a knife, then we cannot blame the knife for this. Actually the fault lies with the person who kept the knife at a place so that a child could get it. These examples make it clear that if statistical methods are misused then the science of statistics cannot be blamed for this misuse.

Various Examples of Misuse of Statistics.

1. Suppose in a private academy there are only two students and they pass the examination. On the other hand, in a well reputed college, out of 100 students 90 pass the examination. On the basis of above it was concluded that level of education is better in a private academy due to its 100 percent result as compared to 90 percent in the well reputed college. This conclusion is wrong.

2. In an enquiry relating to traffic accidents it was found that the number of accidents is lower in foggy weather than on clear weather days. Hence it was concluded that it is safer to drive in fog.

The above conclusion is wrong. It reflects a misuse of statistics. It is because in foggy weather first of all there is no rush of traffic and secondly extra caution is observed while driving. Thus, number of accidents is bound to be less.

3. The ordinary worker's wages in a factory range from Rs. 100 per month to Rs. 500 per month. The managing director gets the salary of Rs. 3000 per month. Now a report that the factory wages vary from Rs. 100 to Rs. 3000 per month is deceptive.

4. Suppose the following facts are known about the performance of two students A and B.

Tests	1st	2nd	3rd	4th	Average score
A's score	80%	70%	60%	30%	60%
B's score	30%	60%	70%	80%	60%

On the basis of average scores it was concluded that the performance of A and B, on the whole, was same. This conclusion is incorrect because whereas A is showing a decline, B is improving. The gap between them is going on widening.

5. The argument that in a country 10,000 vaccinated persons died of small pox, therefore vaccination is useless, is statistically deceptive because it is not known about the number or percentage of persons who were not vaccinated died.

6. Consider the following statement, "Those who drink die before reaching the age of 90 years and hence drinking is harmful to longevity." The above statement cannot be accepted because the number of people who do not drink and die before 90 years are not known.

7. In a report it was stated that the number of deaths in a village dispensary was 100 in one year while the number was 600 for PGI. Therefore, it was concluded that the treatment in a village dispensary is better than the treatment in PGI.

The above statement is also incorrect because normally serious cases are taken to PGI for treatment whose. chance of survival is less.

8. In a report about car accidents it was stated that the number of rich people dying in car accidents is more than the number of poor people dying in car accidents. Therefore, it was concluded that rich people are bad drivers.

This statement is also wrong because only rich people can afford cars and derive them. Since poor people do not have cars, their chances of death in car accidents are also less.

Reasons for the Misuse of Statistics. The distruct of statistics can be due to the following reasons :

1. *Changing Definition.* A slight alteration in the definition may change the results of the enquiry. For example, the 1951 census pointed out that working women constitute 25% of total women in India. The 1961 census came out with the figure of 28%. From this one can conclude that the working women force has increased. But actually this is not true because in 1961 census, the definition of working women was changed. Similarly, a small scale unit of 1979 cannot be compared with a small scale unit of 1989 because of the change in definition.

2. *Selection of Items.* We know that generally any problem is studied by making use of sample method. Now if selected items in the sample are wrong then results of the enquiry will also be wrong. For example, suppose an enquiry relates to the average weight of a player. Now if all the weightlifters are included in the sample and average weight is found to be 110 Kg., then the conclusion based on sample study will be wrong. In other words, it cannot be concluded that the average weight of a player is 110 Kg.

3. *Inappropriate Comparison.* Wrong conclusions may be obtained by comparing statistics which cannot be compared. For example, it is observed that number of people dying in houses after accidents are less than the number of deaths in hospitals. So if we conclude that it is safer to take patients to house after accident, then this conclusion is wrong. It is because our comparison is wrong.

4. *Misinterpretation of Correlation.* In our day to day life we observe a large number of variables related to each other. It is quite possible that variables may be uncorrelated but there exists a high degree of correlation between them. From this we cannot establish any cause and effect relationship between variables. For example, in a study of students it was found that there exists a high degree of relationship between the size of shoe of the student and his average marks. Actually there does not exist any relationship between these

two variables. Now if we conclude that the performance of student can improve with an increase in size of shoe, then this conclusion will be wrong.

5. *Inadequate Sample*. Statistical data may lead to misleading conclusion if they are based on a very small number of items For example, if we want to know about the performance of students and select only two students from a college, then our conclusion based on this sample will be misleading It is due to the fact that the sample is inadequate and unrepresentative. To take another example, suppose we are to decide about the strength of two teams. Now, on the basis of the result of only one match we cannot decide which team is better than other. If we do so, then results are likely to be wrong.

From the above discussion we find that statistics is a tool which has to be handled with care. Its limitations must be kept in mind while using it. In the end we can conclude the discussion with the remarks of *W. I. King*, "*Science of Statistics is a useful servant but only of great value to those who understand its proper use.*"

QUESTIONS

1. Define statistics and discuss its importance.

2. 'Statistics is the science of averages', Do you agree with this view ? If not, give reasons and suggest a proper definition of statistics.

3. Explain and illustrate the use of statistics in economics and planning.

4. "Statistical methods are most dangerous tools in the hands of inexperts". Elucidate.

5. "Statistics are numerical statements of facts, but all numerical statements are not statistics". Comment.

6. Define statistics. What are its limitations ? Can it be misused ?

7. What are the important functions of statistics ? What are its limitations ?

8. Discuss in detail the importance of statistics with special reference to economics and business.

2

Collection of Data

Introduction. By definition statistics is concerned with the collection, classification, presentation, analysis and interpretation of numerical data. Collected data, therefore, provides the basic raw material from which analysis is made, principles are formulated and conclusions are drawn. The collection of data is the first and most important stage in any statistical investigation. The collection of data is the foundation on which the entire superstructure of analysis and interpretation is to be raised. If the collected data is accurate, then correct conclusions can be made. If, on the other hand, the collected data is inaccurate, then the results of the statistical study will also be inaccurate.

Preliminaries to the Collection of Data. It should be kept in mind that the task of collecting data is not easy. It entails various difficulties and hardships. Certain preliminary issues must be decided before undertaking the work of collection of data. In other words, the actual collection of data must be preceded by a well thought out plan. In planning a statistical enquiry the following points need a careful consideration :

1. Purpose of Enquiry
2. Scope of Enquiry
3. Sources of Information
4. Type of Enquiry
5. Determination of Statistical Units
6. Degree of Accuracy.

Let us now dissuss each one of these in detail.

1. Purpose of Enquiry. Before conducting any enquiry the investigator must have a clear idea about the purpose of enquiry. The purpose of enquiry will help in determining the scope of the enquiry, the sources of data, the type of enquiry, the degree of accuracy and a lot of other matters. If the purpose of enquiry is known, then relevant data can be collected. Failure to define clearly the purpose can lead only to misunderstanding and confusion. It will result in the collection of irrelevant data which will mean nothing but the wastage of time, money and energy.

2. Scope of Enquiry. After clearly defining the purpose, the

next important requirement is to define the scope of the enquiry. By scope we mean the coverage of the enquiry with respect to the geographical limits, time and the number of items to be covered. The clear knowledge of these things helps in conducting the enquiry with less cost and less period of time. For example, suppose industrial relations are to be studied. The investigator must know the geographical area—whether enquiry relates to a particular state or whole of India. Similarly, whether data is to be collected for the previous year or last ten years. In the same way it should be decided whether information is to be collected for a particular industry or all the industries. The scope of the enquiry generally depends on three factors (*i*) Object of the enquiry, (*ii*) Availability of Time, and (*iii*) Availability of resources.

3. **Source of Information.** Once the object and scope of the enquiry have been laid down, the investigator has to decide about the sources from where data should be collected. Data can be collected from two sources.

(*i*) Primary sources
(*ii*) Secondary sources.

If the data are collected originally by the investigator for the given enquiry, then it is said to be primary data. If, on the other hand, investigator makes use of data which had been earlier collected by some one else, then it is called secondary data. Such data are generally obtained from newspapers, journals, reports of companies, government publications etc. Primary data is preferred to secondary data. The choice of source of information depends on the purpose, scope of enquiry, availability of time, resources and the agency conducting enquiry.

4. **Type of Enquriy.** After the objective and scope and source of the enquiry is clear, we have to decide what type of enquiry is to be conducted. If we want to study the total employment in Haryana, the type will be that of complete enumeration. If we want to study the yield per acre, we may prefer sample study. It also depends on the institution which is collecting data. If government agency is collecting data, it will have no difficulty. If it is an institution like university or other private research organisation, the data collection will become little difficult. If the data is to be collected by an individual for his own research, the data collection is most difficult. The other factors which affect the type of enquiry are the availability of time, resources and proper investigators etc. Then we have to decide, how the data is to be collected. If the data is primary, we will have different types of problems. In case of secondary data, the definitions and units are fixed. But if the data is primary we shall collect data according to our needs. We have to decide which type of enquiry we are going to do. The various types of enquiries are as follows :

(*a*) **Census or sample.**
(*b*) **Original or repetitive**

(c) Direct or indirect
(d) Open or confidential.

(a) **Census or Sample.** In census enquiry all units in the population are taken into account, whereas in sample enquiry only a few units are selected from the population and it is from these units that data is collected. Whether the enquiry should be based on census data or sample data, it depends upon the object, scope, accuracy required, resources and time available with the investigator. In some cases census may be better enquiry while in other sample enquiry is desirable.

(b) **Original or Repetitive.** The enquiry being made for the first time is original enquiry and if it is made regularly then it is called repetitive. For the original enquiry plan of enquiry is to be made separately, whereas in the case of repetitive enquiry, the plan is already known, but some modifications are made to suit the present requirements. While doing modifications, care should be taken that there is not much change in the meaning of any concept or definition of the terms used earlier.

(c) **Direct or Indirect Enquiry.** By direct enquiry, we mean the conducting of enquiry directly by approaching individual units which have been selected for data collection. This type of enquiry is possible only when the data can be numerically expressed or it is quantitative in nature. But sometimes the enquiry is to be made about some facts which cannot be measured in quantitative terms. Height of students in a class is measurable directly in feet and inches but their intelligence cannot be quantitatively ascertained. In the latter case, reliance may have to be placed on indirect evidences of intelligence, as for example, the marks obtained at a certain examination on intelligence tests etc.

(d) **Open or Confidential.** In an open enquiry the findings are not kept secret, whereas in a confidential enquiry these are kept strictly secret. Some managements Banks and Police intelligence which collect data, keep this confidential. The enquiry which is not confidential is an open enquiry.

5. **Determination of Statistical Units.** Another important requirement before the collection of data is to define the statistical unit or units in which statistical data is to be collected. Statistical unit is necessary not only for the collection of data but also for their analysis, presentation and interpretation. A statistical unit is the measure of an attribute or attributes selected for enumeration, analysis and presentation of data. If the number of students in a class is to be counted, the unit is a student. If study relates to the pocket expenses of students, then the units will be rupees. The definition of units is not as simple as it appears from above examples. Sometimes it becomes very difficult to define units. For example, suppose the size of a business firm is to be known. The size can by known by considering capital invested (unit is rupees); level of output (Unit is Quintals, tonnes

etc.); number of employees (unit is an employee). The following are the main requirements which a statistical unit should possess :

(*i*) Unit should be appropriate to the enquiry ;

(*ii*) Unit should be stable *i.e.*, there should be no change in its definition.

(*iii*) Unit should be specific and unmistakable *i e.*, it should be interpreted in a similar way by different persons.

(*iv*) The definition of the unit should be simple, clear and concise.

Types of Statistical Units. There are two types of statistical units.

(*i*) Units of collection

(*ii*) Units of analysis and interpretation.

(*i*) *Units of Collection*. These are the units in terms of which data are collected. These can be counted or measured. Counted in the case of physical items and measured in the case of qualitative attributes. The units of collection can be divided into two categories (*a*) simple units ; (*b*) composite or complex units.

A simple unit describes only a single characteristics like wage, height, weight, distance etc. Examples of such units are rupees, hours, kilometers, kilograms etc. Such units are commonly used and are not difficult to define.

A composite unit is more complex and consists of a combination of two or more simple units. A simple unit with only one qualifying word is called a compound unit. For example, skilled worker, man-hours, kilowatt hour, monthly wages, educated unemployed etc.

(*ii*) *Units of Analysis and Interpretation*. These are the units in terms of which statistical data are ultimately analysed and interpreted. Such units facilitate comparison between different sets of data with regard to time or space. These units include rates, ratios, percentages and coefficients.

6. **Degree of Accuracy.** Another important step before collecting data is to decide about the degree of accuracy that is to be observed in the collection of statistical material. 100% accuracy is not possible in statistical work because (*i*) statistics are based on estimates, (*ii*) tools of measurement are not always perfect, (*iii*) untrained and biased investigators. Even if perfect accuracy is possible we may not have the time, money and other resources to achieve it. Therefore, it is said that data must possess a reasonable standard of accuracy. The reasonable standard of accuracy depends on the purpose of enquiry and it differs from enquiry to enquiry. For example, while measuring gold even 1/10th of a gram is also considered, whereas while measuring wheat a few grams may not have any value. Similarly, if we are measuring the length of cloth for a shirt, a difference of few centimetres is significant. But if we are measuring the distances between two places say Delhi and Jammu a difference of few metres may be immaterial.

From the above discussion we find that some preliminary steps must be taken before starting the job of actual collection of data. These preliminary steps help any investigator to collect relevant, reliable, appropriate and useful data.

Sources of Data.

Statistics can be classified as primary data and secondary data. The 'primary data' refers to that data which the investigator produces himself for the purpose of enquiry in hand. For example, if it is desired to conduct an enquiry about the wages of workers in a certain mill in Bombay and data pertaining to this enquiry are collected by the investigator from the workers, then such data would be termed as *Primary data*.

The term secondary data, on the other hand, refers to statistical data which does not originate from the investigator himself but is obtained from some one else's records. Thus, if instead of obtaining data from the workers investigator gets the data from records of the mill or trade union office then data will be termed as *secondary data.* In other words, when primary data is utilised for any other purpose or for the same purpose at some subsequent enquiry, it is termed as secondary data.

Difference Between Primary and Secondary Data.

The difference between primary and secondary data is largely one of the degree. Data which are primary to one person are secondary to the other persons. For example, the data collected during census operations are primary to the census department of the government of India but secondary to persons who make use of such data for further research or for any other purpose. But it may be important to note that secondary data is utilised when primary data is either not available or it is too expensive in terms of resources. The problem with secondary data is that it is not available in the form in which researcher needs it. To remove this defect, the object and the methodology of the initial enquiry must be ascertained to assess its usefulness. The utilisation of secondary data, in fact places additional responsibility upon the investigator. He has to examine carefully the available data to find out whether it is usable in its present form or not.

Choice Between Primary and Secondary Data.

It is for the investigator to decide before the start of enquiry whether primary data or secondary data suits his enquiry. This decision he has to take and proceed. This decision is not a casual decision. His decision is based on the following considerations :

(1) Objective and scope of enquiry
(2) Availability of time
(3) Financial resources of the project
(4) Degree of Accuracy
(5) Status of Investigator.

He decides in the light of these considerations, which form of data primary or secondary will be suitable for his enquiry. He may also think that combination of primary and secondary data will be useful in the case of any particular investigations. Now we shall discuss the methods of collecting primary data. These are as follows :

Methods of Collecting Primary Data.

1. Direct Personal Investigation
2. Indirect Investigation
3. Schedules through Investigators
4. Questionnaires by Post
5. By Local Agents.

1. Direct Personal Investigation. Under this method the investigator has to personally contact the sources of information. The investigator asks them questions pertaining to the survey and collects the desired information. This method of data collection is employed where the field of enquiry is small and the need for accuracy is very high.

The success of this method depends largely, among other things, upon the personal qualities of the investigator—his tact, diplomacy, efficiency, and capability of understanding the psychological and instinctive reactions of those whom he is interviewing. He must be conversant with the local conditions such as customs, traditions, language and the method of people's living. It is better if he belongs to the same area. He should put simple questions and try to get precise and accurate answers.

Merits:

(1) This method generally gets higher degree of response from the respondents.

(2) Investigator can clear all doubt and errors on the spot itself. The information is more accurate.

(3) Various kinds of supplementary information can be collected.

(4) No delay occurs in this method to get information.

(5) The possibility of communication according to the status and level of literacy of the informants is more under this method.

(6) If an imformant is sensitive to a particular question, there is every possibility of non-response. A tactful investigator, after studying his reaction carefully can get his response in an indirect manner because of personal interview.

Demerits.

(1) This method is suitable for intensive investigation where area of investigation is small.

(2) It is costly, if the number of persons is large and are spread over vast area.

(3) The quality of an investigator counts much. An untrained investigator may not do justice to the enquiry.

(4) The personal bias of the investigator can come inconsciously into this method.

(5) More time is needed for this method.

Suitability. This method is more suitable for intensive rather than extensive field surveys. It is also recommended where desired degree of accuracy is very high and the investigator has available time and resources.

2. Indirect Investigation. When direct personal investigation cannot be used either on account of the complexity of the fieid of enquiry or on account of the reluctance of informants to supply needed information an indirect oral investigation is conducted. In this method the investigator selects certain sources which are connected with information directly or indirectly. For example, information about the activities of students can be collected from their friends, teachers, parents etc. The investigator interviews the witnesses who are in possession of information about the direct source. This method is generally adopted by the Enquiry Committees or Commissions appointed by the government from time to time. This method is also followed by police and other investigating agencies for getting clues about thefts or murder from third parties.

This method is very popular in practice. Its success depends upon the character of the persons who are interviewed and the ability of the investigators. Views of a large number of witnesses should be collected to judge the real situation.

Merits.

1. With the help of this method a wide area can be covered.
2. This method leads to saving of time, money and labour.
3. Views and suggestions of experts and specialists can be obtained.
4. This method is useful where secret information is required and informants are reluctant to supply the relevant informatior

Demerits.

1. This method depends upon the type of persons who are interrogated. It is applicable only if the witnesses are literate and honest. If witnesses are illiterate. indifferent or unwilling the data callected by this method will be inadequate and erroneous.

2. If the investigators are not trained and biased, then correct information cannot be collected.

Suitability. This method is suitable where field of enquiry is large and actual informants are reluctant to supply the information.

3. Schedules through Investigators. Under this method a list of

questions pertaining to the enquiry is framed. This is known as a Questionnaire. The difference between a questionnaire and a schedule is that a questionnaire is answered by the respondent himself in his own hand writing while a schedule is filled by the investigators in face to face situation with the respondents.

In this method the investigators go to the informants personally with the schedule, ask them questions written in the schedule and record their replies. The success of this method also depends upon the ability, tact, diplomacy and efficiency of the investigator. He should be clear about the purpose and scope of enquiry. Similarly, he should be aware of the local conditions and should know local language also.

Merits.

1. This method of collecting data can be useu even in those cases where informants are illiterate.
2. The problem of non-response is eliminated to a great extent.
3. This technique is useful in extensive enquiries and relevant data can be collected.
4. The accuracy of statements can be checked by some intelligent cross-questioning by the investigator.

Demerits.

1. The method is costly as it requires well trained, experienced and unbiased investigators.
2. If the investigators are untrained, careless and biased, then relevant data cannot be collected.
3. It is a time consuming method.

Suitability. This method is suitable where scope of enquiry is large, high degree of accuracy is desired and informants are illiterate. This method is generally used by big business houses, research institutions and even by government for conducting population census.

4. Questionnaires by Post. In this method a questionnaire is prepared based upon the problem under consideration. Then the questionnaire is sent by post to the persons from whom information is required. The informants are requested to fill various columns and return the questionnaire by post. A covering letter of request should also be sent to the informants to explain them the purpose of enquiry for which data is being collected. In order to ensure quick and better response a self addressed stamped envelope should be sent along with the questionnaire. The success of this method depends upon the skill, efficiency, care and the wisdom with which the questionnaire is framed. The question asked should be easy, clear and brief. It is better if objective type questions are made with four or five alternative answers to each question. The informants will have to indicate (by a tick mark) the answer of their choice only.

Merits.

1. This method is less expensive.

2. A large field of enquiry can be easily covered.

3. All types of informants can be covered.

4. The bias of investigator can be eliminated because of the direct link with informants.

Demerits.

1. This method is suitable only for literate informants.

2. Questions in the questionnaire may convey different meanings to different persons which may make the results unreliable.

3. There can be delay in response.

4. There is a problem of non-response because many people may fail to respond.

Suitability. This method is suitable where informant are spread over a wide area, they are literate and are ready to co-operate.

5. **By Local Agents.** In this method local agents (commonly called *correspondents*) are appointed by the investigator in different parts of the area under study. These correspondents submit their reports periodically to central or head office. This method is employed when there is the need for regular data from a large area. This technique of data collection is generally employed by newspapers or periodical agencies and various departments of government which need regular information from a very wide area. The accuracy and reliability of the collected data depends upon the ability, efficiency, knowledge of the correspondents. Therefore they must be given proper training before giving them the task of data collection.

Merits.

1. A wide field of investigation can be covered.

2. It is less expensive and saves time also.

3. Approximate results become easily available.

Demerits.

1. The data supplied may not be reliable because of the bias of the correspondents.

2. The data of different correspondents may not be comparable because of difference in their area and techniques of data collection.

Suitability. This method is suitable when information is required at regular invervals from a very wide area.

From the above discussion we find that different methods can be used for collecting primary data. Each method has its own merits and demerits. The choice of a method depends upon the purpose of enquiry, scope of enquiry, availability of time, money and other

resources, the degree of accuracy desired and the agency conducting the enquiry.

Collection of Secondary Data. Secondary data are not originally collected, but these are compiled from data collected by some other agency. In fact much statistical analysis is based on secondary data. The difference between primary and secondary data is a matter of degree only. The same set of data may be primary in the hands of one agency and secondary in the hands of other agency. For example, the data relating to population—birth rate, death rate, sex ratio etc. is primary in the hands of agency which actually collects the information. But if this data is used for the purposes of research, then it is said to be secondary data.

Sources of Secondary Data. The sources of secondary data can be broadly classified under two heads :

(A) Published Sources
(B) Unpublished Sources.

(A) **Published Sources.** There are a large number of national and international agencies which collect statistical data regarding prices, unemployment, production, imports, exports, foreign exchange, terms of trade, exchange rate, foreign debt etc. and publish their findings in statistical reports regularly. These publications are most important sources of secondary data. Some of these published sources are as given below :

(*i*) *Official Publications of Central Government* like Monthly Abstract of Statistics ; Annual Survey of Industries. Several Reviews ; National Income Statistics ; Economic Survey ; Indian Population Bulletin etc.

(*ii*) *Reports of Departments in State and Central Government* like reports of Income Tax Department ; Railways ; Post and Telegraphs ; Central Excise Commissioner's reports etc.

(*iii*) *Publications of Semi-Government Statistical Organisations* like The Reserve Bank of India ; The Institute of Foreign Trade etc.

(*iv*) *Publications of Research Institutions* like Indian Statistical Institute ; National Council of Applied Economic Research ; Indian Council of Agricultural Research ; Indian Standards Institute etc.

(*v*) *Publications of Commercial and Financial Institutions* like Institute of Chartered Accountants of India ; Stock Exchanges ; Banks ; Co-operative Societies ; Federation of Indian Chamber of Commerce and Industries etc.

(*vi*) *Reports of Various Commissions and Committees* like Finance Commissions ; Kothari Commission, Pay Commissions, Sarkaria Commission, Land Reforms Committee report etc.

(*vii*) *Newspapers and Periodicals* like The Economic Times ; The Financial Express ; Money ; The Eastern Economist ; Indian Journal of Economics ; Commerce ; Capital etc.

(*viii*) Publications of Political Parties.

(*ix*) *Publications of International bodies* like IMF ; IBRD ; UNCTAD ; IDA ; ADB ; U.N.O. etc.

(B) **Unpublished Sources.** All statistical material is not always published. Statistical data may also be available in unpublished or manuscript form. For example, information contained in the official records and files of the government and private offices, studies made by research institutions and scholars can be important sources of secondary data.

Suitability and Adequacy of Secondary Data.

Primary data, we know is expensive and time consuming. Although primary data is more accurate, but its use is limited. Sometimes researcher is obliged to use the data which already exists. We should have careful scrutiny of published data before use for its suitability, accuracy and adequacy. According to Bowley, it is never safe to take published data at its face value without knowing its nature and limitations.

Before using the data from secondary source, the user must satisfy himself that data available are suitable for the present enquiry. The nature and scope of present enquiry must be compared with the original enquiry from which data are evolved.

Again it is essential for the user to know as to how far the available data are sufficient for the present enquiries. We should find what geographical area was covered by the original investigation, what is the coverage of the present enquiry etc. If both have the same coverage, only in that case, past data should be considered.

For the purpose of present enquiry, the existing data should be reliable. It means that the data must be free from errors. Primary data suffers from some non-sampling errors. These errors are likely to be present in the secondary data also. For reliability of available data, we should study the following points before actually using the data.

(*a*) What was the agency that conducted the enquiry ? What is its reputation in this field ?

(*b*) Sources and methods used for data collection.

(*c*) Definition of terms and units used.

(*d*) Schedules and Questionnaire used for data collection.

(*e*) Who collected the data ? Were they biased ?

(*f*) Process of editing, tabulation and analysis followed.

(*g*) Final result of the enquiry.

(*h*) Degree of accuracy.

After studying these points, if the researcher is satisfied with the data collected earlier, then that secondary data may be used for his present study.

Editing Primary data.

After collecting raw data the information contained in it must be examined. If one statement in the questionnaire contradicts the other statement in it or it is incomplete, it may be returned for correction to the agent. This process is called **Editing** of the data. Editing the schedules is the process of preparing data (got from schedules) for classification, coding and tabulatian. Inspite of precautions, there is always some possibility of errors and irregularities in the large mass of data. This task is known as editing. This work is done by trained and experieneed editor. This indispensability of Editing of data is due to the following reasons.

(1) Non-dependability of the information.

(2) Lack of consistency.

(3) Presence of incomplete schedules.

(4) Lack of homogeneity.

1. Non-dependability of the information.

The information which is collected is supposed to be authentic. In case of wrong information, the conculsion will not be valid. Therefore accuracy will come only from correct answers The editor has to check the questionnaire for any error in totalling or carrying forward. He can correct these figures. Some mistakes cannot be detected at the time of scrutiny and editor has to find ways to check such mistakes.

2. Lack of Consistency.

For editing the data, editor should see that answers to the questions are not contradictory. If mutually contradictory answers are available in a particular schedule they should be removed by refering it to the source or person who has replied like this. For example, if a person is bachelor, he cannot have children. Such contradiction has to be removed and data has to be consistent.

3. Presence of Incomplete Schedulels

Editor has to see that all the questionnaires or schedules are complete in all respects. In case of some incomplete schedules informants should be contacted personally or through agent. If he can not get the desired information, he should leave that question from that schedule and if that question is of vital importance, then that questionnaire should be dropped.

4. Lack of Homogeneity.

Homogeneity of answers will be obtained if all the informants understand and answer the questions in the similar manner. The

editor has to check the schedules in this regard. For example, in relation to a question on income has the informant supplied monthly income or yearly income ? The editor has to ensure homogeneity of schedule in this regard.

QUESTIONS

1. Distinguish between primary and secondary data. What are the various methods by which primary data are collected ?

2. Explain the advantages of direct personal investigation as compared with the other methods generally used in collecting data.

3. Describe the primary and secondary methods of collecting data. In what special circumsiances are the two methods suitable ?

4. What do you mean by collection of data ? What are the preliminary steps before collecting data ?

5. Define 'secondary data'. State their chief sources and point out the dangers involved in their use. What precautions are necessary before using such data ?

6. "It is never safe to take published statistics at their face value without knowing tneir meaning and limitations and it is always necessary to criticise the argument that can be based upon them" Bowley. Elucidate.

7. State the preliminary steps you would take for planning a statistical enquiry.

8. Enumerate the important sources of secondary data. What precautions should one take before using such data ?

3

Sampling and Sampling Methods

Introduction. In the last chapter we have studied different methods of collecting data. There are two methods of collecting data.

1. Census Method
2. Sample Method.

The census method, which we call complete enumeration deals with the entire population for investigation. In the case of sample method, which we call as partial enumeration, a sample is chosen from the universe or population for the purpose of investigation. It should be noted that the word population does not mean the number of people, but it has a technical meaning. Population or Universe means a collection or aggregate of objects in any enquiry. For example, if the enquiry relates to the students of any college, then all the students of that college constitute a population. Similarly, if the study relates to the unemployment situation in India, then all the unemployed persons in India constitute the population. Let us now discuss census method and sample method one by one.

1. Census Method. In this method data are collected from each and every unit of the population. Our unit may be student, unemployed person, factory, household or a shop depending upon the study under consideration. This method of getting information from all the units is also called complete enumeration. For example, if the enquiry relates to the study of average daily pocket expenses of students of a particular college, then in census method information about pocket expenses will be collected from each and every student of that particular college. If the scope of the enquiry is large then census method requires a lot of time, money and other resources. The census method can provide more accurate and exact information as data is collected from each and every unit of the population. In India, Population census is conducted after every ten years.

Merits :

(*i*) The data collected from census method are more accurate and reliable.

(*ii*) Data collected from census method can be widely used as a basis for various surveys.

(*iii*) This method is free from sampling errors. The possibility of personal bias affecting the selection of units and the method of sampling are not there.

(*iv*) Since data is collected from each and every unit of the population, all the characteristics of the population can be studied.

Demerits :

(*i*) This method requires a large number of investigators and other staff involving a lot of money.

(*ii*) It needs a lot of time.

(*iii*) This method becomes useless when results are needed urgently.

(*iv*) In some cases it is not possible to make use of the census method. For example, in the case of enquiries relating to life of candles, electric bulbs, strength of crackers etc., we cannot use census method.

2. **Sampling Method**. Sampling method is a method of knowing about the population on the basis of a *sample* drawn from it. A sample is a part of the population which is taken from it to study the characteristics of the population. It is not always possible to conduct a census enquiry. In actual practice a census enquiry is very rarely undertaken because of the scarcity of time, money and other resources. Then the only alternative is to make use of sample method. The process of sampling involves three elements :

(*a*) Selecting a sample
(*b*) Collecting the information, and
(*c*) Making inferences about the population.

Today sampling is the most important tool and is used extensively in business, research, medical science, administration, defence, planning and in fact, in every field. Most of our decisions, our attitudes and our knowledge of almost every branch of study whether scientific or otherwise, including the ordinary actions of daily life depend very much upon the examination of only a few objects out of a big lot.

Innumerable examples and situations can be quoted to show how much we depend upon sample study even in the elementary affairs of our life where we make use of this type of study without knowing it. A student is examined by asking only a few questions out of a large prescribed course, a consumer purchases vegetables after estimating the quality by testing one or two units out of the seller's basket, the manufacturer asks some of its customers how they liked his commodity and if most of them make positive comment, he assumes that his commodity is generally approved. The housewife usually takes a spoonful of the cooked products to ascertain if it is properly cooked and also to see if it contains proper quantity of salt or sugar. A businessman orders for the products after examining only a sample from it. The error involved in approximations about the population characteristics on the basis of the sample is known as **sampling error and is inherent and unavoidable in any sampling scheme.**

Merits:

(*i*) **Reduced cost.** Sampling results in the reduction of cost in

terms of money and resources because we have not to approach each and every member of the universe.

(*ii*) **Saving in Time.** Sample enumeration requires much less time in comparison to census method. This is one of the important considerations, if we need results urgently.

(*iii*) **Only Method.** In the statistical quality control, when the investigation entails destruction of material, *e.g.* the life of a bulb, biscuits, etc. sampling is the only way with us.

(*iv*) **Organising convenience.** In comparison to census, it is very easy to organise.

(*v*) **Intensive enquiry.** With small number of members in a sample, we can have greater precision and indepth study.

(*vi*) **Reliable.** Conclusions and results obtained by sampling method are more reliable.

(*vii*) **Scientific.** If we select a sample based on some principle that makes the sample representative of the population, the results obtained by this method will be scientific, since no bias has been introduced.

Demerits.

(*i*) If a sample survey is not properly planned and executed, the results given by this method will be inaccurate and misleading.

(*ii*) If sample is not representative, then results of the sample may not be valid for the universe from which the sample was drawn. For this sampling technique used should be scientific and sample should be carefully drawn.

(*iii*) We need trained and qualified personnel for planning, execution and analysis of a sampling enquiry.

(*iv*) We need setting up of a suitable organisation for its execution.

(*v*) Sampling method is more exposed to personal bias and prejudices of the investigators.

(*vi*) If the size of the sample is inadequate, it may fail to bring out the characteristics of the population.

Techniques or Types of Sampling. A technique with the help of which we take a few items from the entire population is called as a sampling technique. Given below are some of the important techniques or types of sampling. For convenience, these are divided into two categories.

A. Probability Sampling Methods,
B. Non-Probability Sampling Methods.

A. Probability Sampling Methods. In these sampling methods the samples are drawn in such a way that each unit of population has some definite chance or probability of being included in the sample. Some of the important probability sampling methods are :

(I) Simple Random Sampling.
(II) Stratified Random Sampling.

(III) Systematic Sampling.
(IV) Cluster Sampling.

(I) Simple Random Sampling. It is a sampling technique in which each unit of the population has an equal chance of being included in the sample. If the unit selected in any draw is replaced before the second draw, then it is known as simple random sampling with replacement (SRSWR) and if it is not replaced, then the sampling plan is called simple random sampling without replacement (SRSWOR).

Let us suppose that there are N units in the population. Then under simple random sampling, each unit of population has an equal probability *i. e.*, $\frac{1}{N}$ of being included in the sample. It should be noted that the word *random* does not mean *haphazard* or *hit or miss.* It rather means that the selection process is such that the chance only determines which items are to be included in the sample. This eliminates the personal bias of the investigator in the selection of sample units. There are two well known methods of selecting a random sample. These are :

(i) **Lottery Method.** This is a very simple and popular method of selecting a random sample. In this method all the units of the population are assigned numbers. Separate slips which are identical in shape, size and colour are made for each unit and a number is written on it. These slips are folded and mixed together. They may be put in a container or drum. From this, the required number of slips are picked out blind folded. The units of the population whose numbers are marked on our selected slips constitute the required random sample.

(ii) **Use of Random Number Tables.** If the size of the population is large, then it becomes difficult and expensive to use the lottery method for taking a random sample. In such a situation random number tables can be used which have been constructed in such a way that each of the digits 0, 1, 2, 3,......, 9 appear with approximately the same frequency and independently of each other. The method of drawing a random sample comprises the following steps :

(*a*) If size of the population is N, then the units of the population are assigned numbers from 1 to N.

(*b*) Select at random any page of the random number tables. Depending upon the value of N, any column or columns are selected. For example if N=90, then any two columns of selected page can be chosen. If N=999, then three columns can be selected.

(*c*) The population units corresponding to the numbers selected in step (*b*) constitute the random sample.

There are many random number tables constructed by different authors. Most common among them are :

(*a*) *Tippet's Random Number Tables.* These tables consist of 10,400 four digited numbers taken from British Census Reports.

(*b*) *Fisher and Yate's Tables.* These tables comprise 15,000 digits arranged into 1500 sets of ten digit random numbers.

(*c*) *Kendall and Babington Smiths Tables.* These tables consist of 1,00,000 digits grouped into 25,000 sets of 4 digits each.

(*d*) *Rand Corporation Tables.* These tables consist of 1,00,000 random digits grouped into 20,000 sets of five digits each.

Merits of Simple Random Sampling.

1. Personal bias of the investigator in sample selection is eliminated.

2. The efficiency of estimates can be ascertained by using the standard errors* of their sampling distributions*.

Demerits of Simple Random Sampling

1. In this method it is required to assign numbers to all the members of the population. But if the population is very large, then numbering becomes a difficult, costly and time consuming process.

2. If the population is heterogeneous in nature, then the random sample will not be representative.

(II) **Stratified Random Sampling.** It is also called as *restricted random sampling*. This sampling method is generally adopted when the parent population is hetrogeneous with respect to the characteristic under study.

In this method the whole population of n units is divided into a number of homogeneous sub-populations of $N_1, N_2, N_3, \ldots\ldots, N_k$ units. These sub-populations are called *stratum* in singular sense and *strata* in plural sense. These strata are non-overlapping and together sum upto the whole of the parent population. In other words,

$$N_1+N_2+N_3+\ldots\ldots+N_k=N.$$

After dividing the parent population into different strata, simple random samples are drawn from each stratum say of size $n_1, n_2, n_3, \ldots\ldots, n_k$. These $n_1+n_2+n_3+\ldots\ldots+n_k$ units taken together will constitute our desired sample of size n. Therefore, the sample which is the aggregate of the sampled units of each of the stratum is termed as *stratified sample* and the technique of drawing such a sample is called *stratified random sampling*.

Merits of Stratified Random Sampling.

1. A properly constructed stratified sample is free from the drawbacks of simple random sampling.

2. It is administratively more convenient. Different persons can be incharge of different strata.

3. Influence of extreme items in the population can be minimum by putting them in a different stratum.

4. Sometimes different amounts of accuracy is desired for different

*These are discussed in detail at the end of this chapter.

parts of the population. This is possible only in stratified random sampling.

Demerits of Stratified Random Sampling.

1. There is possibility of introduction of personal bias of the investigator in the selection of different homogeneous groups.

2. The sample will not be respresentative if the size of sample selected from each stratum is not appropriate.

(III) **Systematic Random Sampling.** This sampling method is adopted when the population units are arranged in some systematic order such as alphabetical, chronological, geographical order etc. Let us suppose that population size is N and population units are arranged in some systematic order with numbers from 1 to N. If required sample size is n, then

$$k = \frac{N}{n},$$ where k is called the sample interval.

The first unit of the sample is selected at random from first 1 to k numbers. If this is the ith unit, then ith, $i+k$, $i+2k$,.........,$i+3k$,.........,$i+(n-1)k$th units of the population will constitute our required systematic sample. This type of sampling is also called *Quasi-Random Sampling* because only first sample unit is selected at random. The other sample units are taken in a systematic order.

Merits of Systematic Random Sampling.

1. Sample selection is easier.

2. It involves less cost, time and labour as compared to simple random sampling and stratified random sampling methods.

3. If the population units are arranged in a random order, then systematic sampling may be more accurate than simple random sampling.

Demerits. If the population has certain hidden periodicities, then systematic random sample will not be true representative of the population.

(IV) **Cluster Sampling.** This sampling method is used when the population is very large and it is scattered over a wide area. In this case the total population is divided into some recognisable categories which are called clusters. A simple random sample of these clusters is then drawn. Our sample constitutes the required data relating to each and every unit in the chosen clusters.

If instead of including all the units in the selected clusters in our sample, we resort to sub-sampling *i.e.* take some of the units from these clusters in our sample, then it is called multi-stage sampling. Let us take an example. Suppose we want to make an enquiry about the daily pocket expenses of college students in a city. First we shall select some colleges from all the colleges in the city. Then we can select some students from each college already selected. From the total students selected in the first two stages we may select a random sample of the

required size. The sampling technique used in this case is called multi-stage sampling because at each stage we take a random sample.

Merits. 1. This method introduces flexibility in the sampling process.

2. It reduces the cost of taking samples because we need the second stage frame for only those units which have been selected in the first stage sample.

Demerits. It is possible that there are more errors in the sample selection by this method as compared to other methods. Moreover, the dispersion of the estimates under this method may be more than the estimates based on simple random sampling method.

B. Non—Probability Sampling Method. The sampling methods discussed above are based on probability theory or random process. However, there are various non-probability or random methods which are widely used for selecting a sample. These are

(*i*) Judgement Sampling
(*ii*) Convenience Sampling
(*iii*) Quota Sampling-

(*i*) **Judgement Sampling.** As the name itself implies the sample is selected on the basis of the judgement of the investigator. Those items are taken in the sample which truely represent the characteristics of the population. This type of sampling is also called purposive or deliberate sampling because here sample units are selected by keeping in mind the purpose of enquiry and not by the random process. For example, suppose the enquiry relates the performance of different classes in college and each teacher is asked to send five students from his class for this purpose. To prove better performance of his class, each teacher is likely to send best five students from his class. If sample selection is made in this way, then it is called purposive or judgement sampling.

Merits. 1. This method is very useful when the size of the population under study is very small. In this case the random sampling process may exclude most important items in the population.

2. This method is generally used in every day business problems and for making urgent decisions.

Demerits. This sampling method is totally based on the investigator. If the investigator has good knowledge about the form of the population, then he can select a good sample, otherwise not. Moreover, if the investigator is biased, then the sample will not be representative and it will also be a biased one.

The utility of this method depends upon the knowledge, impartial approach and the excellence in the judgement of the investigator. Thus, if this method is to be used then trained, mature and impartial investigators should be employed.

(*ii*) **Convenience Sampling.** In this case the sample is selected by considering those units of the population which are conveniently

approachable. For example, suppose the enquiry relates to the drinking habits of the people living in the villages of Ludhiana, Jalandhar and Amritsar districts. So, the villages which are on or near G.T. Road in these districts can be included in the sample. Information may be collected from those persons in these selected villages who are present at the time of survey.

This method is easy and saves a lot of time. However, it is very difficult to obtain a representative sample when this method is used. This method is generally employed in public opinion polls and for this information is collected from persons at bus stand, railway station, cinema houses, offices, colleges etc.

3. **Quota Sampling**. This is a type of Judgement sampling. In this method each investigator is asked to interview a fixed number of persons. This is called Quota. These quotas may be fixed according to some special characteristics which may be sex, occupation, political or religious affiliations etc. Within the quota, the selection of sample items depends on the personal judgement of the investigator. Therefore, like judgement sampling, quota sampling also depends on the ability, training, experience and knowledge of the investigator.

Quota sampling is also generally used in public opinion studies. It provides satisfactory results if the investigators are properly trained and they are impartial. However, because of the risk of personal bias of investigators, this method is not widely used in practice.

QUESTIONS

1. Explain the terms a 'random sample', 'stratified random sample' and 'purposive sample'.

Explain the importance of sampling theory in economics.

2. (*a*) What is sampling ? What precautions would you take in choosing a sample ?

(*b*) Explain why a sample survey is usually preferred to a census survey. Give one example of a situation where a census survey is imperative.

3. (*a*) Discuss the relative merits and demerits of the census method and the sampling method. Describe briefly different sampling methods.

(*b*) What is stratified sampling ? Comment on the relative advantages and disadvantages of simple random sampling and stratified sampling.

4. What is random sampling ? How can a random sample be selected ? Is random sampling always better than other forms of sampling in the context of socio-economic surveys ?

5. Distinguish between the 'Census' and 'Sampling' methods of collecting data and compare their merits.

6. Classify the methods generally employed in the collection of statistical data and state briefly their respective merits and demerits.

4

Diagrammatic Presentation

Introduction. The statistical data can be presented in three different ways (*i*) Tables, (*ii*) Diagrams and, (*iii*) Graphs.

However, figures and numbers are not interesting to all. To many they are dull and confusing ; and if their number is pretty large, it would be difficult to compare them and observe their differences. Therefore, it is necessary to adopt a device which may present huge mass of quantitative data in a way that is at once comparable and appealing both to the eye and to the intellect. One such very simple and effective form of statistical device is to represent the tabular data by drawing diagrams. Diagrams have a more lasting effect on the mind of a person. For example, a doctor finds it easy to judge the condition of a patient by looking at his temperature chart recorded over a period of time rather than at the table of temperatures.

Usefulness of Diagrams.

Diagrams are nothing but geometrical figures like bars, squares, rectangles, circles, cubes etc. According to M.J. Moroney, "**Diagrams help us to see the pattern and shape of any complex situation. Just as a map gives us a bird's eye view of the wide stretch of a country, so diagrams help us to visualize the whole meaning of a numerical complex at a single glance.**" The usefulness of diagrams becomes clear from the following points :

1. The most important advantage of diagrams is that they are very attractive and give an effective impression, one may not like to devote even a minute to the study of a page, a small page—containing a number of quatitative figures, and even if he devotes time, numerical figures may go out of his mind soon after he has studied them. But the same person may not take his eyes away from a picture relating even to the same topic to which the numerical data did. This is based on human psychology, and a successful advertiser always exploits this psychology of the people to win his mark.

2. With the help of diagrams the comparison between two or more sets of any phenomenon can be easily made. For example, imports and exports trends of a country at different times can be easily compared through diagrams. Similarly, the use of diagrams helps us to compare the imports and exports figures of two countries at any time.

3. Diagrams have a great memorising value than mere figures. This is so because the impression left by diagrams is of a lasting nature.

4. Diagrams are very useful where audience concerned is illiterate. With illiterate people we can converse effectively through diagrams.

5. Since no effort is needed to understand diagrams, they save a lot of time which is otherwise required while drawing inferences from a set of figures.

6. Diagrams play an important role in the modern advertising campaigns. The advertisement columns of newspapers and journals are filled with diagrams of all kinds.

7. Diagrams provide, in a summarised form, the whole data and their important characteristics which can be seen at a glance. According to Calvin F Schmid, "Diagrams **make possible the presentation of quantitative data in a simple, clear and effective manner and facilitate comparison of values, trends and relationships.**"

8. Diagrams are extremely useful in all spheres especially business, economic and social studies. Economic laws can be easily explained with the help of diagrams. When a social reformer is addressing an audience mere reading out of figures would make the hearing dull, tedious and tiring. But, if he appears on the platform with pictures, diagrams etc. his talk would be interesting, lively and impressive. A business man, or an administrator, has hardly any time to devote to the study of a huge mass of figures, however well arranged. But, if he is presented with diagrams and pictures it will hardly take him a few minutes to understand the significance of the whole data. It is, thus, evident that diagrams, charts, pictures and similar other visual aids serve a more useful purpose than any other device.

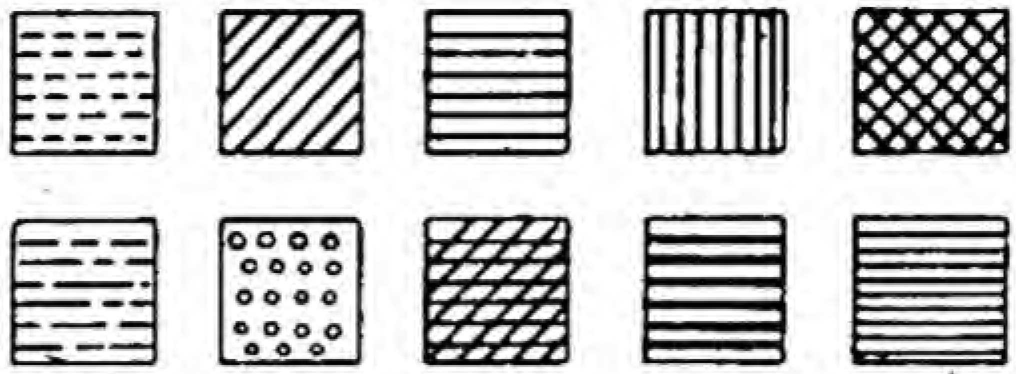
Rules for Constucting a Diagram.

It should be noted that diagrams do not add anything to the meaning of statistics. They give only a method of presenting them. They are only a means to an end, the end being the study of main features of data and making comparisons. Consequently, if there is only one isolated numerical quantity, there is no sense in presenting it diagrammatically. Similarly, if there are many figures, in no way related to one another and, therefore, having no common characteristic, they are incomparable and, therefore, need not be diagrammatically presented. Thus, it is established that the method of diagrammatic representation can be made use of when there are at least two numbers which are similar in nature and character at least in one important respect. The following general rules should be observed while constructing diagrams:

1. Neatness.

A good diagram should be neat and clean. Its presentation should be very attractive and appealing to the eye. Appropriate devices like different colours, shades, dots, dashes, dotted lines, broken lines etc.

can be used to make the presentation of a diagram very attractive. Some commonly used devices are given below :



2. **Title.**

Each diagram should possess a suitable title. Title indicates the subject matter of the diagram. As far as possible, the title should be brief, clear and self explanatory. It can be shown either at the top of the diagram or below it.

3. **Footnotes.**

If there are some points which are not covered in the title, they can be mentioned separately in the footnotes at the left hand bottom of the diagram.

4. **Selection of Scale.**

The size of the diagram changes with the change in the scale to which it is drawn. The scale should be so chosen that the diagram looks neither too small nor too large. However, there are no hard and fast rules for the choice of scale. Preferably, scale should be presented in multiples of 5, 10, 20, 25, 100 etc. The vertical scale should be given on the left hand side of the diagram. The horizontal scale should be given at the botton of the diagram. On each side, the vertical and horizontal, the variable represented should be indicated. If two or more diagrams are to be compared, the same scale should be used in each to draw valid conclusions.

5. **Proportion between width and height.**

There has to be a proper proportion between the width and height of the diagram. Again there are no hard and fast rules for this. Generally the 'root two' rule suggested by Lutz in his book Graphic Representation, is used. In other words, the longer side should be $\sqrt{2}$ or 1·414 times the shorter side.

6. **Index.**

To make clear the meaning of different types of lines, colours, shades, designs used a brief index should be given.

7. **Simplicity.**

As far as possible diagrams should be simple so that even a person without any mathematical or statistical background should be able to understand them and draw valid conclusions from them.

8. Choice of the Diagram.

A large number of diagrams exist for the presentation of statistical data. The choice of a particular diagram depends on the nature of the available data, number of observations and the type of the people for whom the diagrams are needed. Each diagram must suit the purpose of making it.

Types of Diagrams.

There are a large number of diagrams which are used in practice to present the data. For the sake of simplicity and convenience we study the following :

1. Bar Diagrams
2. Pie Diagram
3. Histogram.

Bar Diagrams.

Bar diagrams are very useful, simple to draw and easy to understand. Business and economic data are presented mostly by bar diagrams. Bar diagrams consist of a series of bars of equal width standing on a common base line, the lengths of these bars being proportional to the magnitude of the variables that they represent. A bar is merely a thick line whose width, though shown in the diagram is not taken into consideration. These diagrams are called one-dimensional because only height (length) of the bar is taker into consideration. The following points are noted while making bar di: grams :

(*i*) The width of all the bars is the same.

(*ii*) The gap between any two successive b s is also same.

(*iii*) Bars may be either vertical or horizo ıtal. Generally the vertical bars are preferred.

(*iv*) The scale is adjusted in relation to the magnitude of the biggest item.

(*v*) Figures or magnitudes represented by bars are generally written on the end of the bar to make the value represented very clear.

Types of Bar Diagrams :

(*a*) Simple bar diagrams
(*b*) Multiple bar diagrams
(*c*) Sub-divided bar diagrams
(*d*) Percentage bar diagrams
(*e*) Deviation bar diagrams
(*f*) Broken or split bar diagrams.

(*a*) **Simple Bar Diagrams.** A simple bar diagram represents only one variable. Since one bar represents only one figure, there will be as many bars as the number of figures. For example, data relating to wages, profits, production, national income, imports, exports etc., for different periods may be presented by simple bar diagrams. The scale is adjusted in such a way that the longest bar can be easily adjusted within the available space. All the bars are made equi-distant. If the number of items is very large, the width of bars is reduced and sometimes, when number of items is very very large, bars are replaced by lines.

Example 1. Represent the following data by a suitable diagram.

Years	1951,	1961,	1971,	1981
Population (in crores)	35	44	55	69

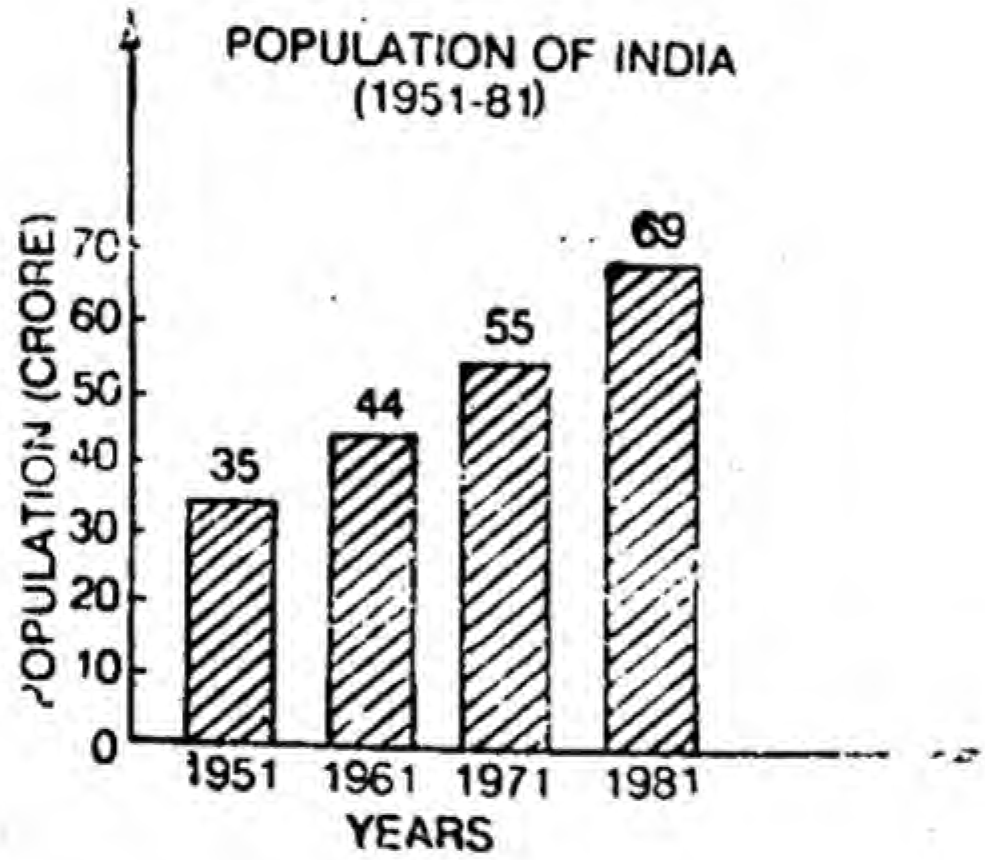


Fig. 1.

The above data is represented by the above simple bar diagrams.

Note. The width of these bars is same and height is equal to birth rate.

(*b*) **Multiple bar diagrams.** These diagrams are prepared on the basis of simple bar diagrams. Multiple bar diagrams represent more than one type of data. For example, if every year number of male and female inhabitants of a certain city are represented by two bars, the diagram would be a multiple bar diagram. Here we construct two or more bars side by side. Proper and equal spacing is given between different sets of bars. Different colours dottings and crossings may be used to distinguish between different bars.

Example 2. Represent the following data by a suitable diagram.

Years	Sales (in cr. Rs.)	Gross Profit (in cr. Rs.)	Net Profit (in cr. Rs.)
1984	200	100	50
1985	175	50	25
1986	250	100	75

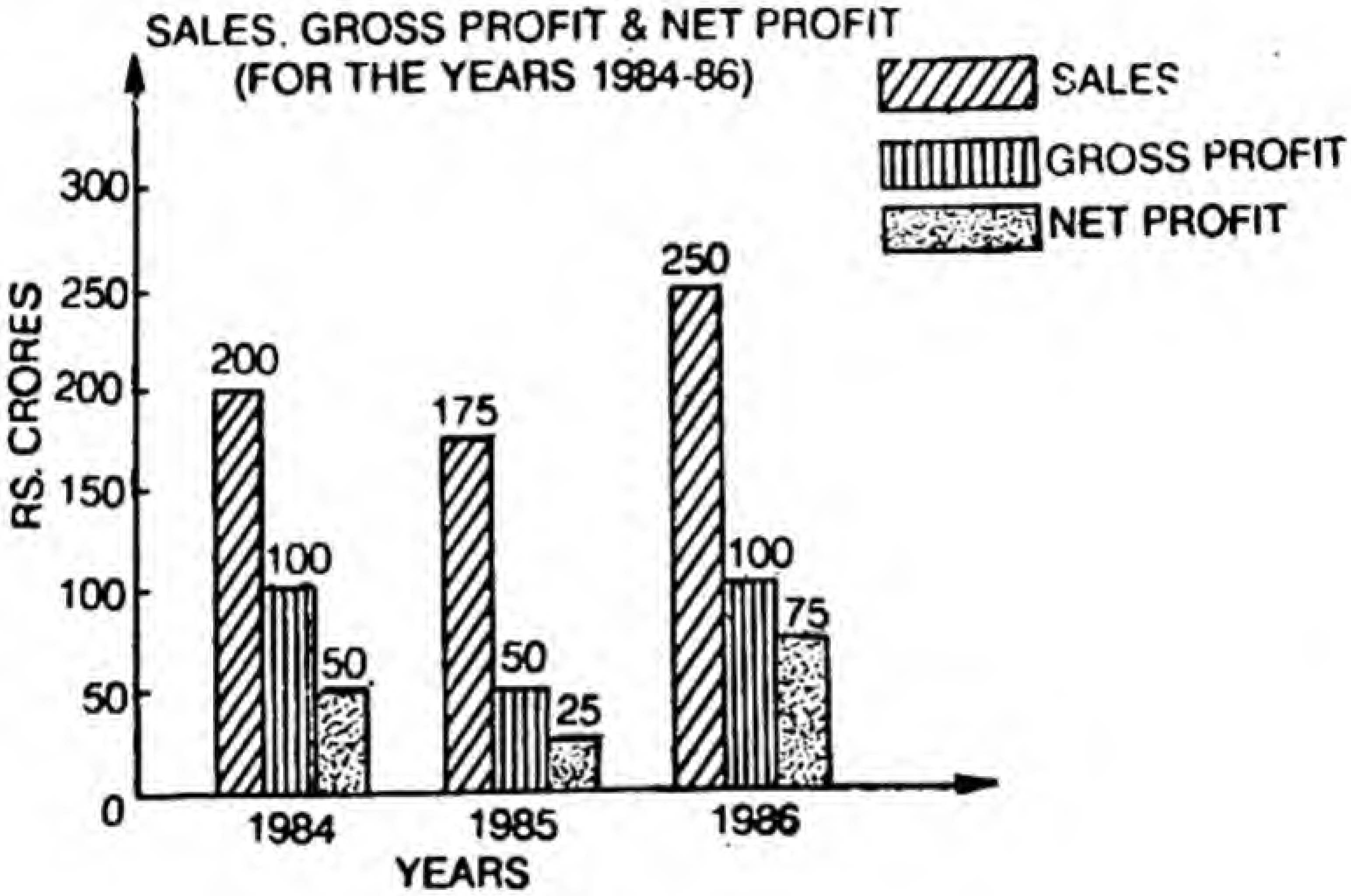


Fig. 2.

(*c*) **Sub-Divided Bar Diagrams.** The third kind of bar diagrams is that in which each bar is divided into certain parts and each part represents a particular phenomena. For example, if the number of inhabitants in a city for last ten years are to be shown on the basis of their caste distribution, sub-divided bar diagrams can be prepared. One bar would represent total number of inhabitants in a city and it will be divided in three, four or five parts depending upon the number of castes, Hindu, Sikhs, Christian etc. in the city. The length of these parts would be proportional to the number of inhabitants in the various castes. Different shades or colours, crossings or dotting are used to distinguish the various components and an index is given to explain these differences.

Example 3. Represent the following data by a suitable diagram.

Year	Males (in cr.)	Females (in cr.)	Total Population (in cr.)
1961	25	10	35
1971	30	20	50
1981	50	25	75

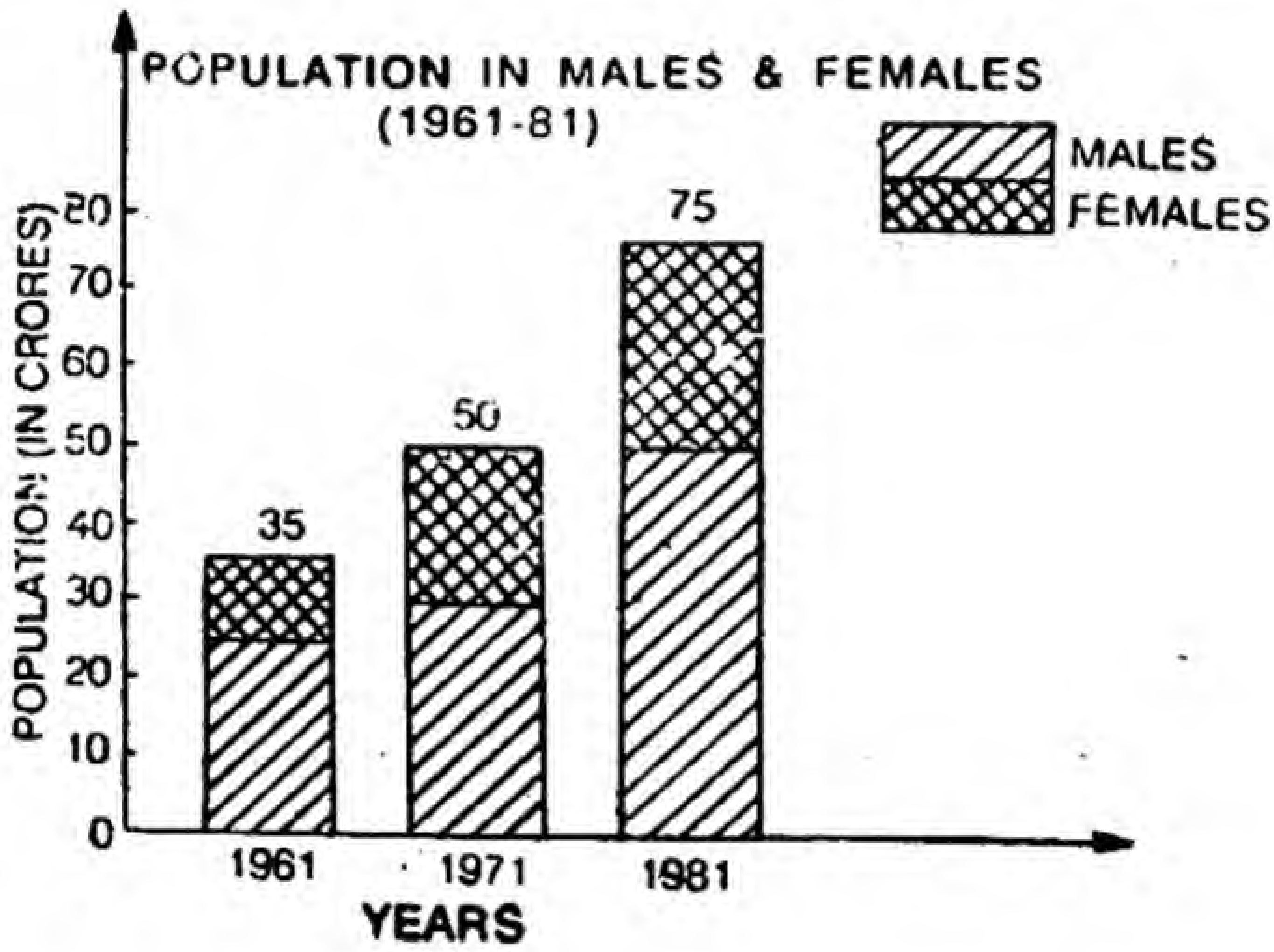


Fig. 3.

(d) Percentage Bar Diagrams. The distribution of an aggregate into its parts may also be shown on a percentage basis. Such diagrams are useful when relative differences or changes in the size of their components are to be shown. The total of each bar is taken as 100 and the value of each component is expressed as a percentage of the respective totals. This implies that the height of all the bars will be the same (indicating 100%). Such diagrams are very useful for comparison purposes. The procedure of constructing a percentages bar diagram is clear from the following example.

Example 4. Represent the following data regarding monthly expenditure of two families by a suitable diagram.

Item of expenditure	Food	Clothing	Rent	Fuel	Miscell.	Total
Family A	200	100	80	40	80	Rs. 500
,, ,, B	250	200	100	50	200	Rs. 800

Sol. Calculation for Drawing Percentage Bars Diagrams

Family A

Items	Total	% Total	Cumulative %
Food	200	$\frac{200}{500} \times 100 = 40$	40
Clothing	100	$\frac{100}{500} \times 100 = 20$	60
Rent	80	$\frac{80}{500} \times 100 = 16$	76
Fuel	40	$\frac{40}{500} \times 100 = 8$	84
Miscell.	80	$\frac{80}{500} \times 100 = 16$	100
Total	500		

Family B

Items	Total	% Total	Cumulative %
Food	250	$\frac{250}{800} \times 100 = 31{\cdot}25$	31·25
Clothing	200	$\frac{200}{800} \times 100 = 25$	56·25
Rent	100	$\frac{100}{800} \times 100 = 12{\cdot}5$	68·75
Fuel	50	$\frac{50}{800} \times 100 = 6{\cdot}25$	75·00
Miscell.	200	$\frac{200}{800} \times 100 = 25$	100·00
Total	800		

The given data is shown by percentage bar diagrams.

(e) **Deviation Bar Diagrams.** Such bar diagrams are generally used to represent net quantities. For example, when we are interested in showing net surplus or net deficit, net profit or net loss, net imports or net exports etc., we make use of deviation bar diagrams. The positive deviations are presented above the base line and the negative

deviations are presented below the base line. The example given below makes clear the procedure of drawing such a diagram.

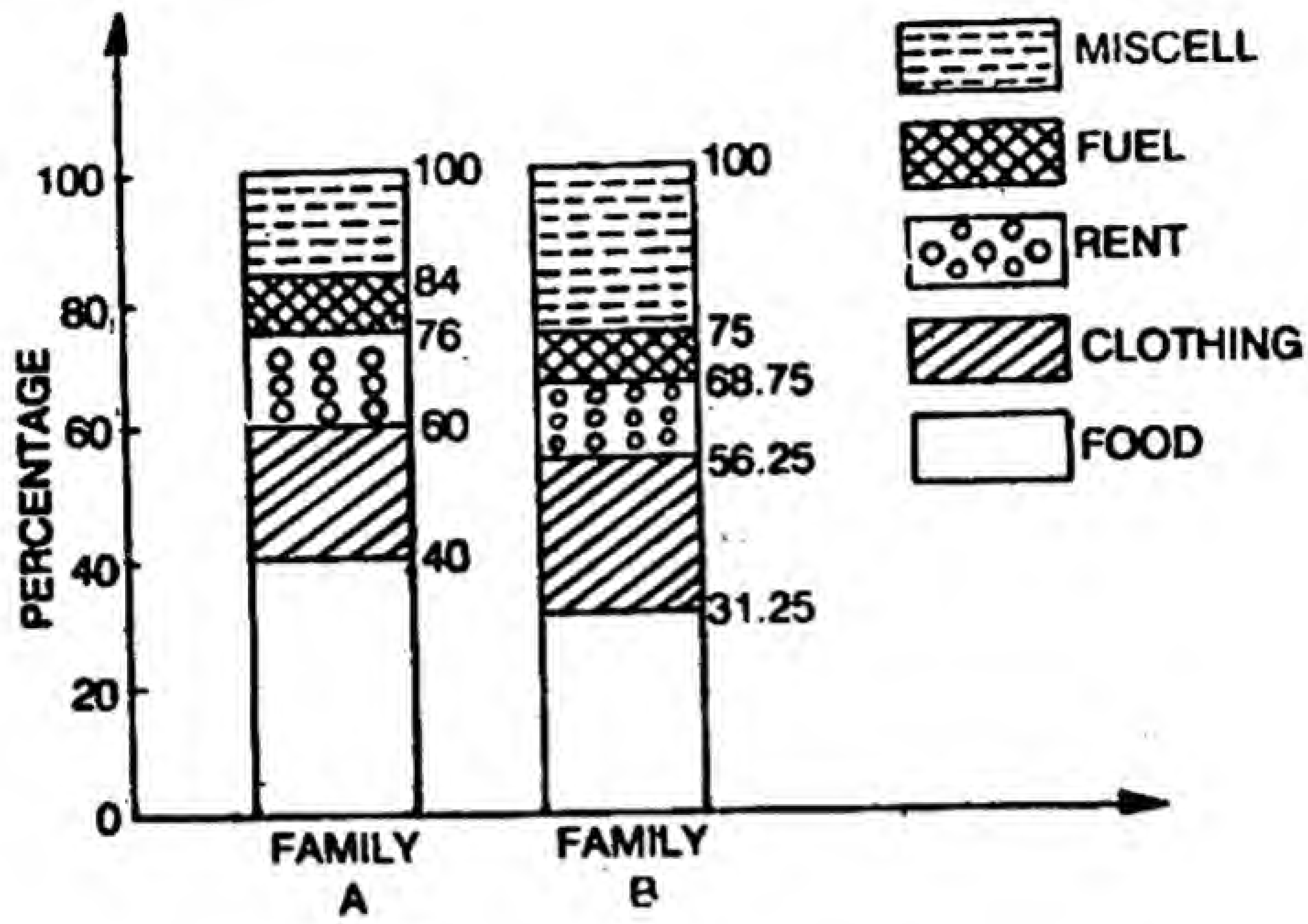


Fig. 4.

Example 5. Present the following data by a suitable diagram showing the difference between receipts and payments.

Year	1983	1984	1985	1986	1987	1988
Receipts	110	135	140	150	160	165
Payments	95	105	150	125	130	170

Sol. We shall draw a Deviation Bar Diagram.

Year	Receipts	Payments	(+) Balance	(—)
1983	110	95	15	
1984	135	105	30	
1985	140	150		10
1986	150	125	25	
1987	160	130	30	
1988	165	170		5

Deviation Bar Diagram

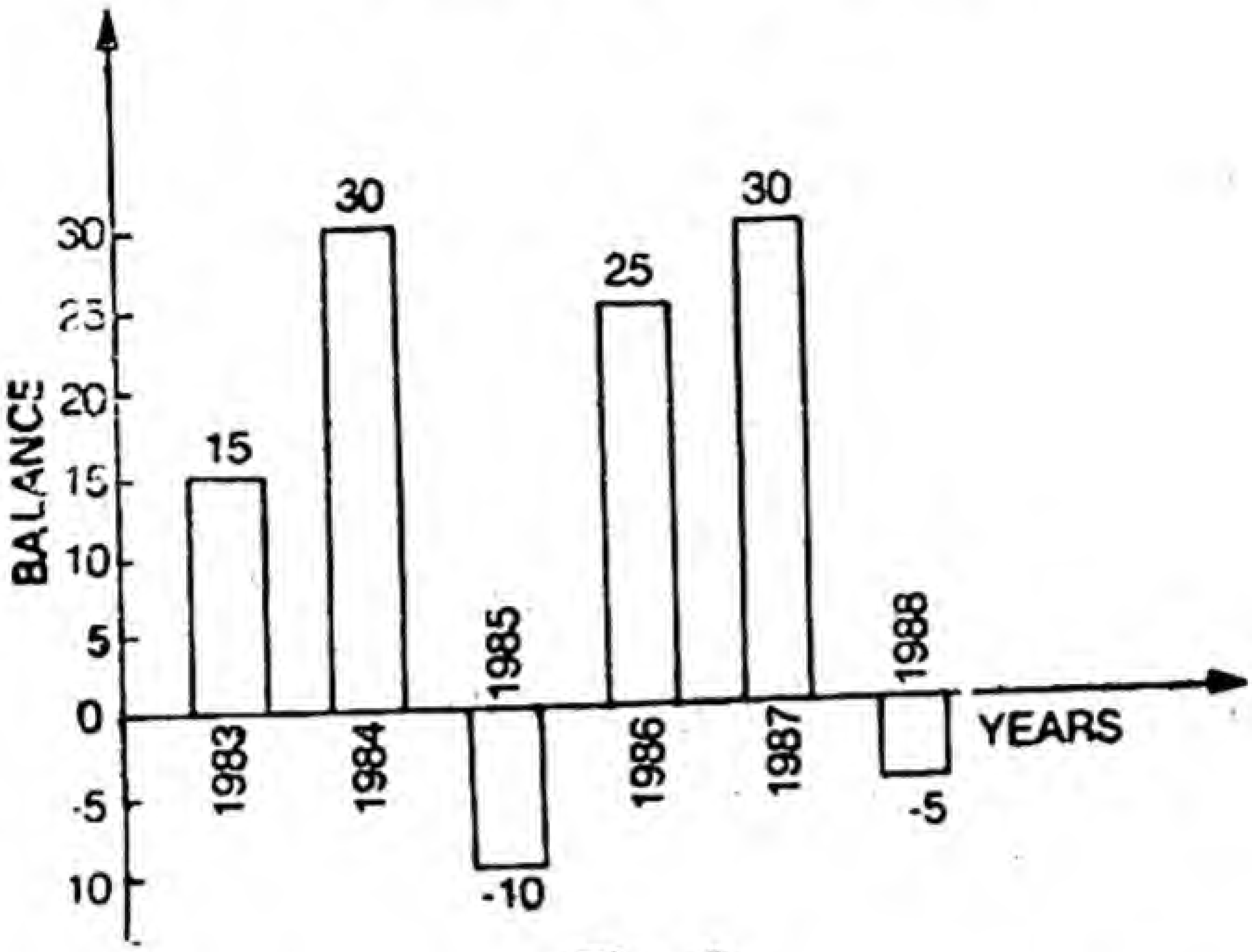


Fig. 5.

(f) Broken or Split Bar Diagrams. Such bar diagrams are made to present the data showing wide variations in the values i.e.. the data containing very large observations along with very small observations. In order to gain space for the smaller bars of the series, the largest bar may be broken. It is clear from the following example.

Example 6. Represent the following data by a suitable diagram.

Division :	First	Second	Third	Fail
No. of Students :	35	40	42	400

Sol. Construction of **Broken Bar Diagram.**

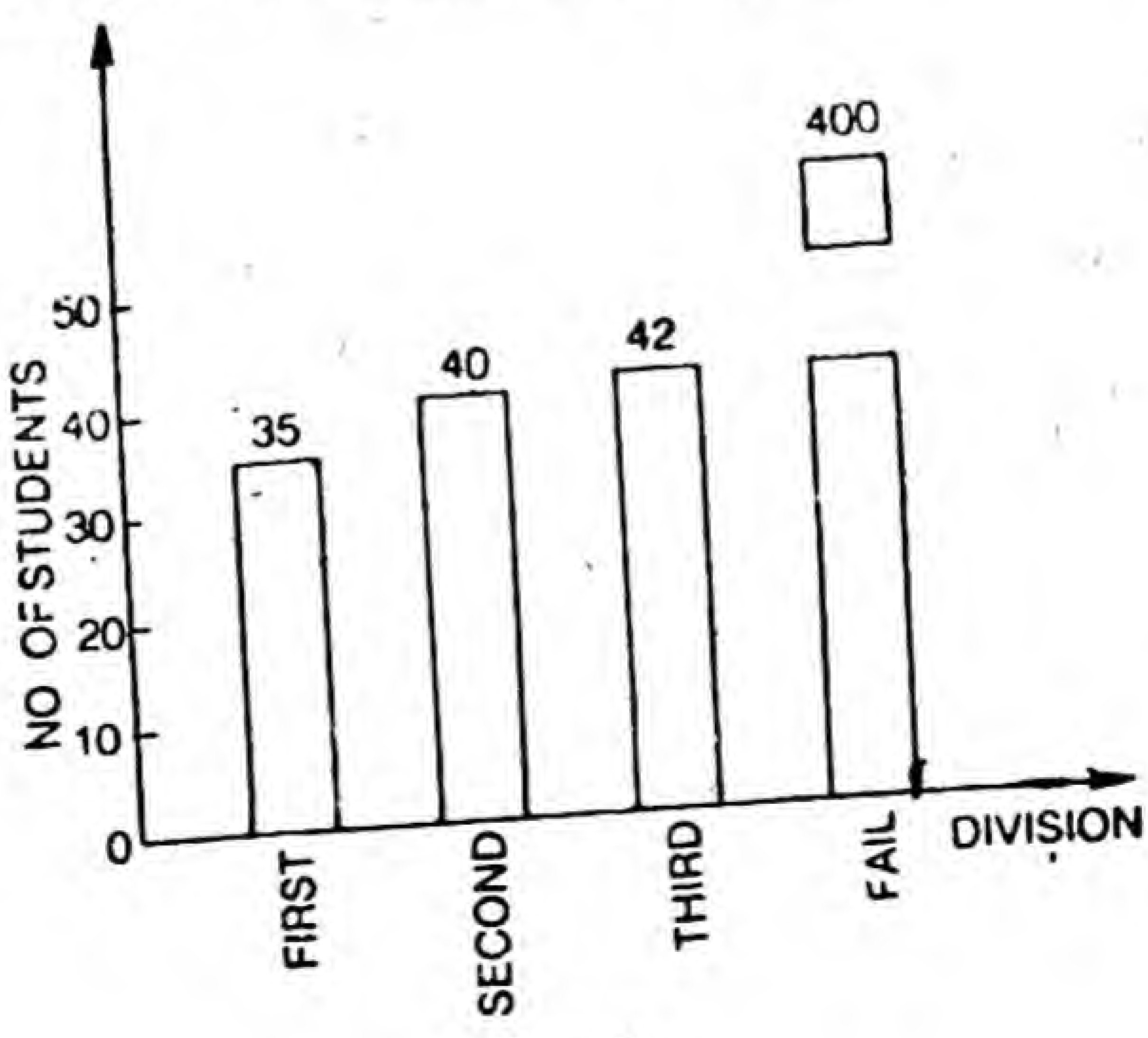


Fig. 6.

2. Angular or Pie Diagram :

A sub-divided circle diagram is known as an angular or pie diagram. Just as we can construct sub-divided or percentage bar diagram or rectangles, in the same way a sub-divided circle diagram is made to represent total magnitude and its various components. A pie diagram is so called because it looks like a pie and the components resemble with slices cut from a pie.

The pie diagram is drawn by first drawing a circle of a suitable radius and then dividing the angle of 360° at its centre in proportion to the figures given under various heads. Thus the areas of the segments made from the circle are in proportion to the number (percentage) of items. The following are the steps involved in constructing a pie diagram :

(*a*) Value of each segment is expressed as a percentage of the total.

(*b*) The total value is taken as 360°. *i.e.*, in other words, 100% is considered as 360°, 1% = 3·6°. The percentages in step (*a*) can be written in degrees by multiplying each percentage value by 3·6.

(*c*) A circle of any radius is drawn if only one characteristic is to be studied. However, if two or more sets of data are to be shown, then the radius of the corresponding circles is taken in proportion to the square roots of their total magnitudes.

(*d*) With the help of a protractor size of each sector (denoted by degrees) is measured on the circle and lines are drawn.

(*e*) Different sectors representing various component parts can be distinguished from one another by using different colours, dottings, shades etc.

The procedure of drawing pie diagram is clear from the following example.

Example 7. Present the following data with the help of a Pie diagram.

Items	*% Share in National Income*
Agriculture	40
Industry	21
Transport	19
Administration	13
Banking	7

Solution. Percentage values are converted into angles of different degrees.

Items	*% Share*	*Angles*
Agriculture	40	$\frac{40}{100} \times 360 = 144°$
Industry	21	$\frac{21 \times 360}{100} = 75·6°$
Transport	19	$\frac{19 \times 360}{100} = 68·4°$

Administration	13	$\frac{13\times360}{100}=46\cdot8^\circ$
Banking	7	$\frac{7\times360}{100}=25\cdot2^\circ$

PIE DIAGRAM

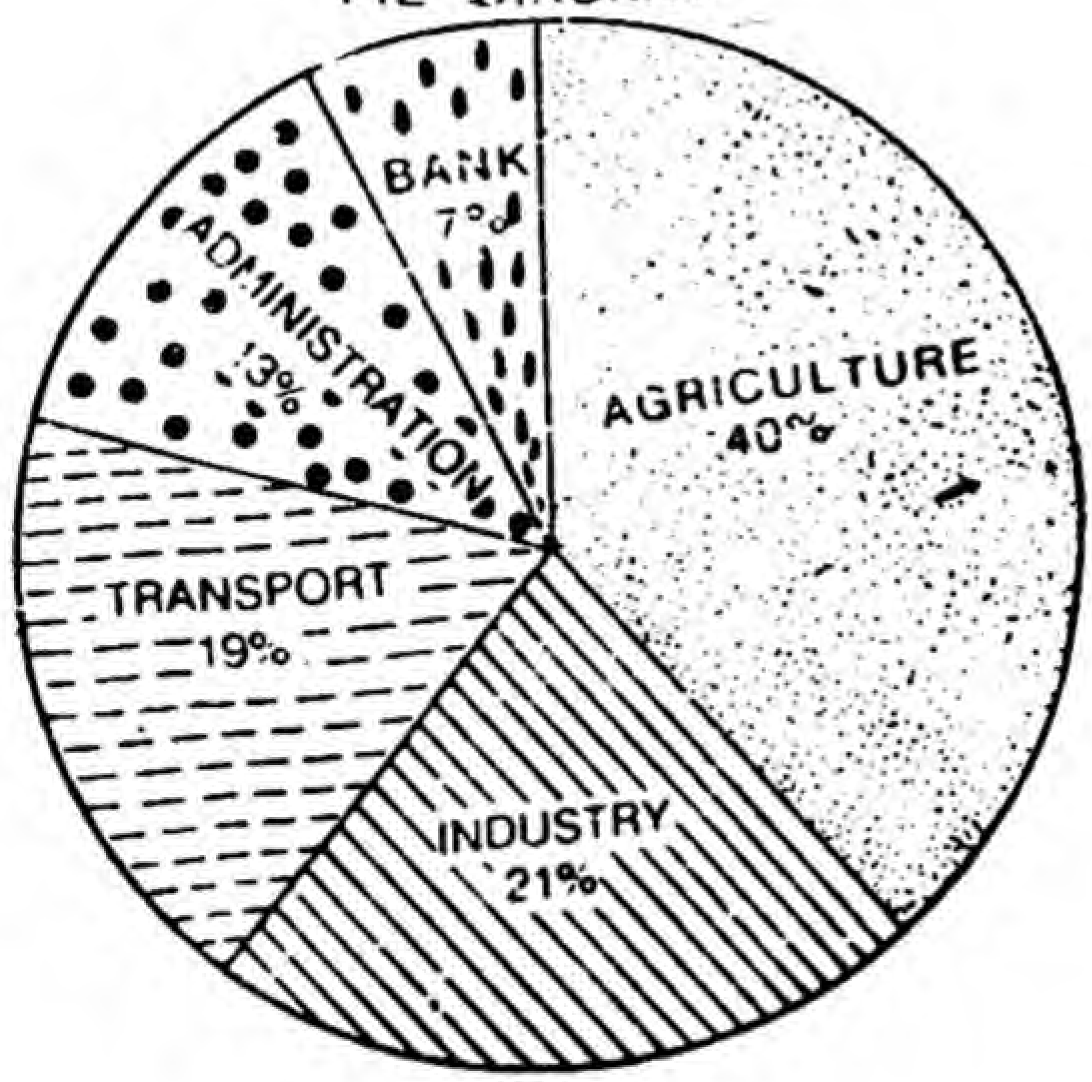


Fig. 7.

Histogram.

The term 'hisrogram' was first used by Karl Pearson in 1895 for a common form of graphic representation. Today histogram is a most popular and widely used method of presenting a continuous frequency distribution graphically. The class intervals are plotted on the X-axis and on each interval, a rectangle is erected with its height proportional to the frequency of that class interval. The altitude of the rectangles vary with the frequency of the interval and the base may be of equal or unequal width according as the frequency distribution consists of equal or unequal class intervals.

Case I : Histogram with equal class intervals.

In this case the class intervals are taken along X-axis and on each class interval a rectangle with height proportional to the corresponding frequency of the cl iss is erected. The series of adjacent rectangles so formed is known as histogram. The procedure is clear from the following example

Example 8. Show the following frequency distribution with the help of histogram.

Classes	10-20,	20-30,	30-40,	40-50,	50-60,	60-70,	70-80.
Frequency	10,	24	40,	32,	20,	14,	5.

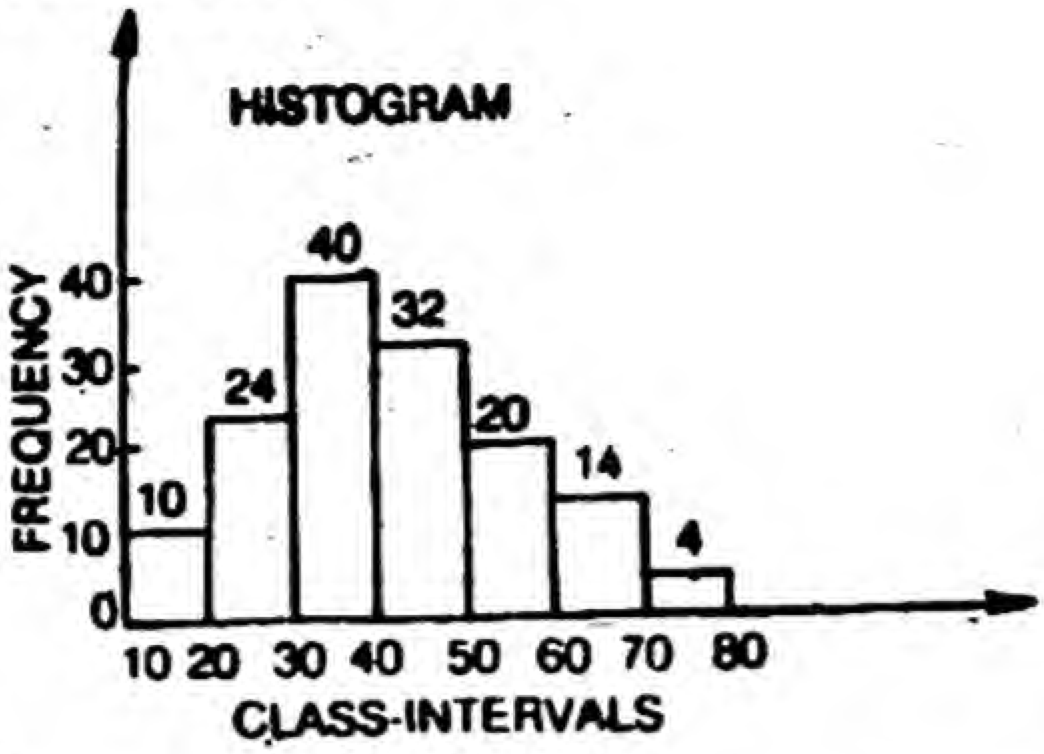


Fig. 8

Case II. Histogram with unequal class Intervals

If a distribution with unequal class intervals is to be presented by means of a histogram, it is necessary to make adjustment for varying magnitudes of the class intervals. This is done by finding for each class the frequency density. The following steps are followed :

1. Make a decision about the class interval in terms of which the frequency density is to be calculated. For this purpose the class which has lowest size of class interval is considered.

2. Adjust the frequencies of other classes with larger class intervals. Thus, if the magnitude of any class interval is twice (thrice) the lowest class interval, the adjustment factor is Two (Three) and the

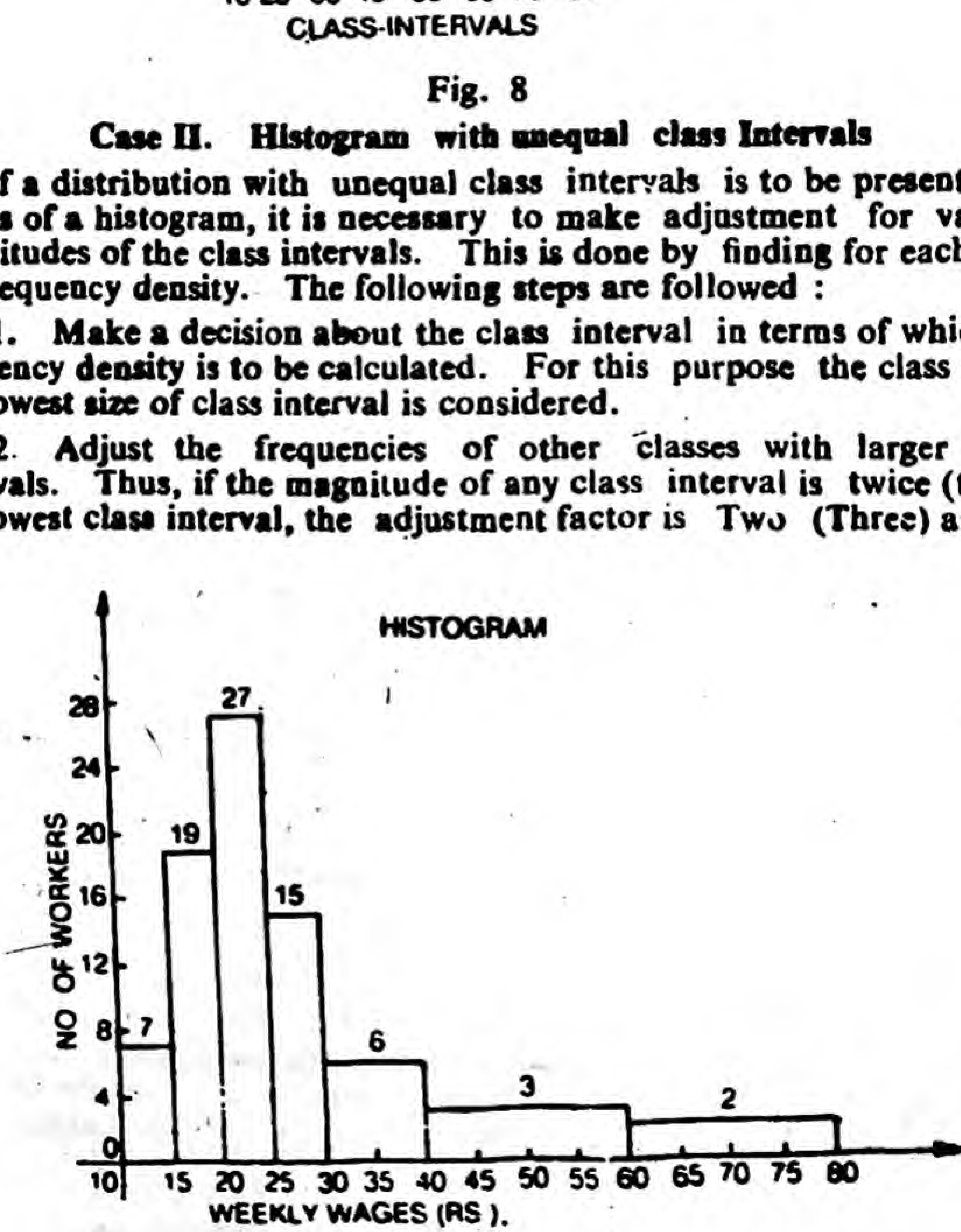


Fig. 9.

height of the rectangle which is represented by the adjusted frequency will be the ½ (⅓rd) of the corresponding class frequency and so on. This is clear from the following illustration.

Example 9. Represent the following data by means of histogram.

Weekly wages (Rs.)	No. of workers	Weekly wages (Rs.)	No. of workers
10—15	7	30—40	12
15—20	19	40—60	12
20—25	27	60—80	8
25—30	15		

Solution. Due to unequal class intervals, the adjustment in frequencies is needed. The lowest class interval is 5. Therefore, the frequency of class 30—40 is divided by 2, of class 40—60 is divided by 4 and also of 60—80 is divided by 4. The histogram is as shown in figure 9.

Questions

1. Show the following data diagrammatically by Multiple Bars.

Pattern of Investment in organised Industrial sector (Rs. crores)

Plan	I	II	III	IV	V
Public Sector	1560	3650	6300	13635	31400
Private Sector	1800	3100	4100	8980	16161

2. Represent the following data by means of percentage sub-divided bar diagrams:

Cost of Scooter	1982 Rs.	1983 Rs.	1984 Rs.
Raw Material	2160	2600	2700
Labour	540	700	800
Direct Expenses	360	200	60
Factory Expenses	360	300	360
Other Expenses	180	200	290
Total Cost	3600	4000	4500

3. Represent the following data by a sub-divided bar diagram. Given price, cost and quantity sold of commodities A and B.

	A	B
Price Per Unit	3	2
Quantity Sold	75	100
Value of Raw Material	175	150
Other Expenses	30	25
Profit	20	25

4. Present the following data by a suitable diagram showing the difference between proceeds and costs.

Year	1980	1981	1982	1983	1984	1985
Total Proceeds (Rs.)	110	135	140	150	160	165
Total Cost (Rs.)	95	105	150	125	130	170

5. Draw a multiple bar diagram for the following data. —

Year	Sales (Rs. Lakhs)	Gross Profit (Rs. Lakhs)	Net Profit (Rs. Lakhs)
1982	125	35	10
1983	140	38	12
1984	160	40	15
1985	200	45	20

6. Represent the following data by simple bar diagram.

Subjects	Economics	History	English	Science	Statistics
Marks (out of 100)	45	38	64	70	75

7. Represent the following data by a deviation bar diagram.

Years	1980	1981	1982	1983	1984
Imports of X (Rs. Crores)	42	65	49	75	100
Exports of X (Rs. Crores)	30	80	64	80	95

8. Given the following data relating to monthly income of five persons. Present the data by a suitable diagram :

Persons	A	B	C	D	E
Monthly income (Rs)	500	800	1000	1500	10,000

9. The following figures relate to the cost of construction of a house in Delhi.

Items	Expenditure	Items	Expenditure
Cement	20%	Timber	15%
Steel	18%	Labour	25%
Bricks	10%	Miscell.	12%

Represent the data by a pie diagram.

10. Draw a pie diagram for the following data :

Items	Percent of total expenditure
Food	65
Clothing	10
Housing	12
Fuel and Lighting	5
Miscellaneous	8

11. Draw histograms from the following data :

(*i*)

Wages (Rs.)	0—100	100—200	200—300	300—400	400—500
Number of marks	10	18	27	35	20

(*ii*)

Marks	0—9	10—19	20—29	30—39	40—49
No of Students	6	12	18	24	16

(Hint : *Make class intervals exclusive*.)

(*iii*)

Mid-Values	10	20	30	40	50	60
Frequency	6	12	18	17	15	4

(Hint : Device class intervals from mid-values.)

(*iv*)

Profits (Rs Thousands)	No. of Firms	Profits (Rs. Thousands)	No. of Firms
0— 5	40	30—40	20
5—10	28	40—60	32
10—20	30	60—80	24
20—30	28		

12. What are the rules for constructing a diagram ? Discuss the usefulness of diagrams.

13. Distinguish between.

(*i*) Bar Diagrams and Pie Diagram

(*ii*) Multiple and Sub-divided bar diagram

(*iii*) Deviation bar diagram and Broken bar diagram.

14. What do you mean by a histogram ? Discuss the procedure of drawing a histogram when class intervals are not equal.

5

Measures of Central Tendency

INTRODUCTION

Measures of central tendency are one of the most important and useful techniques for making analysis of statistical data. The basic objective of statistics is to make wise decisions on the basis of available numerical data. This data is first collected, then classified and presented in the form of tables, diagrams or graphs. After this comes the stage of analysing it. Measures of central tendency or averages provide us with the first and most useful method of making analysis of data.

As the name itself implies, the measures of central tendency tell us about the "central theme" or the essence of the data. They indicate the most representative figure in the data. Since it is difficult for anyone to remember the huge mass of complex data, measures of central tendency or averages are very helpful for analysis and making comparisons. In the words of *Simpson and Kafka*, "*A measure of central tendency is a typical value around which other figures congregate.*" This definition makes it clear that a measure of central tendency is a central value and other observations in the data cluster around it.

In the words of A.L. Bowley, "*Averages are statistical constants which enable us to comprehend in a single effort the significance of the whole.*" This definition explains that the study of averages can help us to know about the entire data.

Objectives or Utility of Measures of Central Tendency (Averages). Averages are very common in these days. Everybody uses them everywhere. We generally talk about average score, average weight, average height, average income, average cost, average revenue, average size of a cap, average rate of growth, average speed etc. All these averages are nothing but different forms of the measures of central tendency. Their utility is clear from the following points :

(i) **Averages enable comparison.** The purpose of averages is to give a single representative value of the data. It can be used to compare the corresponding values and hence the nature of other data. For example, with the help of averages we can compare the marks of students in different colleges. Similarly, we can compare the marks in any college in any year with the marks in the previous years.

(ii) **Averages are very important statistical devices.** The concept of averages is the foundation on which lies the entire structure of statistical analysis. All other tools of statistical analysis like dispersion, skewness, correlation, regression, index numbers, sampling, tests of significance are based on averages.

(iii) **Averages describe the features of entire data.** Measures of central tendency by condensing the mass of data in one single value, enable us to get a bird's eye view of the entire data. Thus average can represent thousands, lakhs or crores of items and describe their characteristics. Their knowledge helps us in making useful decisions.

Properties or Essentials or Requisites of an Ideal Average. **Prof.** Yule and Kendall in their book, "*An Introduction to the Theory of Statistics*" describe the following **requirements** of an ideal average.

1. *It should be easy to understand and simple to compute.* For an ideal average it is essential that its calculation is very simple. In other words, the calculation work should not involve lengthy calculations and advanced mathematical techniques. Moreover, even a common man should be in a position to understand it without any difficulty.

2. *It should be rigidly defined.* For an ideal average it is necessary that it has only one interpretation. Its value should be unique and definite. In other words, the definition of average should not leave anything to the discretion of the investigator or the observer.

3. *It should be based on all the items.* An ideal average should be based on all observations in the data. If some items of the data are not considered while calculating an average, then it will not be true representative of the data.

4. *It should not be unduly affected by the presence of extreme items.* By extreme items we mean very small or very large items. These items should not unduly affect the average. An ideal average is one which is least affected by such items.

5. *It should be capable of further algebraic treatment.* It implies that an ideal average should possess some important mathematical properties so that it can be used in further statistical work. If an average does not possess this quality, its use is bound to be very limited.

6. *It should have sampling stability.* An ideal average should be least affected by the sampling fluctuations. It implies that for different samples from the same population the value of the average should not differ significantly. The value of an ideal average will be either same or have minimum fluctuations for different samples.

Various Measures of Central Tendency. The following are the five measures of central tendency or measures of location or averages which are commonly used in practice :

(*i*) Arithmetic Mean (A.M.) (*iii*) Harmonic Mean (H.M.)
(*ii*) Geometric Mean (G.M.) (*iv*) Median (M).

(v) Mode (Z).

The first three are called *mathematical averages* and the last two are called *positional averages*.

ARITHMETIC MEAN

It is also called simple mean. It is the most popular and widely used measure of central tendency. Its value is obtained by adding all the values of the observations and dividing the sum by the number of observations.

Calculation of Arithmetic Mean.

I. Individual Series.

(*a*) *Direct Method.* Let X be any variable taking values X_1, X_2, X_3,......, X_n. The arithmetic mean of X (denoted by $\overline{X}$) is equal to

$$\overline{X}=\frac{X_1+X_2+X_3+\ldots\ldots+X_n}{n} \quad \text{or} \quad \bar{x}=\frac{\Sigma X}{n}.$$

Where the symbol Σ is called sigma and it is used to denote the sum of values. n implies the number of items.

The calculation of arithmetic mean is clear from the following example.

Example 1. Calculate arithmetic mean for the following data :

X : 15, 17, 20, 35, 40.

Solution.	*X*
	15
	17
	20
	35
	40
	$\Sigma X=127$

$$n=5$$

$$\bar{x}=\frac{\Sigma X}{n}=\frac{127}{5}=25{\cdot}4.$$

(*b*) *Short Cut Method.* In this method any assumed mean (A) is taken and the deviations of items are then taken from the assumed mean. The following formula is used :

$$\overline{X}=A+\frac{\Sigma dx}{n}.$$

Where A is the assumed mean. It can be any value in the data. For the sake of simplicity any value near to the centre of the data is preferred as assumed mean. dx is deviation of values of X from A. In other words, $dx=X-A$ and Σdx is the sum of deviations from assumed mean A. n is the number of items.

Example 2. Calculate arithmetic mean for the following data using short cut method.

X : 20 22 25 30 35 38 46.

Solution. Let us take 30 as the assumed mean.

X	$A=30$ $dx=X-A$ $=X-30$
20	-10
22	-8
25	-5
30	0
35	5
38	8
46	16
	$\Sigma dx = 6$

$dx = x - A$
$20 - 30$
$= -10$

$$\overline{X} = A + \frac{\Sigma dx}{n}$$
$$= 30 + \frac{6}{7}$$
$$= 30 + 0{\cdot}857$$
$$= 30{\cdot}857$$

II. Discrete Series.

In the case of discrete series the values of the variable as well as frequency (*i.e.* number of times any value occurs) is given. The arithmetic mean is calculated as follows :

(*a*) *Direct Method.* In this method each value is multiplied with **the corresponding frequency and the sum ($\Sigma f.X$) is obtained. This** sum is then divided by the sum of frequency (Σf).

Thus $\bar{x} = \dfrac{\Sigma f.X}{\Sigma f}$.

Example 3. Calculate the average marks obtained by students in a class of 50 students.

Marks :	10	15	20	25	30
No. of Students :	12	15	10	8	5

Solution. Let us denote marks by x and the number of students by f. We have,

Marks x	*No. of Students* f	$f.X$
10	12	120
15	15	225
20	10	200
25	8	200
30	5	150
	$\Sigma f = 50$	$\Sigma fx = 895$

$$\overline{X}=\frac{\Sigma fx}{\Sigma f}=\frac{895}{50}=17\cdot9.$$

(*b*) *Short Cut Method.* As in the case of individual series, deviations (dx) are taken from any assumed mean (A). These deviations are then multiplied by their corresponding frequency and the sum ($\Sigma f\,.\,dx$) is obtained. The following formula is used.

$$\overline{X}=A+\frac{\Sigma f\,.\,dx}{\Sigma f}\qquad (dx=X-A)$$

Example 4. The following data shows the profits earned by 100 firms during the year 1986—87. Calculate average profit (use Short cut Method).

Profits (in lakhs) :	5	8	10	12	15
No. of firms :	20	25	30	15	10

Solution :

Profits (in lakhs) *X*	*No. of firms* *f*	*A=10* *dx=X—A* *=X—10*	*f . dx*
5	20	—5	—100
8	25	—2	— 50
10	30	0	0
12	15	2	30
15	10	5	50
	$\Sigma f=100$		$\Sigma fdx=-70$

$$\overline{X}=A+\frac{\Sigma fdx}{\Sigma f}=10+\frac{-70}{100}=10-\frac{70}{100}$$

$$\overline{X}=10-0\cdot7=9\cdot3.$$

Therefore, the average profit earned by any firm is Rs. 9·3 lakh.

III. Continuous Series.

In this series data is given in the form of class intervals. We first calculate the mid value of each class interval and then the same procedure (as in the case of discrete series) is applied for calculating arithmetic mean. The mid value of each class interval is obtained by dividing the sum of lower and upper limit of class interval by two. In other words,

$$\text{M.V.}=\frac{\text{Lower Limit}+\text{Upper Limit}}{2}$$

20+70

25 90/2 45

20+30 50/2

(a) *Direct Method.* The formula used is

$$\overline{X}=\frac{\Sigma fx}{\Sigma f}$$

where Σfx is the sum of the product of mid value of each class interval and the corresponding frequency, Σf is the total frequency.

Example 5. From the following data compute arithmetic mean by direct method.

Age (years) :	20—30	30—40	40—50	50—60	60—70
No. of Persons :	5	8	7	6	4

Solution :

Age (years)	*No. of Persons* f	*Mid Value* X	$f.x$
20—30	5	25	125
30—40	8	35	280
40—50	7	45	315
50—60	6	55	330
60—70	4	65	260
	$\Sigma f=30$		$\Sigma fx=1310$

$$\overline{X}=\frac{\Sigma fx}{\Sigma f}=\frac{1310}{30} \qquad \overline{X}=43\cdot67 \text{ years.}$$

(b) *Short Cut Method.* In this method deviations of the mid values are taken from assumed mean and the following formula is used.

$$\overline{X}=A+\frac{\Sigma fdx}{\Sigma f}$$

where Σfdx is the sum of the product of deviations from assumed mean and corresponding frequencies.

Example 6. Using short cut method calculate mean for data given in example 5.

Solution. Let us suppose that 45 is the assumed mean.

Age (years)	*No. of Persons* f	*Mid Value* X	$A=45$ $dx=X-A$ $=X-45$	$f.dx$
20—30	5	25	—20	—100
30—40	8	35	—10	— 80
40—50	7	45	0	0
50—60	6	55	10	60
60—70	4	65	20	80
	$\Sigma f=30$			$\Sigma fdx=-40$

15

90

895

195

$$\overline{X}=A+\frac{\Sigma fdx}{\Sigma f}$$

or $$\overline{X}=45+\frac{-40}{30}=45-\frac{40}{30}$$

or $$\overline{X}=45-1{\cdot}33$$

or $$\overline{X}=43{\cdot}67.$$

(c) *Step Deviations Method.* In this method the deviations taken from assumed mean are divided by any common factor,say C and the following formula is used.

$$\overline{X}=A+\frac{\Sigma fd'x}{\Sigma f}\times C$$

where $d'x=\frac{X-A}{C}$, C is any common factor. A is the assumed mean.

Example 7. Using step deviations method calculate arithmetic mean for data given in example 5.

Solution :

Age (years)	*No. of Persons f*	*Mid Value X*	*A=45 dx=X—A =X—45*	*C=10 d'x=dx/C*	*fd'x*
20—30	5	25	—20	—2	—10
30—40	8	35	—10	—1	— 8
40—50	7	45	0	0	0
50—60	6	55	10	1	6
60—70	4	65	20	2	8
	$\Sigma f=30$				$\Sigma fd'x=-4$

$$\overline{X}=A+\frac{\Sigma fd'x}{\Sigma f}\times C$$

or $$\overline{X}=45+\frac{-4}{30}\times 10 \quad \text{or} \quad \overline{X}=45-1{\cdot}33=43{\cdot}67.$$

Remarks :

(*i*) The step deviations method is widely used to calculate mean. It has the merit of making calculations very easy.

(*ii*) The value of mean by using direct, short cut or step deviations method is always same. This is also clear from examples 5, 6 and 7 where we get same value of average age *i.e.* 43·67 years for the same data.

Arithmetic mean in the case of Inclusive Class Intervals. In this case there is no need to convert the class intervals into exclusive form. We apply the formula and find mean.

Example 8. Calculate mean from the following data.

Class interval :	10—19	20—29	30—39	40—49	50—59
Frequency :	5	8	7	3	2

Solution :

Class Interval	*Frequency* f	*Mid Value* X	$A=34{\cdot}5$ $dx=X-A$ $=X-34{\cdot}5$	$C=10$ $d'x=dx/C$	$fd'x$
10—19	5	14·5	—20	—2	—10
20—29	8	24·5	—10	—1	— 8
30—39	7	34·5	0	0	0
40—49	3	44·5	10	1	3
50—59	2	54·5	20	2	4
	$\Sigma f=25$				$\Sigma f\,d'x=-11$

$$\overline{X}=A+\frac{\Sigma fd'x}{\Sigma f}\times C$$

$$=34{\cdot}5+\frac{-11}{25}\times 10$$

$$\overline{X}=34{\cdot}5-4{\cdot}4=30{\cdot}1.$$

Arithmetic Mean when Cumulative Frequency Distribution is given. In this case the class intervals can be either in "More than" form or "Less than" form. The class intervals are first converted into simple form and then the mean is calculated.

Example 9. Calculate mean from the data relating to marks of students.

Marks (More than) :	10	20	30	40	50	60
No. of Students :	100	88	72	45	20	5

Solution. Since 100 students get more than 10 marks and 88 students get more than 20 marks, therefore the number of students with marks from 10 to 20 is 100—88=12. Similarly, other values are calculated.

Marks	*No. of students*	*Class intervals*	*No. of students* f	*Mid value* X	$A=35$ $C=10$ $d'x=\frac{X-A}{C}$	$fd'x$
More than 10	100	10—20	12	15	—2	—24
,, ,, 20	88	20—30	16	25	—1	—16
,, ,, 30	72	30—40	27	35	0	0
,, ,, 40	45	40—50	25	45	1	25
,, ,, 50	20	50—60	15	55	2	30
,, ,, 60	5	60—70	5	65	3	15
			$\Sigma f=100$			$\Sigma fd'x=30$

$$\overline{X}=A+\frac{\Sigma fd'x}{f\Sigma}\times C$$

$$=35+\frac{30}{100}\times 10$$

$$\overline{X}=35+3=38.$$

Arithmetic mean in the case of open end class intervals. Class intervals are said to have open ends if the lower limit of first class or upper limit of last class or both are missing. In such case the lower limit of first class is determined on the basis of size of immediately succeeding class and the upper limit of last class is determined on the basis of size of immediately preceding class interval.

Example 10. Calculate average weight from the following data.

Weight in kgm.	Less than 10	10—20	20—40	40—60	60—65	65 &above
Frequency :	12	18	20	35	40	5

Solution. Since size of second class interval is 10, the lower limit of first class is 0 and also since size of last but one class is 5, the upper limit of last class is 70.

Weight in kgm.	*Frequency* f	*Mid value* x	$A=30$ $dx=X-A$ $=X-30$	$C=5$ $d'x=dx/C$	$f.d'x$
0—10	12	5	−25	−5	−60
10—20	18	15	−15	−3	−54
20—40	20	30	0	0	0
40—60	35	50	20	4	140
60—65	40	62·5	32·5	6·5	260
65—70	5	67·5	37·5	7·5	37·5
	$\Sigma f=130$				$\Sigma fd'x=323{\cdot}5$

$$\overline{X}=A+\frac{\Sigma fd'x}{\Sigma f}\times C$$

$$\overline{X}=30+\frac{323{\cdot}5}{130}\times 5=42{\cdot}44.$$

Determination of missing frequency when mean is given.

Example 11. From the following data determine the number of workers earning Rs. 230.

Wages Rs. (Mid values)	210	220	230	240	250	260
No. of workers :	15	35	—	50	42	18

It is given that average wage is Rs. 236·15.

Solution. Let us denote missing frequency by f_1.

Wages (Rs.) Mid values X	*No. of workers* f	$A=230$ $dx=X-230$	$C=10$ $d'x=dx/10$	$f.d'x$
210	15	−20	−2	−30
220	35	−10	−1	−35
230	f_1	0	0	0
240	50	10	1	50
250	42	20	2	84
260	18	30	3	54
	$\Sigma f=160+f_1$			$\Sigma fd'x=123$

$$\overline{X}=A+\frac{\Sigma fd'x}{\Sigma f}\times C$$

$$236{\cdot}15=230+\frac{123}{160+f_1}\times 10$$

or $236{\cdot}15-230=\dfrac{1230}{160+f_1}$

or $(6{\cdot}15)(160+f_1)=1230$

or $984+6{\cdot}15\,f_1=1230$

or $6{\cdot}15\,f_1=246$

or $f_1=\dfrac{246}{6{\cdot}15}=40.$

Therefore, number of workers earning Rs. 230 is 40.

Correction of Incorrect Mean. Sometimes, by mistake wrong items are included in the data which result in the wrong value of mean also. This mean can be corrected by following the procedure given below :

(*i*) Calculate correct Σx.

Correct Σx=Incorrect Σx—Wrong values+Correct values.

We know that $\overline{X}=\dfrac{\Sigma x}{n}$, therefore $\Sigma x=N\,.\,\overline{X}$.

(*ii*) Calculate Correct Number of Items (n).

Correct n=Original n—Wrong number of items+Correct number of items.

Example 12. The average weight of a group of 10 students was 50 kgm. It was later found that a student whose correct weight was 54 was wrongly taken as 45. Find correct mean.

Solution. Let us represent weight by X

Given $n=10$, $\overline{X}=50$ kgm.

Wrong value=45, correct value =54

Incorrect $\Sigma x=n.\,\overline{X}=10\times 50=500.$

Correct Σx=Incorrect Σx—wrong value+correct value

$=500-45+54=509$

Correct n=original n—wrong number of items +correct number of items

$=10-1+1=10$

Correct Mean $\overline{X}=\dfrac{\text{Correct } \Sigma x}{\text{Correct } n}=\dfrac{509}{10}$

or $\overline{X}=50{\cdot}9$ kgm.

Example 13. The mean marks of 50 students was found to be 45. It was later on discovered that marks to two students (42, 52) were wrongly included in this group. These were then deleted. **Find** the correct mean.

6,000

Solution. Given $n=50$, $\overline{X}=45$

Wrong values. 42, 52 Correct values—Nil

Incorrect $\Sigma x = n.\overline{X} = 50 \times 45 = 2250$

Correct $\Sigma x =$ Incorrect $\Sigma x -$ wrong values

$= 2250-(42+52) = 2156$

Correct $n =$ original n—wrong number of items

$= 50-2 = 48$

$$\text{Correct Mean } \overline{X} = \frac{\text{Correct } \Sigma x}{\text{Correct } n} = \frac{2156}{48} = 44{\cdot}92.$$

Combined Mean. It is also known as mean of the means. If $\overline{X_1}$ and $\overline{X_2}$ are means of two groups I and II taking n_1 and n_2 values respectively, then the mean of both the groups is given by

$$\overline{X}_{12} = \frac{n_1 \overline{X}_1 + n_2 \overline{X}_2}{n_1 + n_2}$$

Here $\overline{X}_{12}$ is called combined mean.

Example 14. In a class of 100 students there are 40 boys and 60 girls. The mean marks of boys are 50 and mean marks of girls are 55. Determine the mean marks of the entire class.

Solution. Let us denote marks of boys by X_1 and marks of girls by X_2.

Therefore, given $\overline{X}_1 = 50$ $n_1 = 40$

$\overline{X}_2 = 55$ $n_2 = 60$

We know that $$\overline{X}_{12} = \frac{n_1 \overline{X}_1 + n_2 \overline{X}_2}{n_1 + n_2}$$

or $$\overline{X}_{12} = \frac{40 \times 50 + 60 \times 55}{40+60} = \frac{2000+3300}{100}$$

$$\overline{X}_{12} = \frac{5300}{100} = 53.$$

Example 15. The mean annual salary paid to all employees in a factory is Rs. 500. The average salary paid to male employees is Rs. 520 and to female employees is Rs. 420. Determine the percentage of male and female employees.

Solution. Let n_1 be the number of male employees and n_2 be the number of female employees. Suppose there are 100 employees.

$\therefore$ $n_1 + n_2 = 100$

or $n_2 = 100 - n_1$

Given $\overline{X}_1 = 520$, $\overline{X}_2 = 420$ and $\overline{X}_{12} = 500$

We have $\overline{X}_{12}=\frac{n_1\overline{X}_1+n_2\overline{X}_2}{n_1+n_2}$

or $500=\frac{n_1(500)+(100-n_1)(420)}{100}$ $\because n_1+n_2=100$

or $50000=500n_1+42000-420\,n_1$

or $8000=80\,n_1$

or $n_1=80$

Since $n_2=100-n_1$ $\therefore n_2=20.$

Therefore, percentage of male employees=80

and percentage of female employees=20.

Algebraic Properties of Arithmetic Mean.

1. The sum of deviations taken from actual mean is always equal to zero.

i.e., $\Sigma(X-\overline{X})=0$

For example

X	5	10	15	20	25	$\Sigma X=75$
$X-\overline{X}$ $=(X-15)$	-10	-5	0	5	10	$\Sigma(X-\overline{X})=0.$

2. *The sum of squares of deviations from mean is minimum.*

i.e. $\Sigma(X-\overline{X})^2$ *is minimum,*

For example

X	$X-\overline{X}$	$(X-\overline{X})^2$	X−25	$(X-25)^2$	(X−35)	$(X-35)^2$
10	−20	400	−15	225	−25	625
20	−10	100	−5	25	−15	225
30	0	0	5	25	−5	25
40	10	100	15	225	5	25
50	20	400	25	625	15	225
$\Sigma X=150$ $N=5,\ \overline{X}=30$		$\Sigma(X-\overline{X})^2=1000$		$\Sigma(X-25)^2$ $=1125$		$\Sigma(X-35)^2$ $=1125$

Therefore, sum of squares of deviations from mean is minimum.

3. *Mean is affected accordingly if the observations are given mathematical treatment by any constant term.*

e.g.

X	X+2	X−2	(X).2	X/2
5	7	3	10	2·5
10	12	8	20	5
15	17	13	30	7·5
20	22	18	40	10
25	27	23	50	12·5
$\Sigma X=75$	$\Sigma(X+2)=85$	$\Sigma(X-2)=65$	$\Sigma 2X=150$	$\Sigma(X/2)=37{\cdot}5$
$N=5$	$N=5$	$N=5$	$N=5$	$N=5$
$\bar{X}=15$	$(\overline{X+2})=17$ $=\bar{X}+2$	$(\overline{X-2})=13$ $=\bar{X}-2$	$(\overline{2X})=30$ $=\bar{X}.2$	$(\overline{X/2})=7{\cdot}5$ $=\bar{X}/2$

4. *If the original series is replaced by the actual mean the sum of items will remain the same.*

e.g.						
X	2	5	8	11	14	$\Sigma X=40$
$\bar{X}$	8	8	8	8	8	$\Sigma\bar{X}=40$

5. *Arithmetic Mean is independent of origin i.e. is not affected by any change in origin (assumed mean).*

e.g.

X	5	6	7	8	9	
$A=6$, $dx=X-A$,	−1	0	1	2	3	$\Sigma dx=5$, $\bar{X}=A+\Sigma dx/5 = 6+5/5=7$
$A=9$, $dx=X-A$,	−4	−3	−2	−1	0,	$\Sigma dx=-10$, $\bar{X}=9-10/5=7$

6. *Arithmetic mean can be treated mathematically.*

i.e. when any two of the three values $\bar{X}$, ΣX and **N** are known, we can find the third by using :—

$$\bar{X}=\frac{\Sigma X}{N}, \quad \text{or} \quad N=\frac{\Sigma X}{\bar{X}}, \quad \text{or} \quad \Sigma X=N.\bar{X}.$$

7. *Mean of sum or difference of the observations will be equal to the sum or difference of means provided the number of observations is equal.*

In other words mean of $(X\pm Y)$=Mean of $X\pm$Mean of Y

X	Y	X+Y	X−Y	
5	2	7	3	$\overline{X}=\frac{\Sigma X}{N}=\frac{75}{5}=15$
10	4	14	6	$\overline{Y}=\frac{\Sigma Y}{N}=\frac{30}{5}=6$
15	6	21	9	$\overline{X+Y}=\frac{105}{5}=21=\bar{X}+\bar{Y}$
20	8	28	12	
25	10	35	15	$\overline{X-Y}=\frac{45}{5}=9=\bar{X}-\bar{Y}$
$\Sigma X=75$,	$\Sigma Y=30$,	$\Sigma(X+Y)$, $=105$	$\Sigma(X-Y)$ $=45$	

8. *If there are two or more series with their respective means and number of observations, we can find the combined mean by using the following formula*

$$\overline{X_{12}}=\frac{N_1\overline{X_1}+N_2\overline{X_2}}{N_1+N_2},$$

where N_1 = No. of observations of first series.

N_2 = No. of items of second series. $\overline{X_1}$ = mean of first series

$\overline{X_2}$ = mean of second series

$\overline{X_{12}}$ = combined mean.

e.g. $N_1=50$, $N_2=30$

$\overline{X_1}=40$, $\overline{X_2}=80$

$$\overline{X_{12}}=\frac{50\times40+30\times80}{50+30}=\frac{2000+2400}{80}$$

$$=\frac{4400}{80}=55$$

Merits and Demerits of Arithmetic Mean.

Merits :

1. The calculation of arithmetic mean is very easy.
2. The arithmetic mean is based on all the items in the series.
3. It can be used in further statistical functions.
4. The arithmetic mean is least affected by sampling fluctuations.
5. It is a very useful tool for making comparisons.
6. The value of arithmetic mean is rigidly defined.

Demerits. Inspite of so many merits, arithmetic mean is not free from limitations. The following are some of its demerits :

1. Since mean is based on all the items, it cannot be calculated if any item is not known.

2. Arithmetic mean is adversely affected by the presence of extreme items.

3. It cannot be determined graphically.
4. It can be a figure which does not even exist in the data.

Inspite of above limitations, arithmetic mean is widely used measure of central tendency and it is considered to be one of the best tools of statistics.

MEDIAN

Median is a positional measure of central tendency. It divides the given data into two equal parts. 50% of items are above median and 50% are below it. If the variable under consideration is of qualitative nature like honesty, beauty, friendship, knowledge etc. then median is considered to be the better average. According to Connor, "*Median is that value of the variable which divides the group into two equal parts, one part comprising all values greater, and the other all values less than the median.*"

Calculation of Median

I. **Individual Series.** The steps for calculating median are as follows :

(*a*) Arrange the data in either ascending order or descending order.

(*b*) Apply Median=Size of $\frac{n+1}{2}$ th item. Where n is the number of items.

Example 16. Calculate median value for the data :

4 7 8 2 9 16 12.

Solution. Let us arrange data in ascending order.

X

2

4 $\qquad M = \text{Size of } \frac{n+1}{2}\text{th item}$

7 $\qquad = \text{Size of } \frac{7+1}{2}\text{th item} \quad \because \quad n=7$

8 $\qquad = \text{Size of 4th item}$

9 $\qquad = 8$

12 $\qquad \therefore$ median is 8

16

Example 17. The following data show the runs scored by Kapil Dev in six innings played.

Inning	I	II	III	IV	V	VI
Runs	45	62	3	47	51	18

Calculate Median.

Solution. Let us first arrange the available data of runs scored in ascending order.

Runs

3

18 $\quad M = \text{Size of } \frac{n+1}{2}\text{th item}$

45 $\quad = \text{Size of } \frac{6+1}{2}\text{th item}$

47 $\quad = \text{Size of } 3{\cdot}5\text{th item}$

51 $\quad = \frac{45+47}{2} = 46.$

62

Therefore, the median score of Kapil Dev is 46.

II. Discrete Series. The steps for calculating median are as follows.

(*a*) Arrange the data in either ascending order or descending order.

(*b*) Find cumulative frequeney (denoted as c. f.).

(*c*) Apply Median=Size of N/2th item.
where $N = \Sigma f$ (total of frequency)

(*d*) The value of the variable whose cumulative frequency covers N/2th item is called median value.

Example 18. Find the median from the following data

Variable :	15	19	25	37	45	62
Frequency :	8	7	6	12	20	13

Solution. Since the values of the variable are already arranged in an ascending order, median is calculated as follows.

Variable	*Frequency* *f*	*c.f.*
15	8	8
19	7	15
25	6	21
37	12	33
45	20	53
62	23	76
	N=76	

M=Size of N/2th item
=Size of 76/2th item
=Size of 38th item

Since 38th item lies in the cumulative frequency 53, the median is 45.

III. Continuous Series. The steps for calculating median are as follows.

(*a*) Arrange the class intervals in an ascending order.

(*b*) If the class intervals are given in inclusive form, then convert them to the exclusive form.

(*c*) If cumulative frequency distribution is given, then convert it to simple frequency distribution form.

(*d*) Find cumulative frequencies.

(*e*) Apply M=Size of N/2th item. Where N is the total of the frequency.

(*f*) By using step *e* we shall find only a class interval in which median will lie. To find a unique value of median we use the technique of interpolation and following formula is applied.

$$M = L_1 + \frac{i}{f}\left[\frac{N}{2}\text{th item} - c\right]$$

Where L_1 is the lower limit of the median class interval, i is the size of median class interval, f is the simple frequency of median class interval, N/2th item is obtained in step *e*, and c is the cumulative frequency of the class interval preceding median class interval.

Example 19. Calculate median from the following data.

Class interval :	0—10	10—20	20—30	30—40	40—50
Frequency :	8	18	25	10	9

Solution.

Class Interval	*Frequency* f	c.f.
0—10	8	8
10—20	18	26
20—30	25	51
30—40	10	61
40—50	9	70
	N=70	

Median=Size of N/2th item
=Size of 70/2th item
=Size of 35th item.

Since 35th item lies in the cumulative frequency 51, the median class interval is 20—30.

Applying the technique of interpolation.

$$M = L_1 + \frac{i}{f}\left[\frac{N}{2}\text{th} - C\right]$$

Here $L_1 = 20$, $i = 10$ (size of class interval)
$f = 25$, N/2th item=35, C=26

$$M = 20 + \frac{10}{25}\left[35 - 26\right]$$

$$=20+\frac{10}{25}(9)$$

$$M=23{\cdot}6.$$

Example 20. Calculate median for the data given below :

Marks :	5—9	10—14	15—19	20—24	25—29	30—34
No. of Students :	25	35	55	45	62	28

Solution. Since the class intervals are in inclusive form, we first convert them to the exclusive form.*

Marks	*No. of Students* (f)	*c.f.*
4·5— 9·5	25	25
9·5—14·5	35	60
14·5—19·5	55	115
19·5—24·5	45	160
24·5—29 5	62	222
29·5—34·5	28	250
	N=250	

Median = Size of N/2th item.
= Size of 250/2th item
= Size of 125th item.

$$d=\frac{\text{lower limit of succeeding class}-\text{upper limit of preceding class}}{2}$$

i.e. $$d=\frac{10-9}{2}=\frac{1}{2}=0{\cdot}5.$$

Now we subtract 0·5 from lower limit of each class and add 0·5 to the upper limit of each class. Now the class intervals are exclusive.

This item lies in the cumulative frequency 160 therefore, median class interval is 19·5—24·5.

Applying Interpolation,

$$M=L_1+\frac{i}{f}\left[\frac{N}{2}\text{th}-C\right]$$

or $$M=19{\cdot}5+\frac{5}{45}\left[125-115\right]$$

$$M=19{\cdot}5+\frac{5}{45}(10)=20{\cdot}61.$$

---

*The method of conversion is to find the value of *d*.

Example 21. From the data relating to age of 100 persons calculate median age.

		Less than	Less than	Less than	Less than	Less than
Age (years)	:	10	20	30	40	50
No. of Persons	:	12	20	40	78	100

Solution. The given frequency distribution is cumulative. We first convert it to the simple form and then calculate median.

Age (years)	*No. of Persons* (f)	$c.f.$
Less than 10	12	12
10—20	8	20
20—30	20	40
30—40	38	78
40—50	22	100
	N=100	

M=Size of N/2th item
=Size of 100/2th item
=Size of 50th item.

5 - 9
10 - 74

The 50th item lies in the cumulative frequency 78. Therefore, the median class interval is 30—40.

Now $M=L_1+\frac{i}{f}\left[\frac{N}{2}\text{th}-C\right]$

Here $L_1=30$, $i=10$, $f=38$, $N/2\text{th}=50$, $C=40$.

$\therefore$ $M=30+\frac{10}{38}\left[50-40\right]$

$=30+\frac{10}{38}(10)=30+\frac{100}{38}$

$=32{\cdot}63$.

Example 22. From the following data find out the value of Median.

Mid values	:	5	15	25	35	45
Frequency	:	4	8	12	32	20

14 -

$d = \frac{x-a}{}$

5 · 5

5 · 0
0 · 5
4 · 5

Solution. The class intervals are as follows :
0—10, 10—20, 20—30, 30—40, 40—50.

Classes	*Frequency*	*C.f.*
0—10	4	4
10—20	8	12
20—30	12	24
30—40	32	56
40—50	20	76

$$M = \text{Size of } \frac{N}{2} \text{ th item}$$

$$= \text{Size of } \frac{76}{2}\text{th item.}$$

$$= \text{Size of 38th item.}$$

It lies in C.*f.* 56.

$\therefore$ Median class interval is 30—40.

Applying $M = L_1 + \frac{i}{f}\left[\frac{N}{2}\text{th} - C\right]$

or $M = 30 + \frac{10}{32}\left[38 - 24\right]$

or $M = 30 + \frac{10}{32}\left[14\right]$

$$= 30 + \frac{140}{32}$$

$$= 30 + 4{\cdot}375$$
$$= 34{\cdot}375$$

Determination of Missing Frequency.

Example 23. In the frequency distribution of 100 families given below, the median is known to be 50. Find the missing frequencies.

Expenditure (Rs.)	:	0--20	20—40	40—60	60—80	80—100
No. of Families	:	14	?	26	?	16

Solution. Two frequencies corresponding to class intervals 20—40 and 60—80 are missing. Let us represent frequency corresponding to 20—40 by f_1 and frequency corresponding to 60—80 by f_2. We get

Expenditure (Rs.)	No. of families (f)	c.f.
0– 20	14	14
20– 40	f_1	$14+f_1$
40– 60	26	$40+f_1$
60– 80	f_2	$40+f_1+f_2$
80–100	16	$56+f_1+f_2$
	N=100	

$\therefore$ Given total frequency=100

Since last cumulative frequency is equal to total frequency, we have

$56+f_1+f_2=100$

or $f_1+f_2=44$

or $f_2=44-f_1$

Now M=Size of N/2th item

=Size of 50th item.

But it is given that median is 50.

Therefore, it must lie in the class interval 40—60. Hence median class interval is 40–60.

Applying interpolation :

$$M=L_1+\frac{i}{f}\left[\frac{N}{2}\text{th}-C\right]$$

$L_1=40,\quad i=20,\quad f=26,\quad C=14+f_1,\quad N/2=50$

$$\therefore \quad M=40+\frac{20}{26}\left[50-(14+f_1)\right]$$

$$50=40+\frac{20}{26}\left[50-14-f_1\right] \qquad \Big| \text{ Given } M=50$$

or $$10=\frac{20}{26}\left[36-f_1\right]$$

or $$\frac{10\times 26}{20}=36-f_1$$

or $f_1=36-13$

or $f_1=23$

But $f_2=44-f_1$

$\therefore$ $f_2=44-23=21$

$\therefore$ $f_1=23$ and $f_2=21$.

Mathematical Property of Median. The sum of deviations taken from median (ignoring algebraic signs) is always minimum. In other words, $\Sigma \mid X-M \mid$ is always minimum. Here X is the variable and M is its median value. Let us take an example.

X	$\mid X-15 \mid$	$\mid X-10 \mid$	$\mid X-20 \mid$
5	10	5	15
8	7	2	12
10	5	0	10
15	0	5	5
20	5	10	0
25	10	15	5
30	15	20	10
	$\Sigma \mid X-15 \mid = 52$	$\Sigma \mid X-10 \mid = 57$	$\Sigma \mid X-20 \mid = 57$

$$M = \text{Size of } \frac{N+1}{2} \text{ th item}$$

$$= \text{Size of } \frac{7+1}{2} \text{ th item.}$$

$$= \text{Size of 4th item.}$$

$$\therefore \quad M = 15.$$

From the above example we find that when deviations are taken from 15 *i.e.*, median value, then sum of deviations (ignoring signs) is minimum *i.e.*, equal to 52.

Merits and Demerits of Median.

Merits :

1. Median is not at all affected by the presence of extreme items.

2. It is a very useful measure when data is qualitative in nature *i.e.*, it cannot be expressed in quantitative terms, *e.g.* data relating to beauty, honesty, intelligence etc.

3. The value of median can be determined graphically also. Ogive or Galton's method can be used for this purpose.

4. It is easy to understand as well as calculate.

5. It is used in further statistical analysis, *e.g.*, in the calculation of skewness, mean deviation, sampling etc.

6. It is very useful in the case of open end class **intervals.**

7. It satisfies most of the essentials of an ideal average.

Demerits :

1. Median is not based on all the items. It does not give any importance to extreme items.

2. Median is more affected by sampling fluctuations than arithmetic mean.

3. It is not suitable for further algebraic treatment *i.e.*, we cannot find combined median.

4. The calculation of median involves the additional work of arranging data.

5. In the case of continuous series we do not get exact value of median but only interpolate the most likely value.

Quartiles, Deciles and Percentiles. These are also positional measures (also called partition values). Median divides the given data into two equal parts. On the other hand quartiles, deciles and percentiles divide the data into four, ten and hundered equal parts respectively. Q_1, D_1 and P_1 are respectively called lower quartile, lower decile and lower percentile. Similarly Q_3, D_9 and P_{99} are called upper quartile, upper decile and upper percentile respectively. Q_1 covers 25% of the data, D_1 covers 10% and P_1 covers only 1% of data. Similarly Q_3, D_9 and P_{99} respectively cover 75%, 90% and 99% of the data.

Calculation of Quartiles, Deciles and Percentiles. All the steps involved in their calculation are same as in the case of median. For example, conditions relating to arrangement of data, determination of cumulative frequencies and exclusive class intervals are the same.

I. Individual Series :

$$Q_1 = \text{size of } \frac{n+1}{4} \text{ th item.}$$

$$Q_3 = \text{size of } \frac{3(n+1)}{4} \text{ th item.}$$

$$D_7 = \text{size of } \frac{7(n+1)}{10} \text{ th item.}$$

$$P_{80} = \text{size of } \frac{80(n+1)}{100} \text{ th item and so on.}$$

$$\text{Median} = Q_2 = D_5 = P_{50}.$$

Example 24. From the following data relating to wages of workers find Q_1, Q_3, D_4, P_{60}.

Wages (Rs.)	50	55	40	58	62	80	70

Solution :

Wages (Rs.)
(arranged data)

40 $\quad Q_1 = \text{Size of } \frac{n+1}{4} \text{ th item.}$

50 $\quad = \text{Size of } \frac{7+1}{4} \text{ th item.}$

55 $\quad = \text{Size of 2nd item.}$

58

62

70

80

$n=7$

$\therefore\ Q_1=50.$

$Q_3=\text{Size of } \frac{3(n+1)}{4}\text{th item.}$

$=\text{Size of } 3(2)\text{th item.}$

$=\text{Size of 6th item.}$

$\therefore\ Q_3=70.$

$D_4=\text{Size of } \frac{4(n+1)}{10}\text{th item.}$

$=\text{Size of } \frac{4(7+1)}{10}\text{th item.}$

$=\text{Size of } 3{\cdot}2\text{th item.}$

$=55+{\cdot}2(58-55)$

$D_4=55+{\cdot}6=55{\cdot}6$

$P_{60}=\text{Size of } \frac{60(n+1)}{100}\text{th item.}$

$=\text{Size of } \frac{60(7+1)}{100}\text{th item.}$

$=\text{Size of } 4{\cdot}8\text{th item.}$

$=58+{\cdot}8(62-58)$

$P_{60}=58+3{\cdot}2=61{\cdot}2.$

II. Discrete Series :

$Q_1=\text{Size of } \frac{N}{4}\text{th item.}$

$Q_3=\text{Size of } \frac{3N}{4}\text{th item.}$

$D_1=\text{Size of } \frac{N}{10}\text{th item.}$

$P_{55}=\text{Size of } \frac{55N}{100}\text{th item.}$ and so on.

Example 25. From the following data find Q_3 and D_6.

Variable	:	10	12	15	18	21	24
Frequency	:	5	8	7	9	6	5

Solution :

Variable	*f*	*c. f.*
10	5	5
12	8	13
15	7	20
18	9	29
21	6	35
24	5	40
	N=40	

$$Q_3 = \text{Size of } \frac{3N}{4} \text{ th item.}$$

$$= \text{Size of } \frac{3(40)}{4} \text{ th item.}$$

$$= \text{Size of 30th item.}$$

30th item lies in

c. f. 35, therefore

$$Q_3 = 21.$$

$$D_6 = \text{Size of } \frac{6N}{10} \text{ th item.}$$

$$= \text{Size of } \frac{6(40)}{10} \text{ th item.}$$

$$= \text{Size of 24th item.}$$

It lies in the *c. f.* 29, therefore sixth decile is 18.

III. Continuous Series :

$$Q_1 = \text{Size of } \frac{N}{4} \text{ th item.}$$

After applying interpolation

$$Q_1 = L_1 + \frac{i}{f}\left[\frac{N}{4}\text{th} - C\right].$$

Similarly, $D_8 = \text{Size of } \frac{8N}{10} \text{ th item.}$

After applying interpolation

$$D_8 = L_1 + \frac{i}{f}\left[\frac{8N}{10}\text{th} - C\right].$$

$$P_{78} = \text{Size of } \frac{78N}{100} \text{ th item.}$$

After applying interpolation

$$P_{78} = L_1 + \frac{i}{f}\left[\frac{78N}{100}\text{th} - C\right].$$

Example 26. Calculate twenty sixth percentile from the following data :

Class interval :	0—10	10—20	20—30	30—40	40—50
Frequency :	8	12	30	35	15

Solution.

Class interval	Frequency (f)	$c.f.$
0—10	8	8
10—20	12	20
20—30	30	50
30—40	35	85
40—50	15	100
	$N=100$	

$$P = \text{Size of } \frac{26N}{100}\text{th item}$$

$$= \text{Size of } \frac{26(100)}{100} \text{ th item}$$

$$= \text{Size of 26th item}$$

26th item lies in $c.f.$ 50. Therefore required class interval is 20—30.

Applying interpolation

$$P_{26} = L_1 + \frac{i}{f}\left[\frac{26N}{100}\text{th} - C\right]$$

$$= 20 + \frac{10}{30}\left[26 - 20\right]$$

$$P_{26} = 20 + 2 = 22.$$

MODE

It is another important and generally used measure of central tendency. Mode is the value of the variable which occurs for the maximum number of times or has maximum concentration of items around it. According to Zizek, "*Mode is the value which occurs most frequently in a series*".

The word mode has been derived from the French word "La mode" which implies fashion. Thus mode is a characteristic which is most common. In our day to day life, we generally talk about most common size of shoe, most common colour of a cycle, most common size of ready made garments, most common income, weight, height etc. While talking about all these we have none else but mode in our mind.

In any data there can be either one or more than one mode. When there are two values of the variable having same maximum frequency, then the data is said to be Bimodal. Similarly when there are three or more such values, the data is said to be, *Trimodal* or *Multimodal* respectively.

Calculation of Mode.

(*a*) *Individual Series.* In this case the calculation of mode

is very simple. It can be determined by inspection only. The value occuring for maximum number of times is called modal value.

Example 27. From the following data relating to marks of students find out modal marks.

Roll No :	1	2	3	4	5	6	7	8	9	10	11	12
Marks :	15	18	12	18	20	22	14	18	16	18	20	35

Solution. From the given data we find that there are four students getting 18 marks each. Since this is the maximum number of times of occurrance of any value, mode is 18.

Example 28. Determine mode from the following:

Value 14 18 35 58 67 67 56 72 67
58 95 58 90 78.

Solution. In the given data 58 and 67 both occur three times (maximum number of times), there are two modes 58 and 67. Data is hence bimodal.

(*b*) *Discrete Series.* Sometimes mode can be determined by following the method of inspection in the case of discrete series. The value around which items are heavily concentrated is called the modal value. For example let us have the following data.

Marks	: x	10	15	20	25	30	35	40
No. of students	: f	5	12	30	50	40	20	3

The above example makes it clear that maximum frequency as well as concentration is at value 25 where 50 is the frequency. Therefore, mode is 25. However, the method of inspection cannot be applied when the items are concentrated at more than one value. In such cases we find out the maximum concentration by making a *grouping table* and an *analysis table* and hence mode is calculated.

In a grouping table there are six columns. First column is the frequency column. In the second column frequencies are added in groups of two each. In column third we add the frequencies in two's leaving the first frequency. The fourth column is prepared by adding the frequencies in three's, starting from first. In column fifth we add frequencies in three's leaving the first frequency. The sixth column is prepared by adding the frequencies in three's leaving first two frequencies. The highest total in each column is marked.

In the analysis table we write value of the variable in the extreme left column. Now a bar is placed for each value whose frequency is marked in the grouping table for all the six columns. The value with the highest number of bars is called the modal value.

Example 29. From the following data relating to wages of workers in a factory find out the modal wage.

Wage (Rs.) :	120	140	200	250	270	300
No. of workers :	5	20	25	23	20	15

Solution :

Grouping Table

Wages Rs.	Column I No. of workers *f*	*II*	*III*	*IV*	*V*	*VI*
120	5	25				
140	20		45	50		
200	25	48			68	
250	23		43			68
270	20	35		58		
300	15					

Analysis Table

Wages Rs.	*I*	*II*	*III*	*IV*	*V*	*VI*	*Total*
120							0
140			1		1		2
200	1	1	1		1	1	5
250		1		1	1	1	4
270				1		1	2
300				1			1

Since maximum number of bars is associated with 200, the mode is also 200.

(c) *Continuous Series.* In the case of continuous series we first find modal class interval by using the techniques of grouping table and analysis table. After this we apply the technique of interpolation to find the value of mode (Z).

Mode or $$Z = L_1 + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times i$$

where L_1 is the lower limit of modal class interval.

$$\Delta_1 = |f_m - f_p|$$

where f_m is the frequency of modal class interval and f_p is the frequency of the class interval preceding modal class interval. | | indicate that only absolute value is taken *i.e.*, signs are ignored.

$$\Delta_2 = |f_m - f_s|$$

where f_s is the frequency of the class succeeding modal class interval. i indicates the size of the modal class interval.

Note. While calculating mode in the case of continuous series, the following points should be noted.

(*i*) Class intervals are arranged in an ascending order.

(*ii*) Class intervals should be exclusive. If they are inclusive, they must be converted into exclusive form.

(*iii*) The frequency distribution should be simple. If it is in "more than" or "less than" form, it must be converted to the simple form before calculating mode.

(*iv*) The class intervals must be equal or uniform. If they are not equal, then we make them equal on the assumption that the frequencies are equally distributed throughout the class.

Example 30. From the following data relating to weight of persons find out the modal weight.

Weight (kgm.) :	40–50	50–60	60–70	70–80	80–90
No. of Persons :	18	20	25	30	28
Weight (kgm.) :	90–100	100–110			
No. of Persons :	24	12			

Solution :

Grouping Table

Weight (kgm.)	*I* *No. of Persons* *f*	*II*	*III*	*IV*	*V*	*VI*
40–50	18	38		63		
50–60	20		45		75	
60–70	25	55				83
70–80	30		58	82		
80–90	28	52			64	
90–100	24		36			
100–110	12					

Analysis Table

Weight (kgm.)	*I*	*II*	*III*	*IV*	*V*	*VI*	*Total*
40— 50							0
50— 60					1		1
60— 70		1			1	1	3
70— 80	1	1	1	1	1	1	6
80— 90			1	1		1	3
90—100				1			1
100—110							0

Since maximum concentration is found at class interval 70—80, it is called the modal class interval.

Applying Interpolation

$$Z = L_1 + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times i$$

$L_1 = 70, \quad \Delta_1 = | f_m - f_p |$

$= 30 - 25 = 5$

$\Delta_2 = | f_m - f_s | = 30 - 28 = 2$

$i = 10.$

$\therefore$ Substituting these values in the above formula, we have

$$Z = 70 + \frac{5}{5+2} \times 10$$

$$= 70 + \frac{50}{7}$$

$$= 70 + 7{\cdot}14 = 77{\cdot}14$$

Therefore, modal weight is 77·14 kgm.

Example 31. Calculate mode from the following data.

Mid. Values :	12	18	24	30	36	42	48
Frequency :	100	120	480	580	440	200	80

Solution. Since mid. values are given, we first determine class intervals. Since gap in each mid. value is equal to 6, the class intervals

will be as follows.

Class Interval	*Frequency*
9—15	100
15—21	120
21—27	480
27—33	580
33—39	440
39—45	200
45—51	80

$$\text{Mode}=L_1+\frac{\Delta_1}{\Delta_1+\Delta_2}\times i$$

$$\text{Mode}=27+\frac{100}{100+140}\times 6$$

$$=27+\frac{100}{240}\times 6$$

$$=27+2{\cdot}5$$

$$\text{Mode}=29{\cdot}5.$$

By inspection, modal class interval is 27—33 because it has maximum frequency and also maximum concentration of items.

$$L_1=27$$
$$\Delta_1=|f_m-f_p|$$
$$=580-480$$
$$=100$$
$$\Delta_2=|f_m-f_s|$$
$$=580-440$$
$$=140$$
$$i=6$$

Example 32. The monthly profits in rupees of 100 shops are distributed as below :

Profits Per Shop (Rs.) :	0—100	100—200	200—300	300—400
No. of Shops :	12	—	27	20
Profits Per Shop (Rs.) :	400—500	500—600		
No. of Shops :	—	6		

Determine the missing frequencies if the mode is 256·25.

Solution. Two frequencies are missing. Let us denote them by f_1 and f_2 we have

Profits Per Shop (*Rs.*)	*No. of Shops* f
0—100	12
100—200	f_1
200—300	27
300—400	20
400—500	f_2
500—600	6
	100

We have $65+f_1+f_2=100$

or $f_1+f_2=35$

Since mode is given 256·25 it must lie in the class interval 200—300.

$$\therefore\quad Z=L_1+\frac{\Delta_1}{\Delta_1+\Delta_2}\times i$$

$$L_1=200,\quad \Delta_1=|f_m-f_p|=27-f_1,$$

$$\Delta_2 = |f_m - f_0| = 27 - 20 = 7$$

$$i = 100$$

$$\therefore \qquad 256{\cdot}25 = 200 + \frac{27 - f_1}{(27 - f_1) + 7} \times 100$$

or $$56{\cdot}25 = \frac{2700 - 100f_1}{34 - f_1}$$

or $$56{\cdot}25\,(34 - f_1) = 2700 - 100f_1$$

or $$1912{\cdot}5 - 56{\cdot}25\,f_1 = 2700 - 100f_1$$

or $$43{\cdot}75\,f_1 = 787{\cdot}5$$

or $$f_1 = \frac{787{\cdot}5}{43{\cdot}75} = 18$$

Therefore, $f_1 + f_2 = 35$

or $18 + f_2 = 35$ or $f_2 = 17.$

Relationship between Mean, Median and Mode.

According to *Prof. Karl Pearson,* there exists a relation between mean, median and mode. It is as follows

Mode = 3 Median — 2 Mean

or $Z = 3\,M - 2\overline{X}$

If values of any two are given in a moderately asymmetrical distribution, then the value of third can be calculated by applying above formula. This formula is very helpful in determining mode in the case of data where mode is ill defined *i.e.* it is bimodal.

Example 33. For a moderately asymmetrical data mean and median are 45 and 60 respectively. Find the value of mode.

Solution. We know that mode = 3 median — 2 mean

or Mode = 3(60) — 2(45)

Mode = 180 — 90 = 90.

Merits and Demerits of Mode

Merits

(*i*) It is easy to calculate and understand.
(*ii*) It is suitable in case of open end classes.
(*iii*) It is not affected by the presence of extreme items.
(*vi*) It can be calculated graphically also.
(*v*) It is an actual value of the variable.

Demerits

(*i*) It is not based on all the observations.

(*ii*) It is not capable of algebraic manipulation.

(*iii*) When the number of items is not large, mode loses its significance.

(*iv*) It is not necessary that we get a unique mode for each data. It can be bimodal or multimodal.

(*v*) It does not possess the merit of sampling stability.

GEOMETRIC MEAN

It is a mathematical measure of the central tendency. It is nothing but the *n*th root of the product of *n* observations of the series. For example if there are two observations, say x_1 and x_2, then G.M.$=(x_1.x_2)^{1/2}$ *i.e.* square root of their product.

If there are three items 1, 2 and 4 then G.M.$=(1\times2\times4)^{1/3}$ *i.e.* cube root of their product.

$$\text{G.M.}=(8)^{1/3}=2.$$

Calculation of Geometric Mean

(*a*) *Individual Series.* Suppose x is any variable taking $x_1, x_2, x_3......x_n$ *i.e.* n values. Now Geometric mean of x is the *n*th root of the product of these n items. In other words,

$$\text{G.M.}=(x_1.\ x_2.x_2......x_n)^{1/n}.$$

The calculations can be made very easy by taking log.

$$\therefore \quad \log(\text{G.M.})=\log(x_1.x_2.x_3...x_n)^{1/n}$$
$$=1/n \log(x_1.x_2.x_3......x_n)$$
$$=1/n\,[\log x_1+\log x_2+\log x_3......+\log x_n]$$
$$\log(\text{G.M.})=1/n\ \Sigma \log x$$

Taking Antilog on both sides, we have

$$\text{G.M.}=\text{AL}\left(\frac{\Sigma \log x}{n}\right).$$

Example 34. The marks obtained by four students are 25, 38, 42 and 50. Find the value of geometric mean.

Solution. Let us denote marks by x

x	$\log x$
25	1·3979
38	1·5798
42	1·6232
50	1·6990
	$\Sigma \log x = 6{\cdot}2999$

$$\text{G.M.}=\text{AL}\left(\frac{\Sigma \log x}{n}\right)$$
$$=\text{AL}\left(\frac{6{\cdot}2999}{4}\right)$$
$$=\text{AL}\,(1{\cdot}5749)$$
$$=37{\cdot}58$$

(*b*) *Discrete and Continuous Series.* The formula is,

$$G.M. = AL\left(\frac{\Sigma f \log x}{\Sigma f}\right)$$

where $\Sigma f \log x$ is the sum of the product of log of variable values and the frequency. Σf is the total frequency. In the case of continuous series, the class intervals are represented by mid values and the same formula is applied.

Example 35. From the following data relating to income of 500 families determine geometric mean.

Income (Rs. Per Month) :	100	750	1000	1250	150C
No. of Families :	100	150	75	90	85

Solution :

Income (Rs. Per Month) x	*No. of Families* f	*log x*	$f \cdot \log x$
100	100	2·0000	200·000
750	150	2·8751	431·265
1000	75	3·0000	225·000
1250	90	3·0969	278·721
1500	85	3·1761	269·9685
	$\Sigma f = 500$		$\Sigma f \cdot \log x = \mathbf{1404 \cdot 9545}$

$$G.M. = AL\left(\frac{\Sigma f \log x}{\Sigma f}\right) = AL\left(\frac{1404 \cdot 9545}{500}\right)$$

$$= AL\ (2 \cdot 8099)$$

$$= 645 \cdot 5.$$

Example 36. Compute geometric mean from the following data :

Profits (Rs.) :	1000—2000	2000—3000	3000—4000	4000—5000	5000—6000
No. of firms :	12	18	20	40	10

Solution :

Profits (Rs.)	*No. of firms* f	*M.V.* x	*log x*	f . *log x*
1000—2000	12	1500	3·1761	38·1132
2000—3000	18	2500	3·3979	61·1622
3000—4000	20	3500	3·5441	70·8820
4000—5000	40	4500	3·6532	146·1280
5000—6000	10	5500	3·7404	37·4040
	$\Sigma f=100$			$\Sigma f \log x=353·6894$

$$\text{G.M.}=\text{AL}\left(\frac{\Sigma f \log x}{\Sigma f}\right)=\text{AL}\left(\frac{353·6894}{100}\right)$$

$$=\text{AL}\ (3·5369)$$

$$=3443.$$

Special Use of Geometric Mean. Geometric mean is very useful and appropriate for averaging ratios, percentages and rates of change. It is also useful in determining the rate of growth and constructing index numbers.

Example 37. The price of a commodity increased by 12% from 1983 to 1984, 15% from 1984 to 1985 and 30% from 1985 to 1986. The average increase is quoted as 18·8% instead of 19%. Explain the result.

Solution. Total increase in price for three years is 12%+15%+30%=57%. Therefore, if arithmetic mean is calculated then it comes out to be 19% (57/3). But since we are required to average percentages, the Geometric mean is appropriate measure.

% increase	*Price at the end of year assuming price at the beginning 100.* x	*log x*
12	112	2·0492
15	115	2·0607
30	130	2·1139
		$\Sigma \log x=6·2238$

$$\text{G.M.}=\text{AL}\left(\frac{\Sigma \log x}{n}\right)=\text{AL}\left(\frac{6·2238}{3}\right)$$

$$\text{G.M.}=\text{AL}\ (2·0746)$$

$$=118·8.$$

$\therefore$ Average increase$=118·8-100=18·8\%$.

Combined Geometric Mean : If $G_1, G_2, G_3, \ldots, G_k$ are the geometric means of the k series of sizes $n_1, n_2, n_3, \ldots\ldots, n_k$ respectively, then Geometric mean G of the combined series of sizes $n_1+n_2+n_3+\ldots\ldots+n_k$ is given by

$$G = AL\left[\frac{n_1 \log G_1 + n_2 \log G_2 + \ldots + n_k \log G_k}{n_1+n_2+\ldots\ldots n_k}\right]$$

Example 38. Three groups of items contain 8, 7 and 5 observations. Their geometric means are 8·52, 10·12 and 7·75 respectively. Find the geometric mean of 20 observations.

Solution. Given $n_1=8$, $n_2=7$, $n_3=5$

$G_1=8{\cdot}52$, $G_2=10{\cdot}12$ and $G_3=7{\cdot}75$

Now $$G = AL\left[\frac{n_1 \log G_1 + n_2 \log G_2 + n_3 \log G_3}{n_1+n_2+n_3}\right]$$

$$= AL\left[\frac{8 \,.\, \log(8{\cdot}52) + 7 \,.\, \log(10{\cdot}12) + 5 \,.\, \log(7{\cdot}75)}{8+7+5}\right]$$

$$= AL\left[\frac{8\times 0{\cdot}9304 + 7\times 1{\cdot}0052 + 5\times 0{\cdot}8893}{20}\right]$$

$$= AL\left[\frac{18{\cdot}9261}{20}\right] = AL[0{\cdot}9463]$$

$$= 8{\cdot}837$$

(g) Properties of Geometric Mean.

Geometric mean possesses some very important properties which are as follows. These properties add to the usefulness of the geometric mean.

1. We know that in the case of arithmetic mean if each item is replaced by arithmetic mean, the sum of the items remains the same. Similarly, in the case of geometric mean, if each item is replaced by the geometric mean, the product of items remains the same *e.g.* Mean of 5, 10, 20 is $\frac{35}{3}$ and sum of items is 35. If each items is replaced by arithmetic mean $\frac{35}{3}$, the sum of items remains the same *i.e.* 35. Similarly the G.M. of the above values is 10 and product of values is 1000. If each item is replaced by 10 *i.e.* G.M., then the product $10\times10\times10=1000$ remains the same.

2. Just as in the case of arithmetic mean the sum of the deviations of the items above the mean equals the sum of the deviations of items below the mean (ignoring signs), similarly, in case of geometric mean the products of the corresponding ratios are equal. In other words, the product of the ratios of values less than the geometric mean to the geometric mean is equal to the product of the ratios of the geometric mean to the values greater than the geometric mean.

e.g. we have the values

4, 5, 20, 25, $G.M. = \sqrt[4]{10{,}000} = 10$

Now $\frac{10}{4}\times\frac{10}{5}=\frac{20}{10}\times\frac{25}{10}$, or $5=5$

In other words, $\frac{4}{10}\times\frac{5}{10}=\frac{10}{20}\times\frac{10}{25}$

This property establishes the role of geometric mean as a measure of relative changes. To illustrate, if the price of one commodity increases from Rs. 4 to Rs. 64 and the price of another commodity decreases from Rs. 16 to Rs. 1, there is no relative change in the price level. This is because the increase in price of one is compensated by an equal redution in the price of other commodity. Arithmetic mean is incapable of explaining this.

Commodity	Original Price	New Price
A	4	64
B	16	1
$\overline{X}$	$\frac{20}{2}=10$	$\frac{65}{2}=32\cdot5$
G.M.	$\sqrt{64}=8$	$\sqrt{64}=8$

Since there is no change in G.M., there is no change in the price level. Because of above property, the geometric mean is used while averaging ratios or percentages.

3. The geometric mean of the ratio of corresponding observations in two series is equal to the ratios of their geometric means.

e.g. Suppose we have two series X and Y.

X	Y	X/Y
1	2	·5
5	4	1·25
10	4	2·5
200	8	25
GM.=10	G.M.=4	G.M.= 2·5

Merits and Demerits of Geometric Mean.

Merits :

(*i*) It is based on all the observations of a series.
(*ii*) It is capable of algebraic manipulation.
(*iii*) It is not much affected by extreme items.
(*iv*) It is not much affected by sampling fluctuations.
(*v*) It is very useful for constructing index numbers, averaging percentages, ratios and determining rates of growth.
(*vi*) Since it is a mathematical average, it does not require an arrangement of data before its calculation.

Demerits :

(*i*) It is difficult to calculate.
(*ii*) It cannot be used if any value of the variable is zero.
(*iii*) It gives imaginary value if any item in the data is negative.
(*iv*) Its value cannot be calculated by mere inspection.

HARMONIC MEAN

It is the reciprocal of the arithmetic mean of the reciprocals of the values of the variable. It is very useful for finding average speed. It is a mathematical measure of central tendency and gives more weight to small items.

Calculation of Harmonic Mean.

(*a*) *Individual Series.* If x is any variable taking values $x_1, x_2, x_3, \ldots\ldots, x_n$, then its harmonic mean is given by

$$\text{H.M.}=\frac{1}{\dfrac{\Sigma\left(\dfrac{1}{x}\right)}{n}}$$

or $$\text{H.M.}=\frac{n}{\Sigma\left(\dfrac{1}{x}\right)}$$ *i.e.*, reciprocal of the arithmetic mean of the reciprocals of items.

Example 39. Calculate Harmonic Mean for the following set of values :

10 20 25 40 50 80.

Solution :

X	$1/x$
10	$\frac{1}{10}=\cdot10$
20	$\frac{1}{20}=\cdot05$
25	$\frac{1}{25}=\cdot04$
40	$\frac{1}{40}=\cdot025$
50	$\frac{1}{50}=\cdot020$
80	$\frac{1}{80}=\cdot0125$
	$\Sigma\left(\frac{1}{x}\right)=\cdot2475$

$$\text{H.M.}=\frac{n}{\Sigma\,(1/x)}$$

$$=\frac{6}{\cdot2475}$$

$$=24\cdot242$$

(*b*) *Discrete and Continuous Series.* In the continuous series the class intervals are represented by their mid values and the following formula is used for both the series.

$$H.M. = \frac{\Sigma f}{\Sigma f\left(\frac{1}{x}\right)}$$

Example 40. From the following data determine the value of harmonic mean.

Class Interval :	0—10	10—20	20—30	30—40	40—50	50—60
Frequency :	10	15	75	49	81	110

Solution :

Class Interval	*Frequency* f	*Mid Value* x	$1/x$	$f \cdot 1/x$
0—10	10	5	$\frac{1}{5}$	$10 \times \frac{1}{5} = 2{\cdot}0$
10—20	15	15	$\frac{1}{15}$	$15 \times \frac{1}{15} = 1{\cdot}0$
20—30	75	25	$\frac{1}{25}$	$75 \times \frac{1}{25} = 3{\cdot}0$
30—40	49	35	$\frac{1}{35}$	$49 \times \frac{1}{35} = 1{\cdot}4$
40—50	81	45	$\frac{1}{45}$	$81 \times \frac{1}{45} = 1{\cdot}8$
50—60	110	55	$\frac{1}{55}$	$110 \times \frac{1}{55} = 2{\cdot}0$
	$\Sigma f = 340$			$\Sigma f \cdot \frac{1}{x} = 11{\cdot}2$

$$H.M. = \frac{\Sigma f}{\Sigma f \cdot \frac{1}{x}} = \frac{340}{11{\cdot}2} = 30{\cdot}357.$$

Example 41. A car travels uphill at a speed of 27 km. per hour and while returning covers the distance at a speed of 45 km. per hour. Find the average speed and verify your result.

Solution. Harmonic mean is suitable for averaging speed.

Speed (km./hour) x	$1/x$
27	1/27
45	1/45

$$\text{H.M.} = \frac{n}{\Sigma \frac{1}{x}}$$

$$= \frac{2}{\frac{1}{27} + \frac{1}{45}}$$

$$= \frac{2}{\cdot 03704 + \cdot 02222} = \frac{2}{\cdot 05926}$$

Average speed $= 33 \cdot 75$ km. per hour.

Verification of Result.

Let us suppose that total distance is 100 km.

Speed (km per/Hour)	*Distance (km.)*	*Time Taken (Hours)*
27	100	3·704
45	100	2·222
	200	5·926

We know that speed $= \frac{\text{Total Distance}}{\text{Total time taken}}$

$$= \frac{200}{5 \cdot 926} = 33 \cdot 75 \text{ km. per hour.}$$

Example 42. A cyclist covers his first 5 km. at an average speed of 10 km/hr. another 3 km. at 8 km./hr. and last 2 km. at the speed of 5 km./hr. Find the average speed for the entire journey.

Solution :

Speed (km./hr.) x	*Distance (km.)* f	$1/x$	$f \cdot 1/x$
10	5	$\frac{1}{10}$	$5 \times \frac{1}{10} = 0 \cdot 5$
8	3	$\frac{1}{8}$	$3 \times \frac{1}{8} = 0 \cdot 375$
5	2	$\frac{1}{5}$	$2 \times \frac{1}{5} = 0 \cdot 4$
	$\Sigma f = 10$		$\Sigma\left(f \cdot \frac{1}{x}\right) = 1 \cdot 275$

$$\text{H.M.} = \frac{\Sigma f}{\Sigma\left(f \cdot \frac{1}{x}\right)} = \frac{10}{1 \cdot 275} = 7 \cdot 84.$$

Therefore, the average speed for the entire journey is **7·84 km. per hour.**

Example 43. Suppose bananas are obtained at 2 for 80 Paisa and potatoes at 5 for 60 Paise. If a person spends rupee one on each, find the average price.

Solution. For finding average price the Harmonic mean is more suitable as compared to the Arithmetic mean.

If Arithmetic mean is used.

7 items are obtained for 140 Paisa.

$\therefore$ Average Price$=\dfrac{140}{7}=20$ Paisa.

But this is incorrect because by spending one rupee each a person can get 2·5 bananas and $\dfrac{100}{12}$ *i.e.* $\dfrac{25}{3}$ potatoes. Therefore average price is

$$\frac{200 \text{ Paise}}{2\cdot5+\frac{25}{3}}=18\cdot46 \text{ Paisa}$$

The same result is obtained when Harmonic mean is used.

$$\text{H.M.}=\frac{2}{\frac{2}{80}+\frac{5}{60}}=\frac{2\times240}{6+20}=\frac{480}{26}$$

$$=18\cdot46 \text{ Paisa.}$$

(*i*) A.M>G.M.>H.M., when items are not equal in value.

(*ii*) A.M.=G.M.=H.M., when values of the items are equal.

Proof. (*i*) A.M>G.M.>H.M.

Let us suppose that a and b are two items which are not equal *i.e.*, $a\neq b$.

Now $\text{A.M.}=\dfrac{a+b}{2}$

$\text{G.M.}=\sqrt{ab}$ and $\text{H.M.}=\dfrac{2}{\frac{1}{a}+\frac{1}{b}}$

or $\text{H.M.}=\dfrac{2ab}{a+b}$.

We know that

$(a-b)^2>0$

or $a^2+b^2-2ab>0$

or $a^2+b^2>2ab$.

Adding $2ab$ to both sides, we have,

$a^2+b^2+2ab>2ab+2ab,$

or $(a+b)^2>4ab$

or $\frac{(a+b)^2}{4}>ab$

or $\left(\frac{a+b}{2}\right)^2>ab$

Taking Sq. root, we have.

$$\frac{a+b}{2}>\sqrt{ab} \quad \text{i.e., A.M.} > \text{G.M.} \qquad \text{....I}$$

Again writing $\frac{a+b}{2}>\sqrt{ab}$

$\therefore \quad a+b>2.\sqrt{ab}$

Multiplying both sides by $\sqrt{ab}$, we have,

$(a+b)\sqrt{ab}>2\,.\sqrt{ab}\,.\sqrt{ab}$

or $\sqrt{ab}>\frac{2ab}{a+b}$ i.e., G.M.>H.M. ...(II)

$\therefore$ from I and II

A.M.>G.M.>H.M.

(*ii*) A.M.=G.M.=H.M., when a and b are equal.

We have A.M. $=\frac{a+b}{2}=\frac{a+a}{2}=a.$ $(\because \; a=b)$

G.M. $=\sqrt{ab}=\sqrt{a\,.\,a}=\sqrt{a^2}=a.$

H.M. $=\frac{2ab}{a+b}=\frac{2aa}{a+a}=\frac{2a^2}{2a}=a.$

$\therefore$ When $a=b$, A.M.=G.M.=H.M.

Combining (*i*) and (*ii*) results, we can say that,

A.M. $\geqslant$ G.M. $\geqslant$ H.M.

Merits and Demerits of Harmonic Mean

Merits :

(*i*) It is rigidly defined.

(*ii*) It is based on all the items.

(*iii*) It is capable of algebraic treatment.

(*iv*) It is not greatly affected by sampling fluctuations.

(*v*) It is a very useful measure for calculating average speed.

Demerits :

(*i*) It is difficult to calculate and understand than arithmetic mean.

(*ii*) Its value cannot be calculated if any value is equal to zero.

(*iii*) It gives more weight to small values and less weight to large values.

Which Average to Use : After studying all the measures of central tendency the main question arises "which is the best average"? It is very difficult to answer this question. The usefulness of an average differs from purpose to purpose and availability and nature of the data. No particular average can serve all the purposes. If the variable under consideration is capable of numerical measurement, then we can use arithmetic mean. But here is a difficulty also. If the data contains extreme items or there are open end classes, then use of mean is avoided. In such cases positional measures like median or mode are preferred. Mode indicates the most common value. So it is generally used whenever we are required to find the common size of hat, shoe, readymade garments etc. Geometric mean and harmonic mean are preferred whenever we intend to average percentages or speed respectively.

The fact remains that arithmetic mean is commonly used because of its simplicity and ease in calculation. However, other averages are also used depending upon the purpose and nature of available data.

MISCELLANEOUS PROBLEMS

Example 44. The numbers 3·2, 5·8, 7·9 and 4·5 have frequencies x, $(x+2)$, $(x-3)$ and $(x+6)$ respectively. If the arithmetic mean is 4·876, find the value of x.

Solution :

Number (x)	*Frequency* (f)	$f.x$
3·2	x	$3{\cdot}2\,x$
5·8	$x+2$	$5{\cdot}8x+11{\cdot}6$
7·9	$x-3$	$7{\cdot}9x-23{\cdot}7$
4·5	$x+6$	$4{\cdot}5x+27{\cdot}0$
	$\Sigma f=4x+5$	$21{\cdot}4x+14{\cdot}9$

$$\bar{x}=\frac{\Sigma fx}{\Sigma f}$$

Given $\bar{x}=4{\cdot}876$, Putting the values,

$$4{\cdot}876=\frac{21{\cdot}4x+14{\cdot}9}{4x+5}$$

or $\quad 4{\cdot}876(4x+5)=21{\cdot}4x+14{\cdot}9$

or $\quad 19{\cdot}504x+24{\cdot}380=21{\cdot}4x+14{\cdot}9$

or $19{\cdot}504x - 21{\cdot}4x = 14{\cdot}9 - 24{\cdot}380$

or $-1{\cdot}896x = -9{\cdot}480$

or $$x = \frac{9{\cdot}480}{1{\cdot}896} = 5$$

Example 45. The following are the monthly salaries in rupees of 20 employees of a firm :

130, 62, 145, 118, 125, 76, 151, 142, 110, 98,
65, 116, 100, 103, 71, 85, 80, 122, 132, 95.

The firm gives bonuses of Rs. 10, 15, 20, 25 and 30 for individuals in the respective salary groups exceeding Rs. 60 but not exceeding Rs. 80, exceeding Rs. 80 but not exceeding Rs. 100, and so on upto exceeding Rs. 140 but not exceeding Rs. 160. Find the average bonus paid per employee.

Solution. Let us first express the given data in the form of a continuous frequency distribution and then find average bonus.

Salary (in Rs.)	*Tally Bars*	*Frequency (f)*	*Bonus (Rs.) (x)*	*f.x*
61— 80	~~IIII~~	5	10	50
81—100	IIII	4	15	60
101—120	IIII	4	20	80
121—140	IIII	4	25	100
141—160	III	3	30	90
		$\Sigma f = 20$		$\Sigma fx = 380$

Average Bonus per Employee *i.e.* $\bar{x} = \frac{\Sigma fx}{\Sigma f}$

$$= \frac{380}{20} = 19.$$

Therefore, average bonus paid per employee is Rs. 19.

Example 46. Compute the weighted arithmetic mean of the index number from the following data :

Group	Index No.	Weight
Food	112	6
Clothing	127	4
Fuel and Light	118	5
House Rent	120	2
Miscellaneous	140	3

Solution :

Group	*Index No.* (I)	*Weight* (W)	*W.I.*
Food	112	6	672
Clothing	127	4	508
Fuel and Light	118	5	590
House Rent	120	2	240
Miscellaneous	140	3	420
		$\Sigma W=20$	$\Sigma WI=2430$

$$\text{Weighted Arithmetic Mean}=\frac{\Sigma WI}{\Sigma W}=\frac{2430}{20}=121 \cdot 5.$$

Example 47. For a group of 5000 workers, the weekly wages vary from Rs. 20 to Rs. 80. The wages of 4% of the workers are under Rs. 25 and those of 10% are under 30, 15% of the workers earn Rs. 60 and over, and 5% of them get Rs. 70 and over. The quartile wages are Rs. 40 and Rs. 54, and the sixth decile is Rs. 50. Put this information in the form of a frequency table and calculate the value of arithmetic mean.

Solution. It is given that total number of workers is 5000. The minimum and maximum values of wages are Rs. 20 and Rs. 80 respectively. $Q_1=40$, $Q_3=54$ and $D_6=50$.

$Q_1=40$ implies that 25% of workers get under Rs. 40. $Q_3=54$ implies that 75% of workers get under Rs. 54. Similarly $D_6=50$ means that 60% of workers get wages under Rs. 50.

Weekly wages (*Rs.*)	*Percentage of workers*	*No. of workers* (f)
Under 25	4%	$5000\times\frac{4}{100}=200$
25—30	6%	$5000\times\frac{6}{100}=300$
30—40	15%	$1500\times\frac{15}{100}=750$
40—50	35%	$5000\times\frac{35}{100}=1750$
50—54	15%	$5000\times\frac{15}{100}=750$

54—60	10%*	$5000\times\frac{10}{100}=500$
60—70	10%	$5000\times\frac{10}{100}=500$
70 and over	5%	$5000\times\frac{5}{10}=250$
	100%	$\Sigma f=5000$

Calculation of Arithmetic Mean

Weekly wages (Rs.)	*No. of workers* f	*Mid. Values* x	$f.x$
20—25	200	22·5	4500
25—30	300	27·5	8250
30—40	750	35	26250
40—50	1750	45	78750
50—54	750	52	39000
54—60	500	57	28500
60—70	500	65	32500
70—80	2·0	75	18750
	$\Sigma f=5000$		$\Sigma fx=236500$

$$\bar{x}=\frac{\Sigma fx}{\Sigma f}=\frac{236500}{5000}=47\cdot3.$$

Example 48. The median and mode of the following wage distribution are known to be Rs. 233·5 and Rs. 234 respectively. Determine the missing frequencies. Given that total frequency is 230.

Wages	200-210	210-220	220-230	230-240	240-250	250-260	260-270
Frequencies	4	16	?	?	?	6	4

* Balancing figure.

Solution. Let us denote the missing frequencies by f_1, f_2 and f_3.

Wages (in Rs.)	*Frequency* (f)	$c.f$
200—210	4	4
210—220	16	20
220—230	f_1	$20+f_1$
230—240	f_2	$20+f_1+f_2$
240—250	f_3	$20+f_1+f_2+f_3$
250—260	6	$26+f_1+f_2+f_3$
260—270	4	$30+f_1+f_2+f_3$
	$\Sigma f=230$	

$$30+f_1+f_2+f_3=230$$

or $\quad f_1+f_2+f_3=200 \quad$ or $\quad f_3=200-f_1-f_2 \quad$ ...(i)

We know that Mode $=L_1+\dfrac{\Delta_1}{\Delta_1+\Delta_2}\times i$

Since Mode=234, the modal class interval is 230—240.

$L_1=230,\ i=10,\ \Delta_1=|f_m-f_p|=f_2-f_1$

$\Delta_2=|f_m-f_s|=f_2-f_3.$

Putting the values, we get

$$234=230+\frac{f_2-f_1}{f_2-f_1+f_2-f_3}\times 10$$

or $\quad 4=\dfrac{f_2-f_1}{2f_2-f_1-f_3}\times 10$

or $\quad \dfrac{4}{10}=\dfrac{f_2-f_1}{2f_2-f_1-f_3}$

or $\quad 0{\cdot}4=\dfrac{f_2-f_1}{2f_2-f_1-f_3}$

Now since $f_3=200-f_1-f_2$, we have,

$$0{\cdot}4=\frac{f_2-f_1}{2f_2-f_1-200+f_1+f_2}$$

or $\quad 0{\cdot}4=\dfrac{f_2-f_1}{3f_2-200}$

or $\quad 1{\cdot}2f_2-80=f_2-f_1$

or $\quad f_1=f_2-1{\cdot}2f_2+80$

or $\boxed{f_1 = -0.2 f_2 + 80}$ ...(ii)

We know that

$$M = \text{size of } \frac{N}{2}\text{th item.}$$

$$= \text{size of } \frac{230}{2}\text{th item.}$$

$$= \text{size of 115th item.}$$

Since the value of Median is 233·5, therefore, the median class interval is 230—240.

Applying $M = L_1 + \frac{i}{f}\left[\frac{N}{2}\text{th} - C\right]$, we have

$$233{\cdot}5 = 230 + \frac{10}{f_2}\left[115 - (20 + f_1)\right]$$

or $$3{\cdot}5 = \frac{10}{f_2}\left[115 - 20 - f_1\right]$$

or $$\frac{3{\cdot}5 f_2}{10} = 95 - f_1$$

or $$0{\cdot}35 f_2 = 95 - f_1$$

or $\boxed{f_1 = 95 - 0{\cdot}35 f_2}$ ...(iii)

Solving equations (*ii*) and (*iii*), we have,

$$-0{\cdot}2 f_2 + 80 = 95 - 0{\cdot}35 f_2$$

or $$-0{\cdot}2 f_2 + 0{\cdot}35 f_2 = 95 - 80$$

or $$0{\cdot}15 f_2 = 15$$

or $$f_2 = \frac{15}{0{\cdot}15} = 100.$$

Putting the value of f_2 in equation (*iii*), we have

$$f_1 = 95 - 0{\cdot}35\,(100)$$

$$f_1 = 95 - 35 = 60$$

Now putting the values of f_1 and f_2 in equation (*i*), we have

$$f_3 = 200 - f_1 - f_2$$

or $$f_3 = 200 - 60 - 100$$

$$f_3 = 200 - 160 = 40.$$

Example 49. Find the class intervals if the arithmetic mean of the following distribution is 33 and assumed mean 35.

Step Deviation	—3	—2	—1	0	1	2
Frequency	5	10	25	30	20	10

Sol The values of step deviations are given. It implies the values after taking assumed mean and dividing the deviations by any common factor.

Therefore, we have

$d'x$	f	$f.d'x$
—3	5	—15
—2	10	—20
—1	25	—25
0	30	0
1	20	20
2	10	20
	$\Sigma f=100$	$\Sigma fd'x=-20$

We know that $\bar{x} = A+\frac{\Sigma f d'x}{\Sigma f}\times C$

Given $\bar{x}=33$, $A=35$.

Putting the values, we have

$$33=35+\frac{-20}{100}\times C$$

or $$-2=\frac{-20}{100}\times C$$

or $-200=-20\,C$

or $$C=\frac{200}{20}=10.$$

Therefore, the size of class intervals is 10. From $d'x$, we can now reach at the original class intervals by using the following procedure :

$d'x$	—3	—2	—1	0	1	2
$dx=(d'x)(c)$	—30	—20	—10	0	10	20
$x=(dx+A)$	5	15	25	35	45	55
Class interval	0—10	10—20	20—30	30—40	40—50	50—60

Example 50. The following data relates to the distribution of marks of 50 students.

Marks (More than) :	0	10	20	30	40	50
No. of students :	50	46	40	20	10	3

Find out the median marks. If 60 per cent of students pass this test, find the minimum marks obtained by a pass student.

Solution. For calculating median, first of all the given cumulative frequency distribution is converted to simple form and then cumulative frequencies are calculated.

Marks	*No. of Students*	*C.f.*
0—10	50—46=4	4
10—20	46—40=6	10
20—30	40—20=20	30
30—40	20—10=10	40
40—50	10— 3= 7	47
50—60	3	50
	$\Sigma f=50$	

$$M = \text{Size of } \frac{N}{2} \text{ th item}$$

$$= \text{Size of } \frac{50}{2} \text{th item}$$

$$= \text{Size of 25th item.}$$

25th item lies in cumulative frequency 30

$\therefore$ Median class interval is 20—30

Applying $M = L_1 + \frac{i}{f}\left[\frac{N}{2}\text{ th} - C\right]$.

$$= 20 + \frac{10}{20}\left[25 - 10\right]$$

$$= 20 + \frac{10}{20}(15) = 20 + \frac{150}{20}$$

$$= 20 + 7{\cdot}5 = 27{\cdot}5$$

Hence the median marks are 27·5.

Calculation of Minimum marks of a pass student. Let us suppose that minimum marks of a pass student is x. It is given that 60% students pass the test. Therefore, 40% students get marks less than x. In other words, we calculate the value of P_{40}.

We know that $P_{40} = \text{Size of } \frac{40N}{100} \text{ th item.}$

$$= \text{Size of } \frac{40 \times 50}{100} \text{ th item.}$$

$$=\text{Size of 20th item.}$$

It lies in the C.f. 30.

Therefore, the required class interval is 20—30.

Applying $P_{40}=L_1+\frac{i}{f}\left[\frac{40N}{100}\text{th}-C\right]$

$$=20+\frac{10}{20}\left[20-10\right].$$

$$=20+\frac{10\times 10}{20}=25.$$

Therefore, the minimum marks obtained by a pass student in the test is 25.

QUESTIONS

1. What do you understand by central tendency? Explain with the help of an example. What purpose does a measure of central tendency serve?
2. (a) Explain clearly the relationship between mean, median and mode.
   (b) What are the properties of arithmetic mean?
3. What is meant by an ideal average? Discuss the essentials of an idea average.
4. Compare and contrast arithmetic mean, geometric mean and harmonic mean. Which of them is least affected by extreme items?
5. Define Geometric mean. Explain the method of calculating it with the help of an example.
6. What are the desirable properties of a good average? Under what circumstances the use of median, harmonic and geometric means would give better results than the arithmetic mean? Explain with examples.
7. (a) What are the merits and demerits of arithmetic mean and median?
   (b) What are the merits of mode as an average?
8. The following table gives the monthly income of 9 families in a town.

House No.	1	2	3	4	5	6	7	8	9
Income (Rs.)	220	340	260	180	450	670	720	890	950

($\overline{X}=520$).

9. Calculate the arithmetic mean of following:

Wages (in Rs.)	4	6	8	10	15	16
No. of Workers	5	15	6	7	8	2

($\overline{X}=$Rs. 8·84)

10. The weekly observation on cost of living index in a certain city for the year 1980—81 are given below. Find mean.

Index No.	140—150	150—160	160—170	170—180
No. of Workers	5	10	20	9

Index No.	180—190	190—200
No. of Workers	6	2

($\overline{X}=166.3$)

11. Calculate mean from the following data :

Marks (below)	10	20	30	40	50	60	70	80
No. of students	25	40	60	75	95	125	190	240

($\overline{X}=49.58$)

12. Find the missing frequency if arithmetic mean is 28 of the data given below :

Profit per shop	0—10	10—20	20—30	30—40
No. of shops	12	18	27	—

Profit per shop	40—50	50—60
No. of shops	17	6

(20)

13. The average weight for a group of 25 boys was calculated to be 78·4 lbs. It was later discovered that one weight was misread as 69 lbs, instead of correct value 96 lbs. Calculate the correct mean.

(Correct $\overline{X}=79.48$)

14. The mean wage of 100 labourers working in a factory running two shifts of 60 and 40 workers respectively, is Rs. 38. The mean wage of 60 labourers working in the morning shift is Rs. 40. Find the mean wage of 40 labourers working in the evening shift.

($\overline{X}_2=$ Rs. 35).

15. The pass result of fifty students who took up a class test is given below

Marks	4	5	6	7	8	9
No. of students	8	10	9	6	4	3

If the average marks of all the fifty students was 5·16, find out the average marks of the students who failed. (2·1).

16. From the following data of age of 10 students find the median age.

Age (years) 20, 22, 18, 17, 19, 25, 30, 32, 34, 27.

$$\left(M=\frac{22+25}{2}=23\ 5 \right)$$

17. Find the value of median from data below

Income (Rs.)	1000	1500	800	2000	2500	1800
No. of Persons	24	26	16	20	6	30

(M=Rs. 1500).

18. The following table gives the marks obtained by 65 students in statistics in an exam. Calculate median.

Marks (More than)	70%,	60%,	50%,	40%,	30%,	20%
No. of students	7	18	40	40	63	65

(M=53·4 marks)

(**Hint.** find simple class intervals and arrange them in an ascending order like 20—30, 30—40 and so on.)

19. Calculate median for the data given below :

Income Rs. Below	100	100—200	200—300	300—400
No. of families	50	500	555	100

Income Rs.	400—500	500 & above
No. of families	3	2

(M=209·9).

20. The expenditure of 1000 families is given below :

Expd. (Rs.)	10—20	20—30	30—40	40—50
No. of faimilies	12	30	—	65

Expd. (Rs.)	50—60	60—70	70—80
No. of families	—	25	18

Determine the missing frequencies. It is given that median is 46 and total frequency is 229. **(33·5 ; 45·5)**

21. Determine the median of the distribution.

Class	0—9	10—19	20—29	30—39	40—49
frequency	1	4	15	11	7

Class	50—59	60—69
frequency	4	2

(M=31·32)

(Hint : Make the class intervals exclusive like 0·5—9·5, 9·5—19·5, 19·5—29·5 and so on).

22. From the following data find Q_1, Q_3, D_7, P_{82}.

Marks : Below	10	10—20	20—30	30—40	40—50
No. of Students :	22	38	46	35	20

(Q_1=14·8 ; Q_3=34·21 ; D_7=31·91 ; P_{82}=37·43).

23. The following data shows the sale of shoes of different sizes

during one hour. Find the average size of shoe sold.

Shoe number :	4	8	7	6	8	9	7	8
Sold :	6	5	8	2	10	8	4	3.

(**Mode = 8** Mode is the appropriate average).

24. Determine the modal marks from the data given below :

Marks :	5	10	15	20	25	30	35	40
No. of Students :	4	8	20	30	28	10	5	2

(**Mode = 20**)

25. Calculate the median and mode for the distribution of the weights of 150 students from the data given below :

Weight (in kg.) :	30—40	40—50	50—60	60—70
Frequency :	18	37	45	27
Weight (in kg.)	70—80	80—90		
Frequency	15	8		

(**Median = 54·44 ; Mode = 53·077**).

26. Form an ordinary frequency table from the following cumulative distribution of marks obtained by 22 students and calculate

(*i*) Arithmetic mean, (*ii*) Median and (*iii*) Mode

Marks	Below 10	Below 20	Below 30	Below 40	Below 50
No. of Students	3	8	17	20	22

($\overline{X}$ = 23·18 ; M = 23·33 ; Z = 24.]

27. Calculate mean, median and mode from the following data of the heights in inches of a group of students.
61, 62, 63, 61, 63, 64, 64, 60, 65, 63, 64, 65, 66, 64.
Now suppose that a group of students whose heights are 60, 66, 59, 68, 67, and 70 inches, is added to the original group. Find mean, median and mode of the combined group.

(First Group : **$\overline{X}$ = 63·2 ; M = 63·5 ; Z = 64.**

Combined Group : **$\overline{X}$ = 63·75 ; M = 64 ; Z = 64).**

28. In a moderately asymmetrical distribution the value of median is 42·8 and the value of mode is 40. Find the mean.

($\overline{X}$ = 44·2).

29. Arithmetic mean and median of 50 items are 100 and 95 respectively. At the time of calculations two items 180 and 90 were wrongly taken as 100 and 10. What are the correct values of mean and median ?

($\overline{X}$ = 103·2 ; Median is same i.e. 95)

30. Calculate mode from the following data

Marks :	0—10	10—20	20—40	40—50	50—70
Frequency :	5	15	40	32	28

(Z = 44).

(Hint. Make class intervals equal)

Marks :	0—10	10—20	20—30	30—40	40—50	50—60	60—70
Frequency :	5	15	20	20	32	14	14

31. Calculate geometric mean of the prices :

Commodity :	A	B	C	D	E	F
Price (Rs.) :	207	198	156	124	107	196

(G.M. = 159·8)

32. Find A.M., G.M. and H.M. of the following data and show that A.M.>G.M.>H.M. Values are 32, 35, 36, 37, 39, 41, 43.

(A.M. = 37·56 ; G.M. = 37·52 ; H.M. = 37·25).

33. Find Geometric mean of the following.

Classes : 5—10—15—20—25—30—35—40—45—50

Frequency : 7 9 7 11 13 3 4 13 1

(G.M. = 22·51).

34. The following table gives marks obtained by 70 students in statistics. Calculate arithmetic mean and geometric mean.

Marks (More than) :	70	60	50	40	30	20
No. of Students :	7	18	40	50	63	70

($\overline{X}$ = 50·43 ; G.M. = 48·09).

35. The rates of increase in population of a country during the last three decades are 10%, 20% and 30%. Find the average rate of growth.

(Using G.M., rate of growth is 19·8%).

36. A machine depreciates by 40% in the first year, by 25% in the second year and by 10% per annum for the next three years. Each percentage being calculated on diminshing value. What is the average percentage of depreciation for the entire period ?

(20%)

37. Find the harmonic mean from the data given below

2574 475 75 5 0·8 0·08 0·005 0·0009

(H.M. 0·006)

38. Calculate A.M., Median and Harmonic mean from the following data

Wages (Rs.) :	20	21	22	23	24	25
No. of workers :	8	10	11	1	20	25

($\overline{X}$ = 23·2 ; M = 24 ; H.M. = 23·084).

39. Calculate G.M. and H.M. for the following data.

Value :	0—10	10—20	20—30	30—40	40—50
Frequency :	8	12	20	6	4

(G.M.=18·65 ; H.M.=14·44).

40. A man travels 20 km. at 40 km. per hour, 10 km. at 30 km. per hour and 30 km. at 60 km. per hour. What was his average speed ?

(H M. =45 km. per hour).

41. A motor car covered a distance of 50 miles four times. The first time at 50 m.p.h., the second at 20 m.p.h., the third at 40 m.p.h. and the fourth at 25 m.p.h. Calculate the average speed.
(29·63 m.p.h.)

42. A man having to drive 90 kilometers wishes to achieve an average speed of 30 km. per hour. For the first half of the journey he averages only 22 km. per hour. What must be his average for the second half of the journey ?
(47·14 km per hour)

43. The monthly salaries (in Rs.) of 30 employees of a firm are 139, 126, 114, 100, 88, 62, 77, 99, 103, 144, 148, 63, 69, 148, 132, 118, 142, 61, 123, 104, 95, 80, 85, 1 6, 123, 133, 140, 134, 108, 129. The firm gave bonus of Rs. 10, 15, 20, 25, 30 and 35 for individuls in the respective salary groups 60—75, 75—90 and so on upto 135—150. Find the average bonus paid per employee.

($\overline{X}$=24). Hint. Put the data in a frequency distribution.

44. Find the actual class intervals from the data given below

Step Deviation	—3	—2	—1	0	1	2	3
Frequency	10	15	25	25	10	10	5

It is given that arithmetic mean is 31 and the assumed mean is 35.
(0—10, 10—20, 20—30, 30—40, 40—50, 50—60, 60—70)

45. The following table indicates the increase in cost of living for a working class family in one year. Find out the weighted average increase in the cost of living.

Group.	Food	Rent	Clothing	Fuel and Lighting	Others
% increase	29	54	97·5	75	75
Weights	7·5	2 0	1·5	1·0	0·5

($\bar{x}_w$=46 74)

46. Following is the distribution of marks in law obtained by 50 students :

Marks (More than)	0	10	20	30	40	50
No. of students	50	46	40	20	10	3

Calculate the median marks. If 60 percent of the students pass this test, find the minimum marks obtained by a pass candidate.

(M=27·5 ; Minimum marks by pass candidate=25)

47. 10 percent of the workers in a firm employing a total of 1000 workers earn less than Rs. 5 per day, 200 earn between Rs. 5 and 9·99, 30 percent between 10 and 14·99, 250 workers between 15 and 19·99 and the rest 20 and above. Prepare a frequency distribution and find the median wage.

(M=13·328)

48. A man travelled by car for 3 days. He covered 480 kms. each day. On the first day he drove for 10 hours at 48 kms an hour, on the second day he drove for 12 hours at 40 kms. an hour and on last day he drove for 15 hours at 32 kms. an hour. What was his average speed ?

(H.M=38·92 kms. per hour)

49. Show that for two numbers 16 and 4 A.M>G.M>H.M.

50. The geometric mean of 10 observations on a certain variable was calculated as 16·2. It was later discovered that one of the observations was wrongly recorded as 12.9, in fact it was 21·9. Apply appropriate correction and calculate the correct geometric mean,

(G.M=17·08)

— • —

6

Measures of Dispersion

In the last chapter we have studied about the measures of central tendency. These measures give us one single value (average) representing the entire data. However, an average does not tell us the entire story. From the knowledge of average we cannot construct the entire frequency distribution. It is also possible that different frequency distributions have the same value of average. On the basis of this we cannot conclude that the distributions are identical. Let us take the example of wages paid to five workers in three firms A, B and C

Workers.	*Wages Rs.*		
	Firm A	*Firm B*	*Firm C*
I	100	80	10
II	100	110	20
III	100	120	30
IV	100	100	40
V	100	90	400
Total	500	500	500
Average wage (Rs).	100	100	100

From the above table we find that average wage is same in all firms *i.e.* Rs. 100 but the distribution of wages is different in different firms. In firm A all the workers get same wages, while in firm B and C there is variation in individual wages. This is less in firm B and very high in

the case of firm C. Since measures of central tendency are not in a position to describe these characteristics of the data, they must be supported and supplemented by some other measures for making proper statistical analysis. One such measure is **Dispersion**.

Meaning of Dispersion. The literal meaning of dispersion is **scatteredness.** It may be defined as the extent of scatteredness of items around a measure of central tendency. In the words of **Spiegel**, "*The degree to which numerical data tend to spread about an average value is called the variation or dispersion of the data.*" Similarly, in the words of **Connor** "*Dispersion is a measure of the extent to which the individual items vary*".

Measures of dispersion are also called the "*averages of the second order*". It is because of the fact that, in their calculation, first deviations of items are taken from any measure of central tendency and then these deviations are averaged.

Properties, Characteristics or Essentials of an Ideal Measure of Dispersion. The essentials or requirements for an ideal measure of dispersion are same as those for an ideal measure of central tendency. These are

(*i*) It should be rigidly defined.
(*ii*) It should be simple to compute and understand.
(*iii*) It should be based on all the items.
(*iv*) It sould be capable of algebraic treatment.
(*v*) It should not be affected by exterme items.
(*vi*) It should not be unduly affected by sampling fluctuations.

Importance of Dispersion. Next to averages, dispersion is also widely used in statistical work. As a matter of fact the measures of dispersion support and supplement the information provided by the measures of central tendency. Following are some of the points of importance of dispersion :

(*i*) **They determine the reliability of the measures of central tendency.** Measures of dispersion tell us about the variability in the data. If variability is less, then average is reliable and if the variability in the data is more, then average is not reliable. In other words, in the latter case, it is not the true representative of data.

(*ii*) **Dispersion facilitates comparison of two or more series.** The extent of variability in the data can be used to compare two or more series and hence see the consistency, stability, uniformity or homogeneity of the data. Coefficient of variation is the appropriate measure and it is discussed at a later stage in this chapter.

(*iii*) **Helpful in further statistical analysis.** Other tools of statistical analysis like correlation, regression, tests of significance are based on measures of dispersion.

Types of Dispersion. There are two types of dispersion.

(1) Absolute Dispersion. When dispersion is expressed in terms of the original units of given data like rupees, kgms, metres etc., then it

is called absolute dispersion. Absolute dispersion has limited use because we cannot compare two or more series expressed in different statistical units like weight and height or income and marks etc.

(2) **Relative Dispersion.** When dispersion is expressed as an abstract number such as a percentage or ratio, it is called relative dispersion. It is free from the statistical units in which the original data is expressed. This method enables us to compare the variability in two series having different statistical units. Variability in this case is also expressed by the coefficient of dispersion.

Various Measures of Dispersion. Dispersion may be measured by any of the following measures :

1. Range
2. Inter-Quartile Range
3. Quartile Deviation
4. Mean Deviation
5. Standard Deviation

RANGE

It is the simplest measure of dispersion. Range is equal to the difference between the maximum value and the minimum value in the data. In other words,

Range=Maximum value—Minimum value

The above method is an *absolute measure* of the range. The relative measure of range is given by the ratio of the difference between the maximum and minimum values to the sum of the maximum and minimum values in the data. It is called the *coefficient of range*.

$$\text{Coeff. of Range} = \frac{\text{Maximum value—Minimum value}}{\text{Maximum value+Minimum value}}$$

Example 1. Calculate range and its coefficient from the following data :

Wages. (Rs)	250	310	240	380	420	560

Sol. Minimum value is 240
Maximum value is 560

$\therefore$ Range = Max. value—Min. value
= 560—240=Rs. 320

$$\text{Coeff. of Range} = \frac{\text{Max. value—Min. value}}{\text{Max. value+Min. value}}$$

$$= \frac{560-240}{560+240} = \frac{320}{800} = 0 \cdot 4$$

Example 2. Determine the range of marks and also its coefficient from the data given below.

Marks	10—20	20—30	30—40	40—50	50—60
No. of students	5	8	12	20	10

Sol. Range = Upper limit of the highest class—Lower limit of the lowest class

$$=60-10=50 \text{ marks}$$

$$\text{Coeff. of Range} = \frac{60-10}{60+10}=\frac{50}{70}=0{\cdot}714$$

Merits and Demerits of Range.

Merits.

(*i*) It is simple to compute and easy to understand.

(*ii*) It is rigidly defined.

(*iii*) It has important applications in quality control, weather reports and studies relating to the fluctuations in the prices of shares, stocks, bullion etc.

Demerits.

(*i*) It is a rough measure of dispersion.

(*ii*) It is not based on all the observations in the data.

(*iii*) It is not amenable to algebraic treatment.

(*iv*) It is affected by sampling fluctuations

(*v*) Any change in extreme items changes its value.

INTER-QUARTILE RANGE

The difference of upper and lower quartile is called inter-quartile range. In other words,

Inter-Quartile Range $=Q_3-Q_1$, where Q_3 is upper quartile and Q_1 is lower quartile

$$\text{Coefficient of Inter-Quartile Range} = \frac{Q_3-Q_1}{Q_3+Q_1}$$

Example 3. Calculate inter-quartile range and its coefficient from the following data :

Marks	10	15	20	25	30	35	40
No. of students.	5	8	12	20	10	5	2

Sol.

Marks.	No. of Students (f)	$C.f.$
10	5	5
15	8	13
20	12	25
25	20	45
30	10	55
35	5	60
40	2	62
	N=62	

$Q_1=\text{Size of } \frac{N}{4} \text{ th item}$

$=\text{size of } \frac{62}{4} \text{th item}$

$=\text{size of } 15{\cdot}5\text{th item.}$

It lies in $C.f$ 25

$\therefore \quad Q_1=20$

similarly,

$Q_3= \text{size of} \frac{3(N)}{4} \text{ th item}$

$= \text{size of } \frac{3\times 62}{4} \text{th item}$

$= \text{size of } 46{\cdot}5 \text{ th item}$

46·5th item lies in $C.f.$ 55.

$\therefore\ Q_3 = 30$

Now Inter-quartile range$= Q_3 - Q_1$

$= 30 - 20 = 10$ marks.

Coeff. of Inter-quartile range$= \dfrac{Q_3 - Q_1}{Q_3 + Q_1} = \dfrac{30-20}{30+20} = \dfrac{10}{50} = 0{\cdot}2.$

QUARTILE DEVIATION

If we divide the inter-quartile range by two we get a measure of dispersion known as quartile deviation (Q. D.). In other words,

$$\text{Q. D.} = \frac{Q_3 - Q_1}{2}$$

$$\text{The coefficient of Q. D} = \frac{\dfrac{Q_3 - Q_1}{2}}{\dfrac{Q_3 + Q_1}{2}} = \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

Since range has the limitation that it is based on only extreme items, the quartile deviation avoids this limitation and covers the middle 50% of the data.

Example 4. Calculate quartile deviation and its coefficient for the data given in Example 3.

Sol. We have $Q_1 = 20$ and $Q_3 = 30$.

$$\therefore\ \text{Q.D} = \frac{Q_3 - Q_1}{2} = \frac{30-20}{2} = \frac{10}{2} = 5.$$

$$\text{Coeff. of Q. D} = \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{30-20}{30+20} = 0{\cdot}2$$

Merits and Demerits of Quartile Deviation

Merits :

(*i*) It is easy to calculate and simple to understand.
(*ii*) Its calculation does not require any mathematical difficulties.
(*iii*) Its value is not affected by the presence of extreme items.
(*iv*) It can be easily calculated in the case of open end classes.

Demerits :

(*i*) It is not based on all items of the data. It does not provide any information beyond lower and upper quartiles as it covers only middle 50% of the data.

(*ii*) It is not capable of algebraic treatment.

(*iii*) It is largely affected by sampling fluctuations.

(*iv*) It is only a rough measure of dispersion and cannot be used for accurate results.

MEAN DEVIATION

It is a mathematical measure of dispersion and is based on all the items in the data. It is the arithmetic mean of the deviations taken from any measure of central tendency, say mean, median or mode when signs of the deviations are ignored. The reason for ignoring signs is that the sum of the deviations taken from mean is zero (*i.e.* positive and negative signs cancel) and when the deviations are taken from median or mode, they tend to zero. Mean deviation is also called Average Deviation. In practical problems mode is rarely used because it is sometimes ill-defined. Since sum of deviations from median, when signs are ignored, is minimum, it is frequently used. Again since it is easier to calculate arithmetic mean, generally, for the sake of simplicity, mean is used for taking deviations.

Mean diviation is denoted by δ. If deviations are taken from mean, it is denoted by $\delta_{\bar{x}}$. When deviations are taken about median, it is denoted by δ_M and when deviations are taken from mode, then it is denoted by δ_Z.

Calculation of Mean Deviation

(a) Individual Series (Absolute Method)

(*i*) *When deviations are taken from mean* ($\bar{x}$).

$$\delta_{\bar{x}} = \frac{\Sigma \mid x - \bar{x} \mid}{n}$$

where $\Sigma \mid x - \bar{x} \mid$ indicates the sum of deviations of x from its mean ($\bar{x}$) when signs are ignored and n is called the number of items.

(*ii*) *When deviations are taken from median* (M).

$$\delta_M = \frac{\Sigma \mid x - M \mid}{n}$$

here $\Sigma \mid x - M \mid$ implies the sum of deviations taken from median by ignoring the signs.

(*iii*) *When deviations are taken from mode* (Z)

$$\delta_Z = \frac{\Sigma \mid x - Z \mid}{n}$$

here $\Sigma \mid x - Z \mid$ is the sum of deviations from mode when signs are ignored.

Since all the methods given above are absolute methods, the calculated value of mean deviation is expressed in terms of the original units of the data.

Example 5. Calculate mean deviation from (*i*) mean (*ii*) median and (*iii*) mode for the following data relating to marks of students.

Marks	25	28	32	45	53	60	65

Sol.

Marks x	$\lvert x-\bar{x}\rvert$ $\lvert x-44\rvert$	$\lvert x-M\rvert$ $\lvert x-45\rvert$
25	19	20
28	16	17
32	12	13
45	1	0
53	9	8
60	16	15
65	21	20
$\Sigma x=308$	$\Sigma\lvert x-\bar{x}\rvert=94$	$\Sigma\lvert x-M\rvert=93$

$$M=\text{ size of }\frac{N+1}{2}\text{th item}$$

$$=\text{ size of }\frac{7+1}{2}\text{th item}$$

$$=\text{ size of 4th item}$$

$$=45.$$

mode 65

$\frac{82}{n}$

$\frac{\Sigma|x-2|}{n}$

$n=7$

$$\bar{x}=\frac{\Sigma x}{n}\qquad (i)\ \ \delta\bar{x}=\frac{\Sigma\lvert x-\bar{x}\rvert}{n}\qquad (ii)\ \ \delta_M=\frac{\Sigma\lvert x-M\rvert}{n}$$

$$\bar{x}=\frac{308}{7}\qquad =\frac{94}{7}\qquad =\frac{93}{7}$$

$$\bar{x}=44\qquad =13{\cdot}43\text{ marks}\qquad =13{\cdot}3\text{ marks.}$$

$\Sigma|$

Since in the data given above mode is ill defined, we cannot find mean deviation about mode.

(b) Discrete Series (Absolute Method). In this case, deviations (ignoring signs) are first taken about mean, median or mode and then these deviations are multiplied by the corresponding frequencies. The sum of this product is divided by total frequency to obtain the value of mean deviation.

(*i*) *When deviations are taken from actual mean.*

$$\delta\bar{x}=\frac{\Sigma f\ \lvert x-\bar{x}\rvert}{\Sigma f}$$

(*ii*) *When deviations are taken from median*

$$\delta_M=\frac{\Sigma f\lvert x-M\rvert}{\Sigma f}$$

(*iii*) *When deviations are taken from mode*

$$\delta_Z=\frac{\Sigma f\lvert x-Z\rvert}{\Sigma f}$$

Example 6. Calculate mean deviation from mean, median and mode for the following data

Wages (Rs.) (x)	110	150	200	250	300	350
No. of workers (f)	4	8	10	20	15	8

Sol. (*i*) *Mean deviation from mean.*

Wages (Rs.) x	f	$f.x$	$\lvert x-\bar{x} \rvert$ $\lvert x-236{\cdot}5 \rvert$	$f.\lvert x-\bar{x} \rvert$
110	4	440	126·5	506
150	8	1200	86·5	692
200	10	2000	36·5	365
250	20	5000	13·5	270
300	15	4500	63·5	952·5
350	3	1050	113·5	340·5
	$\Sigma f=60$	$\Sigma fx=14190$		$\Sigma f\lvert x-\bar{x} \rvert=3126{\cdot}0$

$$\bar{x}=\frac{\Sigma fx}{\Sigma f}=\frac{14190}{60}=236{\cdot}5$$

$$\delta\bar{x}=\frac{\Sigma f\lvert x-\bar{x} \rvert}{\Sigma f}=\frac{3126}{60}=\text{Rs. }52{\cdot}1$$

(*ii*) *Mean deviation from Median.*

Wages (Rs.) x	f	$c.f$	$\lvert x-M \rvert$ $\lvert x-250 \rvert$	$f.\lvert x-M \rvert$
110	4	4	140	560
150	8	12	100	800
200	10	22	50	500
250	20	42	0	0
300	15	57	50	750
350	3	60	100	300
	$\Sigma f=N=60$			$\Sigma f\lvert x-M \rvert=2910$

$$M=\text{size of }\frac{N}{2}\text{th item}$$

$$=\text{size of }\frac{60}{2}\text{th item}$$

$=$size of 30th item. This item lies in $c.f$ 42, therefore, median$=250$.

$$\delta_M=\frac{\Sigma f\lvert x-M \rvert}{\Sigma f}=\frac{2910}{60}=\text{Rs. }48{\cdot}5.$$

(*iii*) *Mean deviation from mode.* By inspection we find that the value of mode is 250. Since the value of mode is equal to the value of median, mean deviation about mode is also equal to Rs. 48·5.

(*c*) **Continuous Series.** In this case, the class intervals are represented by their mid values and same formulae and procedure is used as in the case of discrete series.

Example 7. Calculate mean deviation from mean for the following data.

Marks.	0—10	10—20	20—30	30—40	40—50	50—60
No. of students	5	10	20	8	6	1

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Sol. Calculation of mean deviation from mean.

Marks	No. of students f	Mid value x	$f.x$	$\mid x-\bar{x} \mid$ $\mid x-26 \mid$	$f \mid x-\bar{x} \mid$.
0—10	5	5	25	21	105
10—20	10	15	150.	11	110
20—30	20	25	500	1	20
30—40	8	35	300	9	72
40—50	6	45	270	19	114
50—60	1	55	55	29	29
	$\Sigma f=50$		$\Sigma f x=1300$		$\Sigma f. \mid x-\bar{x} \mid =450$

$$\bar{x}=\frac{\Sigma f x}{\Sigma f}=\frac{1300}{50}=26$$

$$\delta_{\bar{x}}=\frac{\Sigma f. \mid x-\bar{x} \mid}{\Sigma f}=\frac{450}{50}=9 \text{ marks.}$$

Example 8. Calculate mean deviation from (*i*) median (*ii*) mode for the following data.

Class Interval : (Less than)	10	20	30	40	50	60	70	80
Frequency :	6	59	144	200	221	237	241	245

Solution. (*i*) *Mean deviation from median.* Let us first prepare simple frequency distribution.

Class Interval.	f	C. f.	M. v x	$\mid x-M \mid$ $\mid x-27\cdot5 \mid$	$f. \mid x-M \mid$	$\mid x-Z \mid$ $\mid x-25.2 \mid$	$f. \mid x-Z \mid$
0—10	6	6	5	22·5	135.0	20.2	121.2
10—20	53	59	15	12·5	662.5	10.2	540.6
20—30	85	144	25	2·5	212.5	0.2	17.0
30—40	56	200	35	7·5	420.0	9.8	548.0
40—50	21	221	45	17·5	367.5	19.8	415.8
50—60	16	237	55	27·5	440.0	29.8	476.8
60—70	4	241	65	37·5	150.0	39.8	159.2
70—80	4	245	75	47·5	**190.0**	49.8	**199.2**
Σf=245					**Σf | X—M | =2577·5**		**Σf | X—Z | =2478·6**

$$M = \text{Size of } \frac{N}{2}\text{th item}$$

$$= \text{Size of } \frac{245}{2}\text{th item}$$

= Size of 122.5th item. It lies in the $C.f.$ 144. Therefore, median class interval is 20—30.

Now $M = L_1 + \frac{i}{f}\left[\frac{N}{2}\text{th} - C\right] = 20 + \frac{10}{85}\left[122\cdot5 - 59\right]$

$$= 20 + \frac{635}{85} = 20 + 7\cdot5$$

$$= 27\cdot5$$

$$\delta m = \frac{\Sigma f.\,|x - M|}{\Sigma f} = \frac{2577\cdot5}{245} = 10\cdot52.$$

(*ii*) *Mean deviation from mode.* By inspection we find that the modal class interval is 20—30.

Now $Z = L_1 + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times i.$

or $Z = 20 + \frac{32}{32+29} \times 10$ $\quad\quad \because \Delta_1 = 85 - 53 = 32$

$= 20 + \frac{320}{61}$ $\quad\quad \Delta_2 = 85 - 56 = 29$

$= 20 + 5.2 = 25\cdot2$

$$\delta_z = \frac{\Sigma f\,|X - Z|}{\Sigma f} = \frac{2478\cdot6}{245} = 10\cdot11.$$

Relative Measure of Mean Deviation. So far we have discussed only an absolute measure of mean deviation. The relative measure is known as the coeff. of mean deviation and it is obtained by dividing the absolute measure of mean deviation by the particular average ($\bar{x}$, M or Z) used to compute mean deviation. In other words,

(*i*) Coeff. of mean deviation about mean $= \frac{\delta \bar{x}}{\bar{x}}$

(*ii*) Coeff. of mean deviation about median $= \frac{\delta m}{M}$.

(*iii*) Coeff. of mean deviation about mode $= \frac{\delta z}{Z}$.

Example 9. Calculate the coeff. of mean deviation from mean and median for data given in example 6.

Sol. We have $\bar{x} = 236\cdot5$, $\quad M = 250$

$\delta \bar{x} = 52\cdot1 \quad \delta_M = 48\cdot5$

Coeff. of mean deviation from mean$=\frac{\delta \bar{x}}{\bar{x}}=\frac{52 \cdot 1}{236 \cdot 5}$

$=0 \cdot 22.$

Coeff. of mean deviation from median$=\frac{\delta m}{M}=\frac{48 \cdot 5}{250}$

$=0 \cdot 194.$

Merits and Demerits of Mean Deviation.

Merits.

(*i*) It is based on all the items in the data.

(*ii*) It is clear and easy to understand because it is the average of variation in the data from any measure of central tendency.

(*iii*) As compared to standard deviation, it is less affected by extreme items.

(*iv*) In a normal distribution 57·5 per cent of items fall in the range of $\bar{x} \pm \delta \bar{x}$ or $M \pm \delta_M$.

Demerits :

(*i*) It is mathematically illogical because while calculating mean deviation the signs are ignored.

(*ii*) It is not capable of algebraic treatment because we cannot compute combined mean deviation of two or more series.

(*iii*) If mean, median or mode is a fractional value, then the calculation of mean deviation becomes difficult.

STANDARD DEVIATION

It is the most important, widely used and considered to be the best measure of dispersion. The concept of standard deviation was given by Karl Pearson in 1893. The main limitation of mean deviation is that it ignores the algebraic signs in its calculation. Standard deviation removes this limitation by squaring the deviations and hence making signs of all the deviations positive. It also possesses many useful properties. In the case of standard deviation, the deviations are taken only from mean. This is due to the fact that the sum of the squares of the deviations from mean is always minimum. The standard deviation, denoted by Greek letter σ, is defined as **the square root of the arithmetic mean of the squares of the deviations taken from mean.** In short it is defined as the "Root-Mean-Square Deviation from Mean."

Calculation of Standard Deviation.

(*a*) **Individual Series.**

(*i*) *When deviations are taken from actual mean.* Let x be any variable and $\bar{x}$ be the mean of x. The standard deviation is given by

$$\sigma=\sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$$

where $\Sigma(x-\bar{x})^2$ is the sum of squares of deviations from mean and n is the number of itmes.

(*ii*) *When deviations are taken from assumed mean.* If actual mean is in fractions, then it becomes difficult to apply the above method. In this case deviations are taken from any assumed mean (A) and following formula is used

$$\sigma=\sqrt{\frac{\Sigma dx^2}{n}-\left(\frac{\Sigma dx}{n}\right)^2}$$

where $dx=X-A$.

Σdx=sum of deviations from A.

Σdx^2 is the sum of squares of deviations from A. n is the number of items.

(*iii*) *When no deviations are taken* (*i.e.* A=0). If the given items in the data are so small that they can be easily squared then the standard deviation is calculated by using actual data and applying the following formula :

$$\sigma=\sqrt{\frac{\Sigma x^2}{n}-\left(\frac{\Sigma x}{n}\right)^2}$$

$$\text{or}\quad \sigma=\sqrt{\frac{\Sigma x^2}{n}-(\bar{x})^2} \qquad \Big| \because \frac{\Sigma x}{n}=\bar{x}$$

Note. The value of standard deviation will be the same for all the methods.

Example 10. Calculate standard deviation for the following data.

x	5	10	15	20	25

Sol. Let us solve the numerical by using all the formulae and see their application.

(*i*) *When deviations are taken from actual mean.*

x	$x-\bar{x}$ $x-15$	$(x-\bar{x})^2$
5	−10	100
10	− 5	25
15	0	0
20	5	25
25	10	100
$\Sigma x=75$		$\Sigma(x-\bar{x})^2$ =250

$$\sigma=\sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}=\sqrt{\frac{250}{5}}=\sqrt{50}=7\cdot07$$

$$\bar{x}=\frac{\Sigma x}{n}=\frac{75}{5}=15.$$

(*ii*) *When deviations are taken from assumed mean.*
Let us take 12 as the assumed mean

x	$dx = X - A$ $= x - 12$	dx^2
5	−7	49
10	−2	4
15	3	9
20	8	64
25	13	169
	$\Sigma dx = 15$	$\Sigma dx^2 = 295$

$$\sigma = \sqrt{\frac{\Sigma dx^2}{n} - \left(\frac{\Sigma dx}{n}\right)^2}$$
$$= \sqrt{\frac{295}{5} - \left(\frac{15}{5}\right)^2}$$
$$= \sqrt{59 - 9} = \sqrt{50}$$
$$= 7{\cdot}07$$

(*iii*) *When no deviations are taken.* Since the observations are small and can be easily squared, we can use this method also.

x	x^2
5	25
10	100
15	225
20	400
25	625
$\Sigma x = 75$	$\Sigma x^2 = 1375$

$$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$$
$$= \sqrt{\frac{1375}{5} - \left(\frac{75}{5}\right)^2}$$
$$= \sqrt{275 - 225} = \sqrt{50}$$
$$= 7{\cdot}07$$

Hence we find that the calculated value of standard deviation is same for all the methods. So we can apply any one of them depending upon our convenience.

(*b*) **Discrete Series**

(*i*) *When deviations are taken from actual mean.*

$\sigma = \sqrt{\dfrac{\Sigma f(x - \bar{x})^2}{\Sigma f}}$, where $\Sigma f(x - \bar{x})^2$ is the sum of the product of squares of deviations from mean and the corresponding frequencies. Σf is the total frequency.

(*ii*) *When deviations are taken from assumed mean.* If actual mean is in fractions, then above method is very difficult to apply. So we use

$$\sigma = \sqrt{\frac{\Sigma f dx^2}{\Sigma f} - \left(\frac{\Sigma f dx}{\Sigma f}\right)^2}$$

where $\Sigma f dx^2$ is the sum of product of the squares of deviations from assumed mean and the frequency ; $\Sigma f dx$ is sum of product of deviations from assumed mean and frequency.

(*iii*) *When no deviations are taken.* If given observations are small which can be easily squared then we use

$$\sigma = \sqrt{\frac{\Sigma f x^2}{\Sigma f} - \left(\frac{\Sigma f x}{\Sigma f}\right)^2}$$

Example 11. Calculate standard deviation from the following data :

x	9	8	7	6	5	4	2
f	1	2	1	2	2	1	1

Solution. *When deviations are taken from mean.*

x	f	$f.x$	$x-\bar{x}$ $(x-6)$	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
9	1	9	3	9	9
8	2	16	2	4	8
7	1	7	1	1	1
6	2	12	0	0	0
5	2	10	−1	1	2
4	1	4	−2	4	4
2	1	2	−4	16	16
	$\Sigma f=10$	$\Sigma fx=60$			$\Sigma f(x-\bar{x})^2=40$

$$\bar{x}=\frac{\Sigma fx}{\Sigma f}=\frac{60}{10}=6$$

$$\sigma=\sqrt{\frac{\Sigma f(x-\bar{x})^2}{\Sigma f}}=\sqrt{\frac{40}{10}}=\sqrt{4}=2.$$

Example 12. Calculate standard deviation of wages from the following data taking deviations from assumed mean Rs. 140. Also find arithmetic mean.

Wages (Rs.)	120	125	130	140	145	150	160
No. of workers.	10	15	12	20	30	9	4

Solution. Given A=140

Wages (Rs.) x	*No. of workers* f	*A=140* $dx=x-A$ $=x-140$	fdx	dx^2	fdx^2
120	10	−20	−200	400	4000
125	15	−15	−225	225	3375
130	12	−10	−120	100	1200
140	20	0	0	0	0
145	30	5	150	25	750
150	9	10	90	100	900
160	4	20	80	400	1600
	$\Sigma f=100$		$\Sigma fdx=-225$		$\Sigma fdx^2=11825$

$$\sigma=\sqrt{\frac{\Sigma fdx^2}{\Sigma f}-\left(\frac{\Sigma fdx}{\Sigma f}\right)^2}$$

$$=\sqrt{\frac{11825}{100}-\left(\frac{-225}{100}\right)^2}=\sqrt{118\cdot25-5\cdot06}$$

$$=\sqrt{113\cdot19} \qquad =10\cdot64$$

$$\bar{x}=A+\frac{\Sigma fdx}{\Sigma f}=140+\frac{-225}{100}$$

$$\bar{x}=140-2\cdot25=137\cdot75$$

Example 13. Without taking any deviations calculate standard deviation.

x	10	12	13	14	15
f	5	10	20	30	5

Solution.

x	f	$f.x$	x^2	$f.x^2$
10	5	50	100	500
12	10	120	144	1440
13	20	260	169	3380
14	30	420	196	5880
15	5	75	225	1125
	$\Sigma f=70$	$\Sigma fx=925$		$\Sigma fx^2=12325$

$$\sigma=\sqrt{\frac{\Sigma fx^2}{\Sigma f}-\left(\frac{\Sigma fx}{\Sigma f}\right)^2}=\sqrt{\frac{12325}{70}-\left(\frac{925}{70}\right)^2}$$

$$=\sqrt{176\cdot07-174\cdot61}=\sqrt{1\cdot46}$$

$$\sigma=1\cdot2$$

(c) **Continuous Series.** The method of calculating standard deviation in continuous series is exactly same as in the case of discrete series. Here the class intervals are represented by their mid values and same formulae as discussed in discrete series can be used. In practice we generally use **Step Deviation Method.** In this method the deviations taken from assumed mean are diyided by any common factor C and the following formula is applied :

$$\sigma=\sqrt{\frac{\Sigma fd'x^2}{\Sigma f}-\left(\frac{\Sigma fd'x}{\Sigma f}\right)^2}\times C$$

where $d'x=\frac{dx}{C}=\frac{X-A}{C}$, A is the assumed mean and C is common factor.

Example 14. Calculate mean and standard deviation of the distribution of ages of 50 persons.

Age (years)	10—20	20—30	30—40	40—50	50—60	60—70
No. of Persons.	5	8	10	12	8	7

Sol. Using step deviation method.

Age (*years*)	*No. of Persons* (f)	*Mid Value* x	$A=35$ $dx=x-A$	$C=10$ $d'x=\frac{dx}{c}$	$f.d'x$	$d'x^2$	$fd'x^2$
10—20	5	15	—20	—2	—10	4	20
20—30	8	25	—10	—1	— 8	1	8
30—40	10	35	0	0	0	0	0
40—50	12	45	10	1	12	1	12
50—60	8	55	20	2	16	4	32
60—70	7	65	30	3	21	9	63
	$\Sigma f=50$				$\Sigma fd'x=31$		$\Sigma fd'x^2=135$

$$\bar{x}=A+\frac{\Sigma fd'x}{\Sigma f}\times C$$

$$=35+\frac{31}{50}\times 10=35+6\cdot 2=41\cdot 2$$

$$\sigma=\sqrt{\frac{\Sigma fd'x^2}{\Sigma f}-\left(\frac{\Sigma fd'x}{\Sigma f}\right)^2}\times C$$

$$=\sqrt{\frac{135}{50}-\left(\frac{31}{50}\right)^2}\times 10$$

$$=\sqrt{2\cdot 7-0\cdot 3844}\times 10$$

$$=\sqrt{2\cdot 3156}\times 10=1\cdot 5217\times 10$$

$$\sigma=15\cdot 217$$

1080

18 × 60

1080

Example 15. The following data relates to temperature on different days in a year. Calculate standard deviation.

Temperature °C	No. of days	Temperature °C	No. of days
—40 to —30	10	0—10	65
—30 to —20	28	10—20	180
—20 to —10	30	20—30	10
—10 to 0	42		

Sol.

Temp °C	*No. of days* f	*Mid Value* x	$A=(-5)$ $dx=x-A$ $=x-(-5)$	$C=10$ $d'x$	$d'x^2$	$fd'x$	$fd'x^2$
—40 to -30	10	—35	—30	—3	9	—30	90
—30 to -20	28	—25	—20	—2	4	—56	112
—20 to -10	30	—15	—10	—1	1	—30	30
—10 to 0	42	—5	0	0	0	0	0
0 to 10	65	5	10	1	1	65	65
10 to 20	180	15	20	2	4	360	720
20 to 30	10	25	30	3	9	30	90
	$\Sigma f=365$					$\Sigma fd'x=339$	$\Sigma fd'x^2$ $=1107$

$$\sigma=\sqrt{\frac{\Sigma fd'x^2}{\Sigma f}-\left(\frac{\Sigma fd'x}{\Sigma f}\right)^2}\times C$$

$$=\sqrt{\frac{1107}{365}-\left(\frac{339}{365}\right)^2}\times 10$$

$$=\sqrt{3\cdot 0328-0\cdot 8626}\times 10$$

$$=\sqrt{2\cdot 1702}\times 10=1\cdot 473\times 10.$$

$$=14\cdot 73$$

Example 16. The mean of 200 items is 48 and the sum of squares is 4,62,600. Find standard deviation.

Sol. We know that $\sigma=\sqrt{\frac{\Sigma x^2}{n}-(\overline{x})^2}$

Given $\Sigma x^2=4,62,600,$ $\overline{x}=48,$ $n=200.$

Putting these values in the formula

$$\sigma=\sqrt{\frac{4,62,600}{200}-(48)^2}=\sqrt{2313-2304}$$

$$\sigma=\sqrt{9}=3$$

Coefficient of Standard Deviation. It is a relative measure of dispersion and can be used for comparing the standard deviation of two or more series expressed in different units.

Coeff. of standard deviation $=\frac{\sigma}{\overline{x}}$.

Example 17. Calculate coeff. of standard deviation from values given in example 16.

Sol. Given $\overline{x}=48$ and $\sigma=3$. We know that coeff. of $\sigma=\frac{\sigma}{\overline{x}}=\frac{3}{48}$

or Coeff. of $\sigma=0 \cdot 0625$.

Coefficient of Variation. It is defined as the standard deviation expressed as the percentage of the arithmetic mean. It is generally denoted by C.V. In other words,

$\text{C.V} = \frac{\sigma}{\bar{x}} \times 100$. It is always expressed in percentages. Coefficient of variation is very useful for measuring consistency, stability, uniformity or homogeneity of the data. Any series with larger value of C.V. is said to be less uniform, less stable, less consistent or heterogeneous. Similarly, a series with lower value of C.V. is called more consistent, uniform, stable and homogeneous.

Example 18. Total students in a class are divided into two groups. In the first group the average score is 45 and standard deviation is 10. In the second group the corresponding values are 56 and 15. Find which group is more consistent ?

Sol. The group having lower value of C.V. will be called more consistent.

Given $\bar{x}_1=45$; $\bar{x}_2=56$

$\sigma_1=10$; $\sigma_2=15$

Now coeff. of variation for first group

$$\text{C.V}_{\text{I}} = \frac{\sigma_1}{\bar{x}_1} \times 100 = \frac{10}{45} \times 100 = 22 \cdot 22\%$$

Coeff. of variation for second group

$$\text{C.V.}_{\text{II}} = \frac{\sigma_2}{\bar{x}_2} \times 100 = \frac{15}{56} \times 100 = 26 \cdot 78\%$$

Since C.V. is less for group I it is said to be more consistent.

Example 19. Comment on the statement "After settlement the average weekly wages in a factory had increased from Rs. 8 to Rs. 12 and the standard deviation had increased from Rs. 2 to Rs. 2·5. After the settlement the wage has become high and more uniform.

Sol. Given Before settlement $\bar{x}=8$, $\sigma=2$

After settlement $\bar{x}=12$, $\sigma=2.5$

Since after settlement there is an increase in average wage we can say that wages have become higher. To decide whether wages have become uniform or not let us compare the coefficients of variation before and after settlement.

C.V. before settlement

$$\text{C.V.} = \frac{\sigma}{\bar{x}} \times 100$$
$$= \frac{2}{8} \times 100$$
$$= 25\%$$

C.V. after settlement

$$\text{C.V.} = \frac{2 \cdot 5}{12} \times 100$$
$$= \frac{25}{12} \times 100 \times \frac{1}{10}$$
$$= 20 \cdot 83\%$$

So after settlement C.V. has decreased. We can say that wages have become more uniform.

Example 20. The scores of two batsmen in six innings are given. Find which batsman is more consistent.

Batsman A :	Runs Scored	12	15	8	10	20
Batsman B :	Runs Scored	11	10	12	13	12

Sol.

Batsman A x (*Runs Scored*)	x^2	*Batsman B* x (*Runs Scored*)	x^2
12	144	11	121
15	225	10	100
8	64	12	144
10	100	13	169
20	400	12	144
$\Sigma x=65$	$\Sigma x^2=933$	$\Sigma x=58$	$\Sigma x^2=678$

Batsman A

$n=5$

$\bar{x}=13$

$$\sigma=\sqrt{\frac{\Sigma x^2}{n}-\left(\frac{\Sigma x}{n}\right)^2}$$

$$=\sqrt{\frac{933}{5}-\left(\frac{65}{5}\right)^2}$$

$$=\sqrt{186.6-169}$$

$$=4{\cdot}195$$

$$\text{C.V.}_A=\frac{\sigma}{\bar{x}}\times 100$$

$$=\frac{4{\cdot}95}{13}\times 100$$

$$=32{\cdot}26\%$$

Batsman B

$n=5$

$\bar{x}=11{\cdot}6$

$$\sigma=\sqrt{\frac{\Sigma x^2}{n}-\left(\frac{\Sigma x}{n}\right)^2}$$

$$=\sqrt{\frac{678}{5}-\left(\frac{58}{5}\right)^2}$$

$$=\sqrt{135{\cdot}6-134{\cdot}56}$$

$$=\sqrt{1{\cdot}04}$$

$$=1{\cdot}02$$

$$\text{C.V.}=\frac{\sigma}{\bar{x}}\times 100$$

$$=\frac{1{\cdot}02}{11{\cdot}6}\times 100$$

$$=8{\cdot}79\%$$

Since coeff. of variation is less for batsman B, he is said be more consistent.

Variance. It is also a measure of variability. Its value can be obtained by squaring the value of standard deviation.. In other words,

$$\text{Variance}=\sigma^2.$$

or $\sigma=\sqrt{\text{Variance}}$

e.g., Variance of data in example 15 is $(3)^2=9$

Combined Standard Deviation. Just as we can find the combined mean of two or more groups, similarly the combined standard deviation can be calculated by using the following formula.

$$\sigma_{12}=\sqrt{\frac{n_1(\sigma_1^2+d_1^2)+n_2(\sigma_2^2+d_2^2)}{n_1+n_2}}$$

Where n_1=number of items in first group.

n_2=number of items in second group.

σ_1=standard deviation of first group.

σ_2=standard deviation of second group.

$d_1=\bar{x}_1-\bar{x}_{12}$ and $d_2=\bar{x}_2-\bar{x}_{12}$

where $\bar{x}_{12}=\frac{n_1\bar{x}_1+n_2\bar{x}_2}{n_1+n_2}$ is the combined mean.

Example 21. For a group of 60 male workers, the mean and standard deviation of their weekly wages are Rs. 63 and Rs. 9 respectively. For a group of 40 female workers these are Rs. 54 and Rs. 6 respectively. Find the standard deviation of the combined group of 100 workers.

Sol. Let us represent male workers by x_1 and female workers by x_2.

Given $\bar{x}_1$=Rs. 63 ; $\bar{x}_2$=Rs. 54

σ_1=Rs. 9 ; σ_2=Rs. 6

$n_1=60$; $n_2=40$

Now
$$\sigma_{12}=\sqrt{\frac{n_1(\sigma_1^2+d_1^2)+n_2(\sigma_2^2+d_2^2)}{n_1+n_2}}$$

$$\bar{x}_{12}=\frac{n_1\bar{x}_1+n_2\bar{x}_2}{n_1+n_2}=\frac{60\times63+40\times54}{60+40}$$

$$=\frac{3780+2160}{100}=\frac{5940}{100}=59\cdot4$$

Now
$$d_1=\bar{x}_1-\bar{x}_{12}=63-59\cdot4=3\cdot6$$
$$d_2=\bar{x}_2-\bar{x}_{12}=54-59\cdot4=-5\cdot4$$

Putting these values in the formula, we have

$$\sigma_{12}=\sqrt{\frac{60[(9)^2+(3.6)^2]+40[(6)^2+(-5\cdot4)^2]}{60+40}}$$

$$=\sqrt{\frac{60(81+12\cdot96)+40(36+29\cdot16)}{100}}$$

$$=\sqrt{\frac{60\times93\cdot96+40\times65\cdot16}{100}}=\sqrt{\frac{5637\cdot6+2606\cdot4}{100}}$$

$$\sigma_{12}=\sqrt{\frac{8244}{100}}=\sqrt{82\cdot44}=9\cdot08$$

Example 22. The combined mean and standard deviation of a group of 75 items is 26 and 6. If one sub-group consists of 30 items

with mean equal to 20 and variance equal to 5, what will be the mean, standard deviation and coefficient of variation of the other sub-group ?

Solution.

Given $\bar{X}_{12}=26, \quad \sigma_{12}=6$

and $N_1+N_2=75$

$N_1=30 \qquad \bar{X}_1=20, \sigma_1^2=5$

$\therefore \qquad N_2=75-30=45.$

Calculation of $\bar{X}_2$

We know that $\bar{X}_{12}=\dfrac{N_1\bar{X}_1+N_2\bar{X}_2}{N_1+N_2}$

or $$26=\frac{(30)(20)+(45)(\bar{X}_2)}{75}$$

or $1950=600+45\bar{X}_2$

or $1350=45\bar{X}_2$ or $\bar{X}_2=30$

Calculation of σ_2

We know that

$$\sigma_{12}=\sqrt{\frac{N_1(\sigma_1^2+d_1^2)+N_2(\sigma_2^2+d_2^2)}{N_1+N_2}}$$

where $d_1^2=(\overline{x_1}=\overline{x_{12}})^2=(20-26)^2=36$

and $d_2^2=(\overline{x_2}-\overline{x_{12}})^2=(30-26)^2=16$

Putting the values in the formula, we have

$$6=\sqrt{\frac{30(5+36)+45(\sigma_2^2+16)}{75}}$$

Squaring on both sides, we get

$$36=\frac{30(41)+45(\sigma_2^2+16)}{75}$$

or $2700=1230+45\sigma_2^2+720$

or $2700=1950+45\sigma_2^2$

or $45\sigma_2^2=750$

or $$\sigma_2^2=\frac{750}{45}=16{\cdot}667$$

or $\sigma_2^2=\sqrt{16{\cdot}667}=4{\cdot}08.$

Calculation of coefficient of variation of second sub-group.

$$\text{C.V. II}=\frac{\sigma_2}{\bar{X}_2}\times 100 \quad =\frac{4{\cdot}08}{30}\times 100=\frac{408}{30}=13{\cdot}6\%.$$

Correction of Incorrect Standard Deviation. Just as we can correct

an incorrect value of mean, in the same way we can correct an incorrect value of standard deviation by taking the following steps :

(*i*) Find **correct** Σx=Incorrect Σx—wrong value + correct value.

(*ii*) Find **correct n** =Original n—wrong number of items+correct number of items.

(*iii*) Find correct Σx^2=Incorrect Σx^2—(wrong values)^2+(correct values)^2

Where **Incorrect** $\Sigma x^2 = n[(\bar{x})^2+\sigma^2]$.

(*iv*) Apply the formula $\sigma=\sqrt{\frac{\Sigma x^2}{n}-\left(\frac{\Sigma x}{n}\right)^2}$ to find the correct value of standard deviation.

Example 23. The arithmetic mean and standard deviation of 20 observations were calculated by a student as 20 and 5 respectively. But while calculating them an item 13 was misread as 30. Find correct mean and standard deviation.

Sol. Given $n=20$, $\bar{x}=20$, $\sigma=5$

Wrong value=30, Correct value=13.

Incorrect $\Sigma x = n.\ \bar{x} = 20\times 20 = 400$

Correct $\Sigma x = 400-30+13 = 383$

Correct $n = 20-1+1 = 20$

Incorrect $\Sigma x^2 = n[(\bar{x})^2+\sigma^2] = 20[(20)^2+(5)^2]$

$= 20[400+25] = 8500$

Correct $\Sigma x^2 = 8500-(30)^2+(13)^2$

$= 8500-900+169 = 7769$

$\therefore$ Correct $\sigma=\sqrt{\frac{\Sigma x^2}{n}-\left(\frac{\Sigma x}{n}\right)^2}=\sqrt{\frac{7769}{20}-\left(\frac{383}{20}\right)^2}$

$=\sqrt{388{\cdot}45-366{\cdot}72}=\sqrt{21{\cdot}73}$

$=4{\cdot}66$

Correct $\bar{x}=\frac{\text{correct } \Sigma x}{\text{correct } n}=\frac{383}{20}=19{\cdot}15$

Interpretation of Total, Mean and Standard Deviation.

Example 24. An analysis of monthly wages paid to workers in two firms A and B gives following data.

	Firm A	Firm B
No. of workers	500	600
Average monthly wage	Rs. 186	Rs. 175
Variance	Rs. 81	Rs. 100

(*i*) Which firm has large wage bill ?
(*ii*) Calculate combined mean.
(*iii*) Calculate combined standard deviation.
(*iv*) In which firm the distribution of wages is consistent ?

Sol. Let us represent wages of firm A by x_1 and wages of firm B by x_2.

Given $n_1=500,\quad n_2=600,\quad \bar{x}_1=186,\quad \bar{x}_2=175$

$\sigma_1^2=81$ or $\sigma_1=\sqrt{81}=9$

$\sigma_2^2=100$ or $\sigma_2=\sqrt{100}=10.$

(*i*) Total wage bill of firm A *i.e.*, $\Sigma x_1=n_1.\ \bar{x}_1$

or $\Sigma x_1=500\times186=$Rs. 93,000.

Total wage bill of firm B *i.e.*, $\Sigma x_2=n_2\ .\ \bar{x}_2$

or $\Sigma x_2=600\times175=$Rs. 1, 05, 000.

Therefore, firm B has larger wage bill.

(*ii*) Combined Mean $\bar{x}_{12}=\dfrac{n_1\ \bar{x}_1+n_2\ \bar{x}_2}{n_1+n_2}$

or $$\bar{x}_{12}=\frac{500\times186+600\times175}{500+600}=\frac{93,000+1,05,000}{1100}$$

$$=\frac{1,98,000}{1100}=\text{Rs. }180.$$

(*iii*) Combined Standard Deviation

$$\sigma_{12}=\sqrt{\frac{n_1(\sigma_1^2+d_1^2)+n_2(\sigma_2^2+d_2^2)}{n_1+n_2}}$$

Now, $d_1^2=(\bar{x}_1-\bar{x}_{12})^2=(186-180)^2=36$

$d_2^2=(\bar{x}_2-\bar{x}_{12})^2=(175-180)^2=25$

$$\therefore\quad \sigma_{12}=\sqrt{\frac{500(81+36)+600(100+25)}{500+600}}$$

$$=\sqrt{\frac{500(117)+600(125)}{1100}}=\sqrt{\frac{58500+75000}{1100}}$$

$$=\sqrt{\frac{133500}{1100}}=\sqrt{121\cdot36}=11\cdot02$$

(*iv*) To find consistency in the distribution of wages we calculate coefficients of variation.

$$\text{C.V}_1=\frac{\sigma_1}{\bar{x}_1}\times100=\frac{9}{186}\times100=4\cdot84\%$$

$$\text{C.V}_2=\frac{\sigma_2}{\bar{x}_2}\times100=\frac{10}{175}\times100=5\cdot71\%$$

Since the coefficient of variation is less in the case of Firm A, its distribution of wages is said to be more consistent.

Mathematical Properties of Standard Deviation

Standard deviation is considered to be the most useful measure of dispersion because of the presence of some important properties which are given as under :

1. *Like the arithmetic mean, standard deviation also lends itself to algebraic treatment.* It implies that we can find out the standard deviation of series from the standard deviations of its component parts and their means. Let us suppose that N_1, N_2, σ_1, σ_2 and $\overline{X}_1$, $\overline{X}_2$ represent the number of observations, standard deviations and means of two groups respectively. The combined standard deviation σ_{12} is given by :

$$\sigma_{12}=\sqrt{\frac{N_1(\sigma_1^2+d_1^2)+N_2(\sigma_2^2+d_2^2)}{N_1+N_2}},$$

where $d_1=\overline{X}_1-\overline{X}_{12}$, $d_2=\overline{X}_2-\overline{X}_{12}$

If the number of observations and means of the two component parts are equal, then

$$\sigma_{12}=\sqrt{\frac{\sigma_1^2+\sigma_2^2}{2}}.$$

The idea can be extended to find the combined standard deviation of n component parts

2. *The standard deviation of first 'n' natural numbers 1, 2, 3, 4,...n is given by*

$$\sigma=\sqrt{\frac{n^2-1}{12}}.$$

Proof. We know that $\sigma=\sqrt{\frac{\Sigma n^2}{n}-\left(\frac{\Sigma n}{n}\right)^2}$

But $\Sigma n^2=\frac{n(n+1)(2n+1)}{6}$, and $\Sigma n=\frac{n(n+1)}{2}$

$$\therefore \quad \sigma=\sqrt{\frac{n(n+1)(2n+1)}{6n}-\left(\frac{n(n+1)}{2n}\right)^2}$$

$$=\sqrt{\frac{2n^2+3n+1}{6}-\frac{n^2+2n+1}{4}}=\sqrt{\frac{n^2-1}{12}}$$

e.g. standard deviation of first five natural numbers 1, 2, 3,4, 5 is given by

$$\sigma=\sqrt{\frac{5^2-1}{12}}=\sqrt{\frac{24}{12}}=\sqrt{2}=1{\cdot}414$$

3. The variance and consequently standard deviation of a distribution is independent of change of origin.

i.e. **Whatever be the value of the assumed mean, the value of standard deviation remains the same.**

Proof :—We know that

$$\sigma_x^2=\frac{\Sigma(X-\overline{X})^2}{N}=\frac{\Sigma(X^2+\overline{X}^2-2X\overline{X})}{N}$$

$$=\frac{\Sigma X^2}{N}+\frac{N\overline{X}^2}{N}-2\overline{X}.\frac{\Sigma X}{N}=\frac{\Sigma X^2}{N}+\overline{X}^2-2\overline{X}^2$$

$$\sigma_x^2=\frac{\Sigma X^2}{N}-(\overline{X})^2,\text{ or }\sigma_x^2=\frac{\Sigma X^2}{N}-\left(\frac{\Sigma X}{N}\right)^2$$

Now let us take deviations from assumed mean A and let $d=X-A$.

$$\therefore\quad \sigma_x^2=\frac{\Sigma[(X-A)-\text{Mean of }(X-A)]^2}{N}$$

$$=\frac{\Sigma[(X-A)-(\overline{X}-A)]^2}{N}$$

$$=\frac{\Sigma[(X-A)^2+(\overline{X}-A)^2-2(X-A)(\overline{X}-A)]}{N}$$

$$\text{or}\quad \sigma_x^2=\frac{\Sigma(X-A)^2}{N}+\frac{N(\overline{X}-A)^2}{N}-2\frac{(\overline{X}-A).\Sigma(X-A)}{N}$$

$$=\frac{\Sigma d^2}{N}+\left(\frac{\Sigma d}{N}\right)^2-2\left(\frac{\Sigma d}{N}\right)\left(\frac{\Sigma d}{N}\right)\qquad \because\ \overline{X}=A+\frac{\Sigma d}{N}$$

$$\sigma_x^2=\frac{\Sigma d^2}{N}-\left(\frac{\Sigma d}{N}\right)^2=\sigma_d^2\qquad \text{or }\ \overline{X}-A=\frac{\Sigma d}{N}$$

Hence we get $\sigma_x^2=\sigma_d^2$

which shows that variance of X is same as the variance of d. In other words, standard deviation is independent of origin.

4. Variance or standard deviation is not independent of the change of scale.

Let us take $d=\frac{X-A}{h}$ where h is any common factor

$$\text{or}\quad hd=X-A.$$

$$\therefore\quad \sigma_x^2=\frac{\Sigma[(X-A)-(\overline{X}-A)]^2}{N}$$

$$=\frac{\Sigma[(hd)-(\overline{X}-A)]^2}{N}=\frac{\Sigma[h^2d^2+(\overline{X}-A)^2-2hd(\overline{X}-A)]}{N}$$

$$=\frac{\Sigma h^2d^2}{N}+\frac{N(\overline{X}-A)^2}{N}-\frac{2(\overline{X}-A)\Sigma hd}{N}$$

$$=\frac{h^2\Sigma d^2}{N}+\left(\frac{h.\Sigma d}{N}\right)^2-\frac{2h.\Sigma d}{N}.\frac{h.\Sigma d}{N} \qquad \because \overline{X}=A+\frac{\Sigma d}{N}.h$$

$$=\frac{h^2\Sigma d^2}{N}+h^2.\left(\frac{\Sigma d}{N}\right)^2-2h^2.\left(\frac{\Sigma d}{N}\right)^2. \qquad \overline{X}-A=\frac{h.\Sigma d}{N}$$

$$=h^2\left[\frac{\Sigma d^2}{N}+\left(\frac{\Sigma d}{N}\right)^2-2\left(\frac{\Sigma d}{N}\right)^2\right]$$

$$\sigma_X{}^2=h^2\left[\frac{\Sigma d^2}{N}-\left(\frac{\Sigma d}{N}\right)^2\right]=h^2.\sigma_d{}^2$$

This shows that the variance or standard deviation is not independent of scale.

5. *Standard deviation is the minimum root-mean-square deviation.*

i.e. $$\sqrt{\frac{\Sigma(X-\overline{X})^2}{N}}<\sqrt{\frac{\Sigma(X-A)^2}{N}}$$

where A is any assumed average.

6. *If the values of mean* $(\overline{X})$ *and standard deviation* (σ) *are given, we can very easily find the sum of observations* (ΣX) *and the sum of squares of observations* (ΣX^2).

We know that $\overline{X}=\frac{\Sigma X}{N}$, or $\Sigma X=N.\overline{X}$.

Similarly $$\sigma_X=\sqrt{\frac{\Sigma X^2}{N}-\left(\frac{\Sigma X}{N}\right)^2}$$

or $$\sigma_X{}^2=\frac{\Sigma X^2}{N}-\left(\frac{\Sigma X}{N}\right)^2$$

or $$\sigma_X{}^2=\frac{\Sigma X^2}{N}-(\overline{X})^2, \text{ or } \sigma_X{}^2+(\overline{X})^2=\frac{\Sigma X^2}{N}.$$

or $$\Sigma X^2=N[\sigma_X{}^2+(\overline{X})^2]$$

7. *If, by taking wrong values we calculate an incorrect value of standard deviation, then it can be easily corrected without repeating the entire process.* We have to find only the corrected values of ΣX, ΣX^2 and N.

8. *If all the values of the variable are the same, then the standard deviation is equal to zero.*

Let us suppose that all the values of X are equal to K. Then $\overline{X}=K$,

$$\therefore \quad \sigma_X=\sqrt{\frac{\Sigma(X-\overline{X})^2}{N}}$$

But $X-\overline{X}=0$ for all the value of X

$\therefore \quad \Sigma(X-\overline{X})^2=0.$

or $\quad \sigma_X=\sqrt{\frac{0}{N}}=0.$

9. ***If a constant quantity say k is added to or subtracted from each value of the variable, the standard deviation remains the same. Similarly,*** **if each item is multiplied or divided by a constant quantity, the value standard deviation also gets multiplied or divided by the same quantity**

i.e. $\sigma'_{K.X}=K.\sigma_X$

or $\sigma_{X/K}=\frac{1}{K}.\sigma_X.$

10. *The standard deviation bears a fixed area relationship to arithmetic mean in a normal distribution or symmetrical distribution.* The total area below a normal curve is taken as 100 percent.

(*i*) $\overline{X}\pm\cdot6745\,\sigma$ covers 50% of the data.

(*ii*) $\overline{X}\pm1\sigma$ covers 68·27% of the data.

(*iii*) $\overline{X}\pm2\sigma$ covers 95·45% of the data.

(*iv*) $\overline{X}\pm3\sigma$ covers 99·73% of the data.

Thus for practical purposes the range of a normal curve is taken $\overline{X}\pm3\sigma$.

Merits and Demerits of Standard Deviation.

Merits.

(*i*) It is the most important and widely used measure of dispersion.

(*ii*) It is based on all the observations.

(*iii*) It is amenable to algebraic treatment.

(*iv*) It can be used to find consistency, stability, uniformity in the data and hence compare two or more series so far as their variability is concerned.

(*v*) It is least affected by sampling fluctuations.

(*vi*) Since it gives the minimum root-mean-square deviations, it is called the best measure of dispersion.

(*vii*) Various statistical concepts like correlation, regression, skewness, tests of significance etc. are based on standard deviation.

Demerits.

(*i*) As compared to other measures of dispersion, standard deviation is difficult to compute and understand.

(*ii*) In the case of open end classes its value cannot be calculated.

(*iii*) By squaring the deviations, it gives more weight to items away

from mean and less weight to items near to mean.

Despite the above limitations, there is no denying the fact that standard deviation is the best and widely used measure of dispersion. It is very precise and suitable for fields requiring high degree of accuracy.

Difference between Mean Deviation and Standard Deviation. The important points of difference between them are given below :-

1. In mean deviation signs are ignored, whereas it is not so in the case of standard deviation.

2. In mean deviation the deviations can be taken from mean, median or mode. On the other hand, in the case of standard deviation the deviations are taken only from mean.

3. Mean deviation is not capable of algebraic treatment. Standard deviation can be treated algebraically and hence combined standard deviation can be calculated.

4. Mean deviation has a limited use, while standard deviation is widely used.

5. Mean deviation is less affected by extreme items. On the other hand, standard deviation gives more weight to extreme items and less weight to items near to mean.

Relationship Between Various Measures of Dispersion

In a normal distribution there exists a fixed relationship between the three most commonly used measures of dispersion *i.e.*, Quartile Deviation, Mean Deviation and Standard Deviation. These relationships are as given below :

(1) The Quartile Deviation is 0·6745 times the value of the standard deviation.

In other words, $Q.\ D=\frac{2}{3}\ \sigma$ **(approximately)**

or $3\ Q.D=2\sigma.$

(2) The mean deviation is equal to 0·7979 times the value of standard deviation.

In other words,

$M.D=0{\cdot}7979\ \sigma$ **or** $M.D=\frac{4}{5}\sigma$ **(approximately)**

or $5\ M.D=4\sigma.$

(3) The Quartile Deviation is equal to 0·8459 times the value of Mean Deviation.

In other words,

$$Q.D=0{\cdot}8459\ M.D$$

or $Q.D=\frac{5}{6}M.D.$ **(approximately).**

or $\quad 6\,Q.D = 5\,M.D$

Combining all the three relationships given above, we get

$$4\,\sigma = 5\,M.D = 6\,Q.D.$$

MISCELLANEOUS PROBLEMS

Example 25. In an industrial establishment, the co-efficients of variation of wages of male and female workers were 55% and 70% respectively. The standard deviations were Rs. 22·00 and Rs. 15·40 respectively. Calculate the combined average wages of all the workers if 80% of the workers were males.

Sol. Let us denote the coefficients of variation of Males and Females by C.V (M) and C.V (F) respectively.

$\therefore$ Given, $\quad C.V\,(M) = 55\%$

and $\quad C.V\,(F) = 70\%$

Let us denote means and standard deviations of wages of males and females by $\bar{x}_1$, $\bar{x}_2$ and σ_1, σ_2 respectively.

$\therefore$ Given $\quad \sigma_1 = 22, \quad \sigma_2 = 15{\cdot}40$

We know that $\quad C.V = \dfrac{\sigma}{\bar{x}} \times 100$

$$\text{or} \quad \bar{x} = \frac{\sigma}{C.V} \times 100$$

$$\text{or} \quad \bar{x}_1 = \frac{\sigma_1}{C.V\,(M)} \times 100 = \frac{22}{55} \times 100 = \text{Rs. } 40$$

$$\text{and} \quad \bar{x}_2 = \frac{\sigma_2}{C.V(F)} \times 100 = \frac{15{\cdot}40}{70} \times 100 = \text{Rs. } 22$$

Let us suppose that the total number of workers is 100.

Given, no. of males = 80% of total workers

$\therefore \quad n_1 = 80 \quad$ and $n_2 = 20$

Now combined Mean $\bar{x}_{12} = \dfrac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

$$\text{or} \quad \bar{x}_{12} = \frac{(80)(40) + (20)(22)}{80 + 20}$$

$$\bar{x}_{12} = \frac{3200 + 440}{100} = \frac{3640}{100} = \text{Rs. } 36{\cdot}4.$$

Example 26. If the values of the mean and standard deviation of the following frequency distribution (obtained by step. Deviation Method) are 135·3 and 9·6 respectively. Determine the actual class intervals.

$d'x$	-4	-3	-2	-1	0	1	2	3
f	2	5	8	18	22	13	8	4

Sol. First of all we shall find the value of C and A and by using these values, class intervals will be determined.

$d'x$	f	$f.d'x$	$d'x^2$	$fd'x^2$
−4	2	−8	16	32
−3	5	−15	9	45
−2	8	−16	4	32
−1	18	−18	1	18
0	22	0	0	0
1	13	13	1	13
2	8	16	4	32
3	4	12	9	36
	$\Sigma f=80$	$\Sigma fd'x=16$		$\Sigma fd'x^2=208$

We know that $\sigma=\sqrt{\frac{\Sigma fd'x^2}{\Sigma f}-\left(\frac{\Sigma fd'x}{\Sigma f}\right)^2}\times C$

or $9{\cdot}6=\sqrt{\frac{208}{80}-\left(\frac{-16}{80}\right)^2}\times C$

or $9{\cdot}6=\sqrt{2{\cdot}6-0\ 04}\times C$

or $9{\cdot}6=\sqrt{2{\cdot}56}\times C$

or $9{\cdot}6=1{\cdot}6\ C$

or $C=\frac{9{\cdot}6}{1{\cdot}6}=6.$

Now $\bar{x}=A+\frac{\Sigma fd'x}{\Sigma f}\times C$

or $135{\cdot}3=A+\frac{-16}{80}\times 6$

or $135{\cdot}3=A-\frac{96}{80}$

or $135{\cdot}3+1{\cdot}2=A$ or $A=136{\cdot}5$

The class intervals will be determined as given below :

$d'x$:	−4	−3	−2	−1	0	1	2	3
$dx=(d'x)(c)$:	−24	−18	−12	−6	0	6	12	18
$x=(dx+A)$:	112·5	118·5	124·5	130·5	136·5	142·5	148·5	154·5

Class interivals can be determined as follows :

$$X\pm\frac{C}{2} \quad \text{or} \quad X\pm\frac{6}{2} \quad \text{or} \quad X\pm 3$$

Class intervals are :

109·5—115·5 ; 115·5—121·5 ; 121·5—127·5 ; 127·5—133·5;
133·5—139·5 ; 139·5—145·5 ; 145·5—151·5 ; 151·5—157·5.

Example 27. Find the mean and standard deviation of first n natural numbers.

Sol. The first n natural numbers are

$$1, 2, 3, 4, 5, \ldots\ldots\ldots, n$$

Let $x = 1, 2, 3, 4, 5, \ldots\ldots\ldots, n$

Now $\Sigma x = \frac{n(n+1)}{2}$ and $\Sigma x^2 = \frac{n(n+1)(2n+1)}{6}$

We know that

$$\bar{x} = \frac{\Sigma x}{n} = \frac{n(n+1)}{2 \cdot n} = \frac{n+1}{2}$$

$$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2} \quad \text{or} \quad \sigma = \sqrt{\frac{\Sigma x^2}{n} - (\bar{x})^2}$$

$$= \sqrt{\frac{n(n+1)(2n+1)}{6 \cdot n} - \left(\frac{n+1}{2}\right)^2}$$

$$= \sqrt{\frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}} = \sqrt{\frac{2(n+1)(2n+1) - 3(n+1)^2}{12}}$$

$$= \sqrt{\frac{(n+1)(4n+2-3n-3)}{12}} = \sqrt{\frac{(n+1)(n-1)}{12}}$$

$$= \sqrt{\frac{n^2-1}{12}}.$$

Example 28. The mean, standard deviation and range of a distribution of weights of 12 children are 9 lbs, 2 lbs and 6 lbs respectively. The median of the distribution is same as mean. Find the mean and standard deviation of the group if the lightest and the heaviest children are not considered.

Solution :

Given $\bar{x} = 9$ lbs., $\sigma = 2$ lbs., Range $= 6$ lbs.,
and $n = 12$.

Range $= 6$ lbs.

$\therefore$ Heaviest Weight $-$ Lightest Weight $= 6$ lbs. ...I

Given $\bar{x} = M = 9$ lbs.

$\therefore \frac{\text{Lightest} + \text{Heaviest}}{2} = 9$ lbs.

or Lightest+Heaviest = 18 lbs. ...II

By solving equations I and II, we get.

Weight of Lightest child = (L) = 6 lbs.

anc Weight of Heaviest child = (H) = 12 lbs.

We know that, Incorrect $\Sigma x = n.\overline{x} = 12 \times 9 = 108$

$\therefore$ Correct Σx = Incorrect $\Sigma x - L - H$
$= 108 - 6 - 12 = 90$ lbs.

Correct $n = 12 - 2 = 10$

$\therefore$ Correct $\overline{x} = \frac{\Sigma x}{n} = \frac{90}{10} = 9$ lbs.

Again we know that,

$$\text{Incorrect } \Sigma x^2 = N[\overline{x}^2 + \sigma^2]$$
$$= 12[(9)^2 + (2)^2] = 12(81+4)$$
$$= 12(85) = 1020.$$

$$\text{Correct } \Sigma x^2 = \text{Incorrect } \Sigma x^2 - (L)^2 - (H)^2$$
$$= 1020 - (6)^2 - (12)^2 = 1020 - 36 - 144$$
$$= 840.$$

$$\therefore \text{ Correct } \sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$$
$$= \sqrt{\frac{840}{10} - \left(\frac{90}{10}\right)^2} = \sqrt{84 - 81}$$
$$= \sqrt{3} = 1{\cdot}732 \text{ lbs.}$$

Example 29. In a partially destroyed data given below two frequencies are missing. Determine their value and also calculate the value of mean and standard deviation.

Class Interval	*Frequency*	*Class Interval*	*Frequency*
100—110	4	150—160	—
110—120	7	160—170	16
120—130	15	170—180	10
130—140	—	180—190	6
140—150	40	190—200	3

It is given that median is equal to 146·25 and total frequency is equal to 150.

Solution. Let us first determine the missing frequencies by using the given value of median. We suppose that missing frequencies are f_1 and f_2 :

Class Interval	*f*	*c. f.*	*Class Interval*	*f*	*c.f.*
100—100	4	4	150—160	f_2	$66+f_1+f_2$
110—120	7	11	160—170	16	$82+f_1+f_2$
120—130	15	26	170—180	10	$92+f_1+f_2$
130—140	f_1	$26+f_1$	180—190	6	$98+f_1+f_2$
140—150	40	$66+f_1$	190—200	3	$101+f_1+f_2$
				$\Sigma f=150$	

Now $101+f_1+f_2=150$

or $f_1+f_2=150-101$

or $f_1+f_2=49$

or $f_2=49-f_1$.

We know that $M=\text{Size of } \frac{N}{2} \text{ th item}$

$=\text{Size of } \frac{150}{2}\text{-th item}$

$=\text{Size of 75 th item}$

Since value of median is equal to 146·25, it lies in the class interval 140—150.

Applying :

$$M=L_1+\frac{i}{f}\left[\frac{N}{2}\text{ th}-C\right], \quad \text{we get}$$

$$146{\cdot}25=140+\frac{10}{40}[75-(26+f_1)]$$

$$\text{or } 6{\cdot}25=\frac{1}{4}[75-26-f_1]$$

$$25=49-f_1$$

$$\text{or } f_1=49-25=24$$

$$\therefore\ f_2=49-f_1$$

$$=49-24=25.$$

Calculation of Mean and Standard Deviation

$A=145 \quad C=10$

Class Interval	f	M.V. (X)	$dx=X-A$	$d'x=\frac{dx}{c}$	$f.\ d'x$	$d'x^2$	$fd'x^2$
100—110	4	105	—40	—4	—16	16	64
110—120	7	115	—30	—3	—21	9	63
120—130	15	125	—20	—2	—30	4	60
130—140	24	135	—10	—1	—24	1	24
140—150	40	145	0	0	0	0	0
150—160	25	155	10	1	25	1	25
160—170	16	165	20	2	32	4	64
170—180	10	175	30	3	30	9	90
180—190	6	185	40	4	24	16	96
190—200	3	195	50	5	15	25	75
	$\Sigma f=150$				$\Sigma fd'x=35$		$\Sigma fd'x^2=561$

$$\bar{x}=A+\frac{\Sigma fd'x}{\Sigma f}\times C$$

$$=145+\frac{35}{150}\times 10$$

$$\bar{x}=145+2{\cdot}33=147{\cdot}33$$

$$\sigma=\sqrt{\frac{\Sigma fd'x^2}{\Sigma f}-\left(\frac{\Sigma fd'x}{\Sigma f}\right)^2}\times C$$

$$=\sqrt{\frac{561}{150}-\left(\frac{35}{150}\right)^2}\times 10$$

$$=\sqrt{3{\cdot}74-0{\cdot}054}\times 10$$

$$=\sqrt{3{\cdot}686}\times 10=1{\cdot}9189\times 10$$

$$\sigma=19{\cdot}189.$$

QUESTIONS

1. What do you mean by dispersion ? What are the requisites of a good measure of dispersion ?
2. What are the principal measures of dispersion ? Discuss their merits and demerits. Under what situations would you use each of these measures ?
3. Define coefficient of variation. What is its use in statistical data analysis ?

4. Distinguish between :
   (i) Absolute and Relative measures of dispersion.
   (ii) Mean deviation and Standard deivation.
   (iii) Quartile deviation and Mean deviation
   (iv) Measures of Central Tendency and Measures of Dispersion.
5. Define mean deviation. Is it the best measure of dispersion ?
6. Critically examine the various methods of measuring dispersion. Which of these methods do you think is the best and why ?
7. Define quartile deviation and its coefficient. How does it differ from mean deviation ?
8. Is it possible to find combined standard deviation and also correct an incorrect value of the standard deviation ? Explain steps.
9. What is the need of measures of dispersion ? Which is the best measure of dispersion ? Explain reasons.
10. Calculate range and its coefficient for the data

Value :	12	15	18	20	10	25	35	50

(Range = 40 ; Coeff. of Range = 0·67)

11. Compute the coefficient of quartile deviation from the following data. Also find the coeff. of range.

Wages (Rs.)	10	20	30	40	50	80
No. of workers.	4	7	15	8	7	2

(Coeff. of Q. D = 0·333 ; Coeff. of Range = 0·78)

12. Find quartile deviation and its coefficient from the following data :

Marks	20—25	25—30	30—35	35—40	40—45
No. of students.	2	10	25	16	7

(Q.D. = 3·45 ; Coeff. of Q.D. = 0·101)

13. Find mean deviation and its coefficient for the following data (Use median)

Value	5,	8,	9,	12,	15,	18,	20

(δ_M = 4·43 ; Coeff. of δ_M = 0·369)

14. Calculate mean deviation from mean of the following :

Marks	0—10	10—20	20—30	30—40	40—50
No. of students	5	8	15	16	6

(M.D. = 9.44)

15. With median as base calculate the mean deviation and compare the variability of the two series A and B.

Series A : 3484, 4572, 4124, 3682, 5624, 4388, 3680, 4308

Series B : 487, 508, 620, 382, 408, 266, 186, 218

(Series A : δ_M = 490·25 ; Coeff. of δ_M = 0·116.

Series B : δ_M = 121·38 ; Coeff. of δ_M = 0·307)

16. Calculate mean deviation from median for data given below :

Marks (less than) :	10	20	30	40	50	60	70	80
No. of students :	5	13	20	32	60	80	90	100

($\delta_M = 14{\cdot}286$)

17. Calculate the standard deviation of marks of 10 students. Marks are 7, 10, 12, 13, 15, 20, 21, 28, 29, 35.
($\sigma = 8{\cdot}76$)

18. Calculate arithmetic mean and standard deviation.

Class interval :	5—15	15—25	25—35	35—45	45—55
Frequency :	8	12	15	9	6

($\bar{x} = 28{\cdot}6$; $\sigma = 12{\cdot}3$)

19. Calculate standard deviation for the following data :

Wages (Rs.)	15	30	45	60	75	90	105	120
No. of workers	12	30	65	107	157	202	222	230

($\sigma = 26{\cdot}02$)

20. The following table shows the prices of two shares A and B (in rupees). Find which share is more consistent.

Share A :	16,	14,	28,	31,	35,	40,	5,	30,	48,	7
Share B :	9,	15,	24,	26,	34,	45,	5,	31,	20,	40

($C.V_A = 53{\cdot}53\%$; $C.V_B = 49{\cdot}59\%$. B is more consistent)

21. Calculate mean, standard deviation and coefficient of variation from data given below :

	under	under	under	under	under	under	under	under
Age (in years) :	10	20	30	40	50	60	70	80
No. of persons dying.	15	30	53	75	100	110	115	125

($\bar{x} = 35{\cdot}16$; $\sigma = 19{\cdot}758$; $C.V = 56{\cdot}19\%$)

22. Coefficients of variation of two series are 60% and 80%. Their standard deviations are 20 and 16. What are their arithmetic means ?
($\bar{x}_1 = 33{\cdot}3$; $\bar{x}_2 = 20$)

23. You are given below the daily wages paid to workers in two factories x and y.

Daily wages (Rs.)	2—3	3—4	4—5	5—6	6—7	7—8	8—9
No. of workers in x	15	30	44	60	30	14	7
No. of workers in y	25	40	60	35	20	15	5

(a) Which factory pays higher average wages ?
(b) In which factory are wages more variable ?

($\bar{x}_1 = 5{\cdot}15$; $\bar{x}_2 = 4{\cdot}75$. Factory x pays higher average wages. $C.V_x = 28{\cdot}25\%$; $C.V_y = 31{\cdot}85\%$ ...

24. Goals scored by two teams A and B in a football season were as follows :

No. of goals scored in a match	*Number of Matches*	
	Team A	*Team B*
0	27	17
1	9	9
2	8	6
3	5	5
4	4	3

By calculating the coeff. of variation in each case, find which team may be considered more consistent.

($C.V_A = 123{\cdot}24\%$; $C.V_B = 109\%$. Team B is more consistent)

25. In a cricket season batsman A gets an average of 64 runs per inning with standard deviation of 15 runs while batsman B gets an average score of 43 runs with standard deviation of 9 runs in about equal number of innings. Discuss the efficiency and consistency of both the batsmen ?

[Batsman A is more efficient (average score is more)

Batsman B is more consistent (C.V is less)]

26. For two firms A and B belonging to the same industry the following data is given :

	Firm A	Firm B
No. of wage earners	586	648
Average monthly wage	Rs. 52·5	Rs. 47·5
Variance	Rs. 100	Rs. 121

(*a*) Which firm pays out larger amount as wages ?

(*b*) In which firm is there greater variability in individual wages ?

(*c*) Find the combined mean and standard deviation.

[(*a*) Firm B (*b*) Firm B (*c*) $\bar{x}_{12} = 49{\cdot}87$; $\sigma_{12} = 10{\cdot}83$]

27. A student obtained the mean and standard deviation of 100 observations as 40 and 5·1 respectively. It was later discovered that he had wrongly copied down an observation as 50 instead of 40.

Calculate the correct mean and standard deviation.

(Correct $\bar{x}=39\cdot9$; Correct $\sigma=5$)

28. The mean and standard deviation of a group of 100 observations were found to be 20 and 3 respectively. After the calculations were made, it was found that three of the observations were incorrect, which were recorded as 21, 21 and 18. Find the mean and standard deviation if the incorrect observations were omitted.

(Correct $\bar{x}=20$; Correct $\sigma=3\cdot9$)

29. For a frequency distribution of marks in statistics of 100 candidates (grouped in class intervals of 0-10, 10-20, 20-30 etc.) the mean and standard deviation were found to be 40 and 16. Later it was discovered that score 44 was misread as 54. Find the correct mean and standard deviation.

(Correct $\bar{x}=39\cdot9$; Correct $\sigma=15\cdot937$)

[**Hint.** Values 44 and 54 lie in classes 40-50 and 50-60. These classes are represented by mid values 45 and 55 respectively. So we consider 45 and 55 instead of 44 and 54].

30. The means of two samples of sizes 50 and 100 respectively are 54·1 and 50·3 and the standard deviations are 8 and 7. Obtain the standard deviation of the sample of size 150 obtained by combining the two samples.

($\sigma_{12}=7\cdot5625$)

31. For a group containing 100 observations, $\bar{x}=8$ and $\sigma=\sqrt{10\cdot5}$. For 50 observations selected from these 100 observations, the mean and standard deviation are 10 and 2 respectively. Find the mean and standard deviation of the other half.

($\bar{x}_2=6$; $\sigma_2=3$)

32. An association doing charity work decided to give old age pension to people over sixty years of age. The scales of pension were fixed as follows :

Age group (years)	60—65	65—70	70—75	75—80	80—85
Pension (Rs. Per month)	25	30	35	40	45

The ages of 25 persons who secured the pension rights are 74, 62, 84, 72, 61, 83, 72, 81, 64, 71, 63, 61, 60, 63, 74, 66, 64, 79, 73, 75, 76, 69, 68, 78, 67.. Calculate the monthly average pension payable and the standard deviation.

($\bar{x}=33$; $\sigma=6\cdot93$)

[**Hint.** Find frequency distribution using tally bars.]

Age group. (years)	60-65	65-70	70-75	75-80	80-85
Pension (Rs.) (x)	25	30	35	40	45
No. of Persons. (f)	8	4	6	4	3

33. Given below is the frequency distribution of marks obtained by 90 students. Find mean and standard deviation.

Marks	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99
No. of students.	5	12	15	20	18	10	6	4

($\bar{x}=56{\cdot}5$; $\sigma=17{\cdot}65$)

[Hint. There is no need to convert the class intervals to the exclusive form.]

34. The standard deviation for a set of 32 observations is 5. If the sum of the observations is 80, what is the sum of the squares of these observations ?

($\Sigma x^2=1000$)

35. **The arithmetic mean of runs scored by three batsman A, B and C in the same series of 10 innings are 50, 48 and 12 respectively. The standard deviations of their runs are respectively 15, 12 and 2. Find**

(*i*) Who is a consistent batsman ?

(*ii*) Who is more efficient batsman ?

[(*i*) *Batsman C is more consistent because its C.V. is less.* (*ii*) *Batsman A is more efficient because its average score is maximum.*]

36. **The following data give the mean and standard deviations of three sub-groups. Calculate the mean and the standard deviation of the whole group.**

Sub-group	***No. of men***	***Mean***	***Standard Deviation***
A	**50**	**61**	**8**
B	**100**	**70**	**9**
C	**120**	**80·5**	**10**

($\bar{x}_{ABC}=73$, $\sigma_{ABC}=11{\cdot}9$)

37. **Calculate coefficient of quartile deviation and coefficient of variation from the following data :**

Marks	***No. of Students***
Below 20	**8**
,, 40	**20**
,, 60	**50**
,, 80	**70**
,, 100	**80**

(Coeff. of Q. D=0·273 ; C.V.=42·65%)

38. **Find the actual class intervals from the data given ahead when mean is 35·16 and standard deviation is 19·76.**

$d'x$	-3	-2	-1	0	1	2	3	4
f	15	15	23	22	25	10	5	10

(A=35 C=10 and 4 class intervals are 0—10, 10—20, 20—30, 30—40, 40—50, 50—60, 60—70 and 70—80)

39. The mean of 5 items is 4.4 and standard deviation is $\sqrt{8{\cdot}24}$. If three items are 1, 2, 9, find the other two items.
(6,4)

40. For two unequal values of x and y mean is 20 and standard deviation is 3. Find the value of x and y.
($x=23$, ; $y=17$).

— • —

7

Measures of Skewness

In the last two chapters we have studied about the measures of central tendency and dispersion. Measures of central tendency give us the most representative figure in the data and measures of dispersion give an average of variability around the measures of central tendency. The measures of dispersion do not tell whether the dispersal of items on either side of the average is symmetrical or not. In other words, they do not say anything about the shape of the frequency distribution. We may have two or more frequency distributions of different shapes with same value of mean and standard deviation, as given below :

Distribution A

Class Intervals	Frequency
0—5	1
5—10	3
10—15	6
15—20	6
20—25	3
25—30	1

Distribution B

Class Intervals	Frequency
0—5	1
5—10	4
10—15	3
15—20	9
20—25	2
25—30	1

Both these distributions have the value of mean equal to 15 and standard deviation equal to 6. Inspite of this the two distributions differ with each other. The distribution A is symmetrical, i.e., the scatter of items above and below mean is the same. On the other hand, distribution B is skewed. By skewness we mean the presence of lopsidedness or lack of symmetry. In the words of Spiegel, "*Skewness is the degree of asymmetry or departure from symmetry, of a distribution.*"

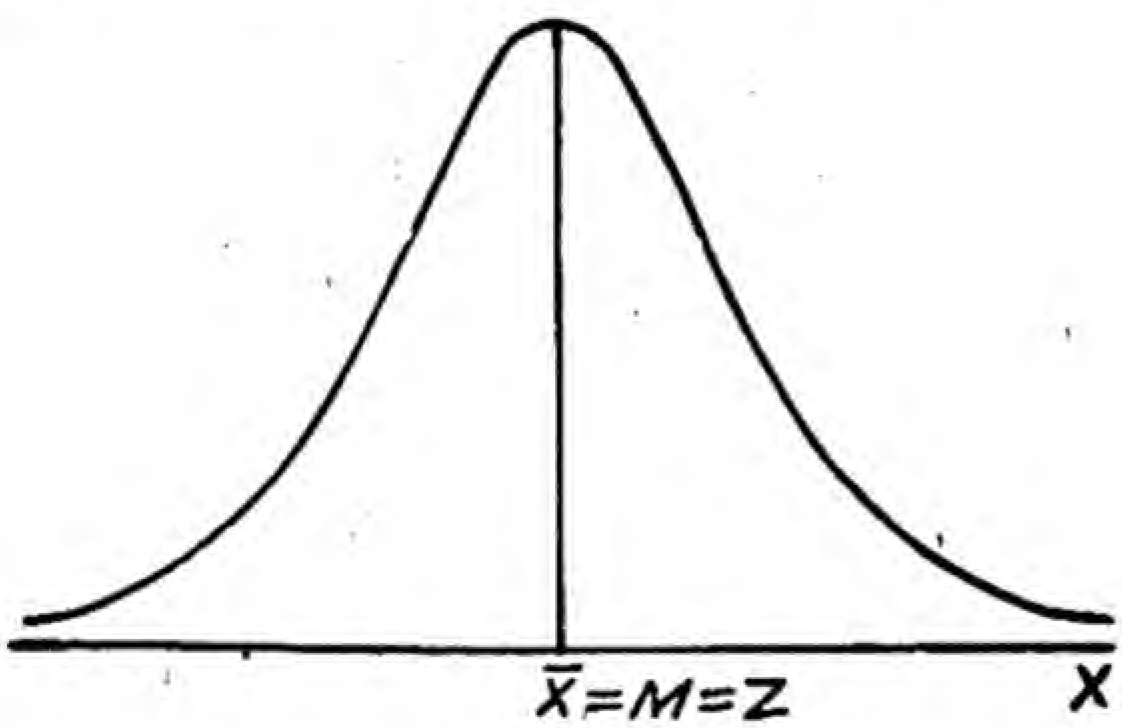
In a symmetrical distribution there is no skewness. The values of mean, median and mode are equal. The quartiles are equidistant from median *i.e.*, $Q_3 - M = M - Q_1$. The sum of positive deviations from median is equal to the sum of negative deviations. The frequency curve of a symmetrical distribution is bell shaped. It is called a normal or ideal curve. If it is folded about the ordinate at mean, the two halves will coincide with each other. It is unimodal and possesses many properties. Let us take an example.

Class Interval	Frequency
0—10	5
10—20	10
20—30	15
30—40	20
40—50	15
50—60	10
60—70	5

$\bar{x}=35$, $M=35$, $Z=35$,

$Q_1=23\cdot33$, $Q_3=46\cdot67$

$\therefore \quad \bar{x}=M=Z\ ;\ Q_3-M=M-Q_1=11\cdot67$

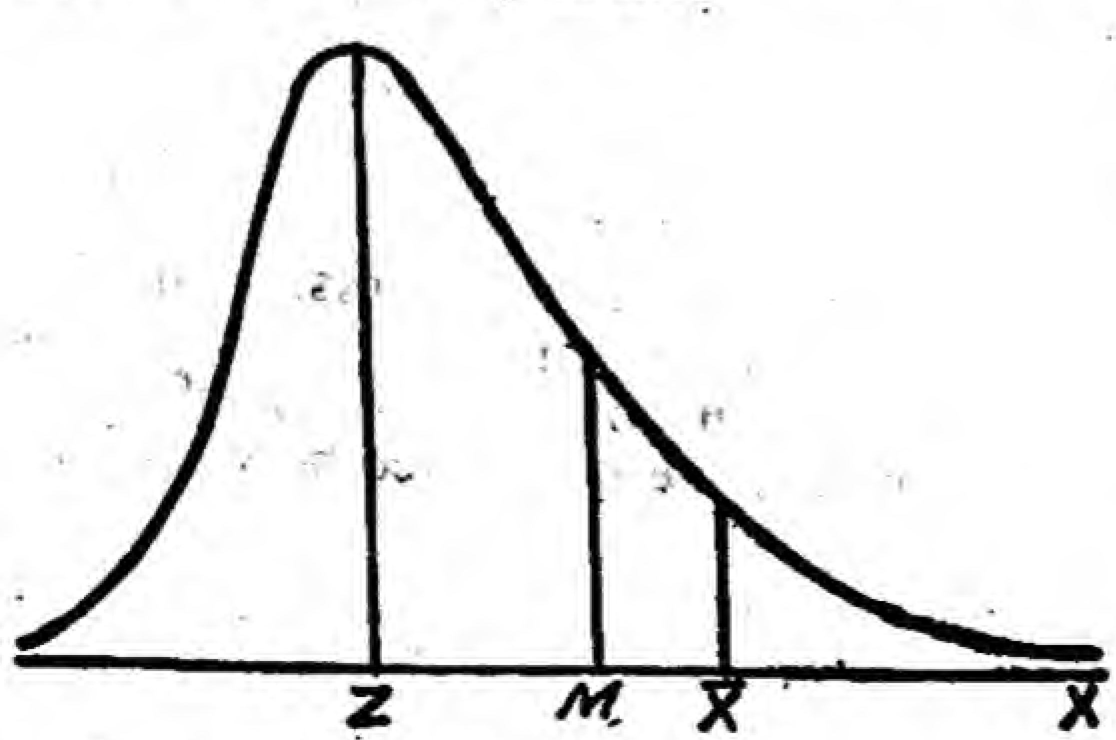


Normal Curve
(*Symmetrical Distribution*)

Note : Any distribution which does not possess any of the above characteristics is called a skewed distribution.

Types of Skewness. Skewness can be Positive or Negative.

(*a*) **Positive Skewness.** A distribution is said to be positively skewed if frequency at the smaller values of the variable is very large. In other words, the excess tail is towards the right hand side. The value of mean is more than median and mode. Median lies between mode and mean. The difference of third quartile and median is more than the difference of median and first quartile.

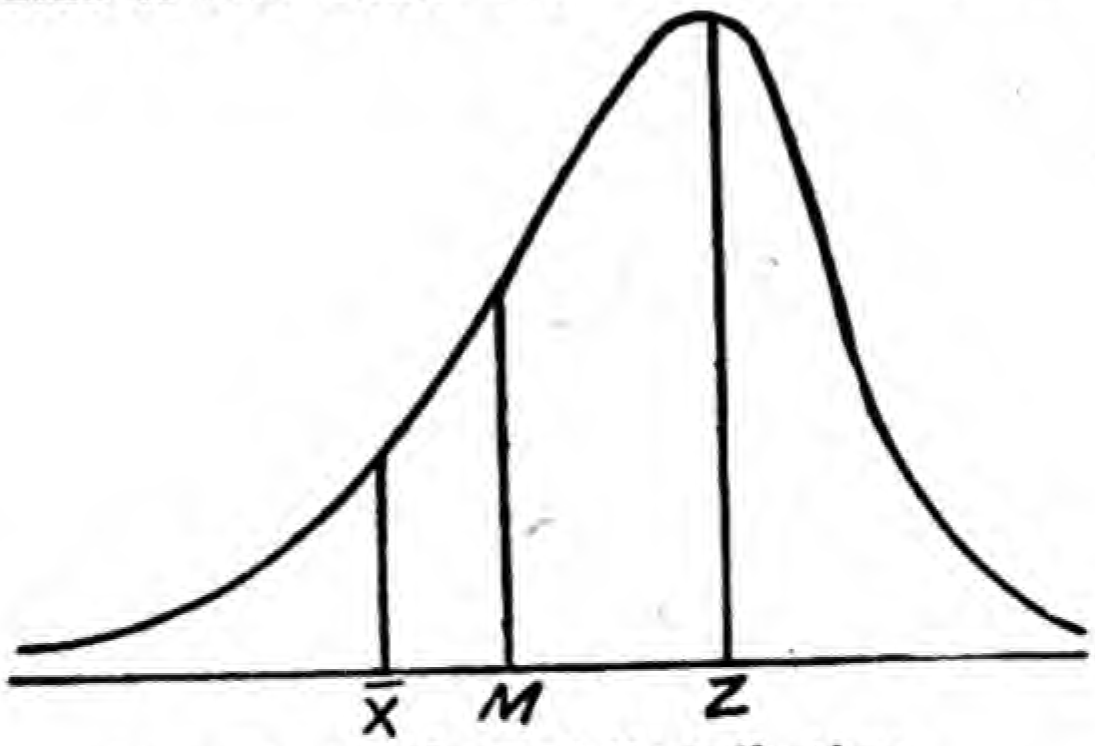


Positively Skewed Distribution

$\bar{x} > M > Z$

and $Q_3 - M > M - Q_1$

(*b*) **Negative Skewness.** A frequency distribution is said to be negatively skewed if the maximum frequency occurs at the higher values of the variable. In other words, the excess tail is towards the left hand side. Value of mean is minimum and that of mode is maximum. Median lies between mean and mode. The difference of third quartile and median is less than the difference of median and first quartile.



Negatively Skewed Distribution

$\bar{x} < M < Z$

and $Q_3 - M < M - Q_1$

Tests of Skewness. Skewness in any distribution can be tested from any of the following tests :

(*i*) Mean, median and mode are not equa in a skewed distribution

i.e., $\bar{x} \neq M \neq Z$

(*ii*) Quartiles are not equidistant from median

i.e., $Q_3 - M \neq M - Q_1$

(*iii*) The corresponding pairs of deciles and percentiles are not equidistant from median *i.e.*,

$$M - D_1 \neq D_9 - M \quad \text{or} \quad M - P_1 \neq P_{99} - M$$

(*iv*) When plotted on a graph paper, a skewed distribution does not give bell shaped normal curve.

(*v*) The sum of positive deviations from median is not equal to the sum of negative deviations.

Measures of Skewness : Important measures are.

(*i*) Karl Pearson's measure of skewness.

(*ii*) Bowley's measure of skewness.

(*iii*) Kelly's measure of skewness.

(*i*) **Karl Pearson's Measure of Skewness.** This measure is based on the relationship between mean, median and mode in a symmetrical distribution. In such distribution, the values of mean, median and mode are equal. Therefore, the difference between them is used as the

absolute measure of skewness in an asymmetrical distribution. The following formulae are used :

(*a*) Skewness = Mean — Mode

or (*b*) Skewness = Mean — Median

or (*c*) Skewness = Median — Mode

Since these are only absolute measures of skewness, they cannot be used for comparing skewness in distributions having different statistical units. Karl Pearson has developed a *relative measure* of skewness known as the coefficient of skewness. It is equal to the ratio of difference between mean and mode to the standard deviation. In other words,

$$\text{Karl Pearson's Coeff. of Skewness} = \frac{\bar{x} - Z}{\sigma}$$

It is a pure number independent of units. Its value usually lies between ± 1. Sometimes it is not possible to find mode *e.g.*, in the case of bimodal data. In such cases we can use the empirical relationship between mean, median and mode to find the value of mode.

$$\text{Mode} = 3\ \text{Median} - 2\ \text{Mean}$$

or $$Z = 3\,M - 2\,\bar{x}$$

$\therefore$ $$\text{Karl Pearson's Coeff. of Skewness} = \frac{\bar{x} - (3M - 2\bar{x})}{\sigma}$$

$$= \frac{\bar{x} - 3M + 2\bar{x}}{\sigma} = \frac{3(\bar{x} - M)}{\sigma}$$

It is also a pure number free from units of the data. Its value can lie between ± 3, but these limits are rarely attained in practice. Normally the value of coeff. of skewness lies between ± 1.

Example 1. Calculate Karl Pearson's Coefficient of Skewness from the following data.

Marks	0—10	10—20	20—30	30—40	40—50	50—60
No. of students	5	8	10	20	4	3

Sol. Calculation of Mean and standard deviation.

Marks	*No. of Students* (*f*)	*M.V* *x*	*A=25* *dx*	*C=10* *d'x*	*f.d'x*	*d'x*^{2}	*fd'x*^{2}
0—10	5	5	—20	—2	—10	4	20
10—20	8	15	—10	—1	—8	1	8
20—30	10	25	0	0	0	0	0
30—40	20	35	10	1	20	1	20
40—50	4	45	20	2	8	4	16
50—60	3	55	30	3	9	9	27
	$\Sigma f = 50$				$\Sigma f d'x = 19$		$\Sigma fd'x^2 = 91$

$$\bar{x} = A + \frac{\Sigma fd'x}{\Sigma f} \times c$$

$$= 25 + \frac{19}{50} \times 10 = 25 + 3{\cdot}8$$

$$\bar{x} = 28{\cdot}8$$

$$\sigma = \sqrt{\frac{\Sigma fd'x^2}{\Sigma f} - \left(\frac{\Sigma fd'x}{\Sigma f}\right)^2} \times c$$

$$= \sqrt{\frac{91}{50} - \left(\frac{19}{50}\right)^2} \times 10$$

$$= \sqrt{1{\cdot}82 - 0{\cdot}1444} \times 10 = \sqrt{1{\cdot}6756} \times 10$$

$$\sigma = 1{\cdot}294 \times 10 = 12{\cdot}94.$$

Calculation of Mode. Let us first make grouping table.

Grouping Table

Marks	*frequency I*	*II*	*III*	*IV*	*V*	*VI*
0—10	5	13		23		
10—20	8		18		38	
20—30	10	30				34
30—40	**20**		24	27		
40—50	4	7				
50—60	3					

Analysis Table

Marks	*I*	*II*	*III*	*IV*	*V*	*VI*	*Total*
0—10							0
10—20					1		1
20—30		1			1	1	3
30—40	1	1	1	1	1	1	6
40—50			1	1		1	3
50—60				1			1

It is clear that modal class interval is 30—40.

Applying Interpolation $Z=L_1+\frac{\Delta_1}{\Delta_1+\Delta_2}\times i$

or $Z=30+\frac{10}{10+16}\times 10$ $\quad\because\ \Delta_1=20-10=10$

$=30+\frac{100}{26}$ $\quad \Delta_2=20-4=16$

$Z=30+3\cdot846=33\cdot846$

Now Coeff. of Sk. $=\frac{\bar{x}-z}{\sigma}=\frac{28\cdot8-33\cdot846}{12\cdot94}$

$=\frac{-5\cdot046}{12\cdot94}=-0\cdot39$

The distribution is negatively skewed.

Example 2. The sum of 20 observations is 360 and its sum of squares is 8000. Find the coeff. of skewness and coeff. of variation if median is 15.

Sol. Given $n=20$, $\Sigma x=360$, $\Sigma x^2=8000$ and $M=15$. Since mode is not given,

We use Coeff. of Sk. $=\frac{3(\bar{x}-M)}{\sigma}$

Where $\bar{x}=\frac{\Sigma x}{n}=\frac{360}{20}=18$

$$\sigma=\sqrt{\frac{\Sigma x^2}{n}-\left(\frac{\Sigma x}{n}\right)^2}=\sqrt{\frac{8000}{20}-\left(\frac{360}{20}\right)^2}$$

$$=\sqrt{400-324}=\sqrt{76}=8\cdot72$$

Now Coeff. of Sk. (Karl Pearson's) $=\frac{3(18-15)}{8\cdot72}$

$=\frac{9}{8\cdot72}=1\cdot032$

Coeff. of Variation $=\frac{\sigma}{\bar{x}}\times 100=\frac{8.72}{18}\times 100=48\cdot44\%$

(*ii*) **Bowley's Measure of Skewness.** Bowley's measure is based upon median and quartiles. We know that in a symmetrical distribution, the quartiles are equidistant from median i.e. $Q_3-M=M-Q_1$. Therefore, in a skewed distribution the difference of (Q_3-M) and $(M-Q_1)$ is called Bowley's absolute measure of skewness. In other words, $S_K=(Q_3-M)-(M-Q_1)$

Or $S_K=Q_3+Q_1-2M$.

Bowley's Coefficient of Skewness can be obtained by dividing Bowley's absolute measure of skewness by the sum of (Q_3-M) and $(M-Q_1)$. In other words, Bowley's Coeff. of Skewness

$$=\frac{(Q_3-M)-(M-Q_1)}{(Q_3-M)+(M-Q_1)}$$

$$\text{or Coeff. of Sk}=\frac{Q_3+Q_1-2M}{Q_3-Q_1}$$

It is a pure number independent of units of original data. Its value lies between ± 1.

Notes. (*a*) It should be noted that the value of skewness from Bowley's method is not comparable with the value of skewness from Karl Pearson's method. It is due to the fact that both the methods are based on different measures. It may happen that one method gives positive skewness while the other method shows negative skewness.

(*b*) Bowley's method is very useful in the following cases :

(*i*) When mode is ill defined

(*ii*) There are open end classes.

(*iii*) There are extreme items which unduly affect the values of mean and standard deviation.

Example 3. From the following data determine coefficient of skewness based on Bowley's method.

Wages (Rs.)	100-200	200-300	300-400	400-500	500-600	600-700
No. of workers	15	20	20	10	8	7

Sol.

Wages (Rs.)	*No. of workers* (f)	*C.f*
100—200	15	15
200—300	20	35
300—400	20	55
400—500	10	65
500—600	8	73
600—700	7	80
	$\Sigma f=80$	

$Q_1=\text{Size of }\frac{N}{4}\text{th item}$

$=\text{Size of }\frac{80}{4}\text{th item}$

$=\text{Size of 20th item}$

It lies in *C.f.* 35

$\therefore$ required class interval is 200—300

Now $$Q_1=L_1+\frac{i}{f}\left[\frac{N}{4}\text{ th}-C\right]$$

$$=200+\frac{100}{20}[20-15]$$

$$Q_1 = 200 + 25 = \text{Rs. } 225.$$

Q_3 = Size of $\frac{3N}{4}$th item.

or Q_3 = size of 60th item. It lies in *C.f.* 65
Therefore, the required class interval is 400—500.

Now $Q_3 = L_1 + \frac{i}{f}\left[\frac{3N}{4}\text{th} - C\right]$

$$= 400 + \frac{100}{10}[60 - 55] = 400 + 50$$

$$Q_3 = \text{Rs. } 450$$

M = Size of $\frac{N}{2}$ th item.

= Size of 40th item. It lies in *C.f.* 55

Therefore, median class interval is 300—400.

Now $M = L_1 + \frac{i}{f}\left[\frac{N}{2}\text{th} - C\right]$

$$= 300 + \frac{100}{20}[40 - 35]$$

$$= \text{Rs. } 325.$$

Bowley's Coeff. of Sk. $= \dfrac{Q_3 + Q_1 - 2M}{Q_3 - Q_1}$

$$= \frac{450 + 225 - 2(325)}{450 - 225}$$

$$= \frac{675 - 650}{225} = \frac{25}{225} = 0{\cdot}111.$$

Example 4. If the sum of upper and lower quartiles is 100 and their difference is 40, find Bowley's coeff. of skewness if median is 35.

Sol. Given $Q_3 + Q_1 = 100$, $Q_3 - Q_1 = 40$, $M = 35$.

Now Bowley's Coeff. of Sk. $= \dfrac{Q_3 + Q_1 - 2M}{Q_3 - Q_1}$

$$= \frac{100 - 2(35)}{40} = \frac{30}{40} = 0{\cdot}75$$

(*iii*) **Kelly's Measure of Skewness.** This method is based on the fact that the corresponding pairs of deciles and percentiles are equidistant from median in a symmetrical distribution. Therefore, their difference is taken as a measure of skewness.

Kelly's Absolute Measure of Sk. $=(D_9-M)-(M-D_1)$

$=D_9+D_1-2M.$

or Kelly's Absolute measure based on percentiles is

$$Sk=(P_{90}-M)-(M-P_{10})$$
$$=P_{90}+P_{10}-2M.$$

$$\text{Kelly's Coeff. of Sk. (based on deciles)}=\frac{D_9+D_1-2M}{D_9-D_1}$$

$$\text{Kelly's Coeff. of Sk. (based on percentiles)}=\frac{P_{90}+P_{10}-2M}{P_{90}-P_{10}}$$

It is independent of units and its value lies between ± 1.

This method is not much popular in practice and generally Karl Pearson's method is used.

Example 5. Calculate Kelly's Coeff. of skewness when it is given that $D_9=80$, $D_1=20$, $M=40$.

Sol.

$$\text{Kelly's Coeff. of Skewness}=\frac{D_9+D_1-2M}{D_9-D_1}$$

$$=\frac{80+20-2(40)}{80-20}=\frac{20}{60}=0\cdot 33$$

Difference between Dispersion and Skewness. Dispersion and skewness indicate two different characteristics of a frequency distribution. Dispersion tells us about the scatterness of the distribution. Measures of dispersion show the extent of deviation of individual items from any measure of central tendency. On the other hand, skewness tells whether the dispersal of items is symmetrical or not. In other words, skewness tells whether extent of deviation of items below mean and above mean is same or not. If it is same, then there is no skewness and distribution is called symmetrical otherwise it is called asymmetrical *i.e.*, having skewness.

MISCELLANEOUS EXAMPLES

Example 6. Calculate Karl Pearson's coefficient of Skewness from the following data :

Marks	*No. of Students*	*Marks*	*No. of Students*
above 0	150	above 50	70
,, 10	140	,, 60	30
,, 20	100	,, 70	14
,, 30	80	,, 80	0
,, 40	80		

Solution. Let us first make simple class intervals.

Marks.	*f*	*c.f.*	*Mid value (x)*	$A=35$ $dx=x-A$	$C=10$ $d'x=\frac{dx}{C}$	$fd'x$	$d'x^2$	$fd'x^2$
0—10	10	10	5	—30	—3	—30	9	90
10—20	40	50	15	—20	—2	—80	4	160
20—30	20	70	25	—10	—1	—20	1	20
30—40	0	70	35	0	0	0	0	0
40—50	10	80	45	10	1	10	1	10
50—60	40	120	55	20	2	80	4	160
60—70	16	136	65	30	3	48	9	144
70—80	14	150	7	40	4	56	16	224
	$\Sigma f=150$				$\Sigma fd'x=$	—130 +194 64	$\Sigma d'x^2=808$	

$M=$ size of $\frac{N}{2}$th item

$=$ size of $\frac{150}{2}$ th item

$=$ Size of 75th item

It lies in *c.f.* 80

$\therefore$ Median class Interval is 40—50

Applying $M=L_1+\frac{i}{f}\left[\frac{N}{2}\text{th}-C\right]$

or $M=40+\frac{10}{10}\left[75-70\right]$

or $M=40+5=45$

Now $\bar{x}=A+\frac{\Sigma fd'x}{\Sigma f}\times C$

$=35+\frac{64}{150}\times 10$

$=35+4{\cdot}27=39{\cdot}27$

$$\sigma=\sqrt{\frac{\Sigma fd'x^2}{\Sigma f}-\left(\frac{\Sigma fd'x}{\Sigma f}\right)^2}\times C$$

$$=\sqrt{\frac{808}{150}-\left(\frac{64}{150}\right)^2}\times 10$$

$$=\sqrt{5\cdot3867-0\cdot182}\times 10=\sqrt{5\cdot2047}\times 10$$

$$=2\cdot281\times 10=22\cdot81$$

Since the given frequency distribution is bi-modal, we use

$$\text{Co-efficient of } S_k=\frac{3(\bar{x}-M)}{\sigma}=\frac{3(39\cdot27-45)}{22\cdot81}$$

$$=\frac{3(-5\cdot73)}{22\cdot81}=\frac{-17\cdot19}{22\cdot81}=-0\cdot75$$

Example 7. The median, Mode and co-efficient of skewness for a certain distribution are respectively 17.4, 15.3 and 0·35. Calculate the co-efficient of variation :

Solution. Given $M=17\cdot4$, $Z=15\cdot3$ and

Co-efficient of $S_k=0\cdot35$

We know that $Z=3M-2\bar{x}$

or $2\bar{x}=3M-Z$

or $$\bar{x}=\frac{3M-Z}{2}=\frac{3(17\cdot4)-15\cdot3}{2}=\frac{3(17\cdot4)-15\cdot3}{2}$$

or $$\bar{x}=\frac{52\cdot2-15\cdot3}{2}=\frac{36\cdot9}{2}=18\cdot45$$

We know that Karl Pearson's coefficient of Skewness is given by

$$S_k=\frac{\bar{x}-Z}{\sigma}$$

or $$0\cdot35=\frac{18\cdot45-15\cdot3}{\sigma}$$

or $$0\cdot35\,\sigma=3\cdot15$$

or $$\sigma=\frac{3\cdot15}{0\cdot35}=9$$

Now $$C.V.=\frac{\sigma}{\bar{x}}\times 100=\frac{9}{18\cdot45}\times 100=48\cdot78\%.$$

Example 8. Calculate Bowley's co-efficient of Skewness from the following data :

Mid-Value :	75	100	125	150	175	200	225	250
Frequency :	35	40	48	100	125	80	50	22.

Solution. Since mid values are given, we first determine the class intervals for finding quartiles and median. Since the size of class intervals is 25, the class intervals can be determined by using

$$M.V. \pm \frac{25}{2}.$$

or Class Interval = Mid value $\pm$ 12·5

Class Interval	*f*	*c.f.*
62·5—87.5	35	35
87·5—112·5	40	75
112·5—137·5	48	123
137·5—162·5	100	223
162·5—187·5	125	348
185·7—212·5	80	428
212·5—237·5	50	478
237·5—262·5	22	500
	$\Sigma f=500$	

$M=$ size of $\frac{N}{2}$ th item

$=$ size of $\frac{500}{2}$ th item

$=$ size of 250 th item.

250th item lies in *c. f.* 348

$\therefore$ Median class interval is 162·5—187·5

Applying

$$M=L_1+\frac{i}{f}\left[\frac{N}{2}\text{th}-C\right]$$

We have $M=162{\cdot}5+\frac{25}{125}\left[250-223\right]$

$=162{\cdot}5+\frac{1}{5}(27)=162{\cdot}5+5{\cdot}4=167{\cdot}9$

$Q_1=$ size of $\frac{N}{4}$ th item

$=$ size of $\frac{500}{4}$ th item

$=$ size of 125 th item.

It lies in *c. f.* 223.

$\therefore$ Q_1 class interval is 137·5—162·5

or $Q_1=L_1+\frac{i}{f}\left[\frac{N}{4}\text{th}-C\right]$

$=137{\cdot}5+\frac{25}{100}\left[125-123\right]=137{\cdot}5+0{\cdot}5=138$

Similarly,

$Q_3=$ size of $\frac{3N}{4}$ th item

=size of 375 th item

375 th item lies in *c. f.* 428

$\therefore$ Q_3 class interval is 187·5—212·5.

Applying,

$$Q_3 = L_1 + \frac{i}{f}\left[\frac{3N}{4}\text{ th} - C\right]$$

$$= 187{\cdot}5 + \frac{25}{80}\left[375 - 348\right]$$

$$= 187{\cdot}5 + \frac{5}{16}(27) = 187{\cdot}5 + 8{\cdot}44 = 195{\cdot}94.$$

Bowley's co-efficient of $S_k = \frac{Q_3 + Q_1 - 2M}{Q_3 - Q_1}$

$$= \frac{195{\cdot}94 + 138 - 2(167{\cdot}9)}{195{\cdot}94 - 138} = \frac{-1{\cdot}86}{57{\cdot}94}$$

$$= -0{\cdot}032.$$

Example 9. You are given the position in a factory before and after the settlement of an industrial dispute. Comment on the gains or losses from the point of view of workers.

	Before	After
No. of workers	2400	2300
Mean wages	Rs. 45·5	Rs. 47·5
Median wages	Rs. 48 0	Rs. 45·0
Modal wages	Rs. 53·0	Rs. 40·0
Standard Deviation	Rs. 12.0	Rs. 10.0

Solution :

Employment. Since the number of workers employed afrer the settlement has decreased, so the settlement has gone against the interests of workers.

Wages. There has been an increase in the average wages to workers employed in the factory from Rs. 45·5 to Rs. 47·5. Moreover, the total wage bill has increased from 1, 09, 200 *i.e.* 2400×45·5 before settlement to Rs. 1, 11, 625 *i.e.*, 2350×47·5 after the settlement. So workers employed are better off now than before settlement.

Co-efficient of Variation :

C.V. before settlement$=\frac{\sigma}{\bar{x}} \times 100 = \frac{12}{45{\cdot}5} \times 100$

$=26{\cdot}37\%$

C.V. after settlement$=\frac{\sigma}{\bar{x}} \times 100 = \frac{10}{47{\cdot}4} \times 100 = 21{\cdot}05\%$

Since C.V. has decreased after the settlement, we can say that wages have become more uniform after the settlement. This position is good for workers.

Pattern of wage structure :

Before settlement, Median > Mean. It means that 50% of the workers were getting wages more than mean wages. This trend has reversed after the settlement. Moreover, before settlement maximum number of workers were getting Rs. 53 (because mode is 53). But after the settlement, the maximum number of workers get only Rs. 40. So the pattern of wage structure after settlement is against the workers.

Co-efficient of Skewness :

Before Settlement.

$$\text{Coefficient of } S_k = \frac{\bar{x} - Z}{\sigma} = \frac{45 \cdot 5 - 53}{12} = \frac{-7 \cdot 5}{12} = -0 \cdot 625$$

After Settlement.

$$\text{Co-efficient of } S_k = \frac{\bar{x} - Z}{\sigma} = \frac{47 \cdot 5 - 40}{10} = \frac{7.5}{10} = 0 \cdot 75$$

So we find that co-efficient of skewness has changed from —ve to +ve after the settlement. It implies that, after settlement there are a large number of workers getting lower wages. So after settlement, inequalities in the distribution of wages have increased.

Thus, there is a mixed picture from the point of view of workers after settlement.

QUESTIONS

1. What is skewness ? How does it differ from dispersion ? Describe the various measures of skewness.
2. List the tests that are applied to find out the presence or absence of skewness in a distribution.
3. Distinguish between :
   (*i*) Absolute and Relative skewness.
   (*ii*) Positive and Negative Skewness.
   (*iii*) Karl Pearson's and Bowley's measures of skewness.
4. Define and discuss the quartiles of a distribution. How are they used for measuring skewness ?
5. Explain the term skewness. Distinguish between absolute and relative measures of skewness. Mention the various measures you know.
6. Calculate Pearson's measure of skewness on the basis of mean, mode and standard deviation.

x	14·5	15·5	16·5	17·5	18·5	19·5	20·5	21·5
f	35	40	48	100	125	87	43	22

($\bar{x}$ = 18.07 ; z = 18.4 ; σ = 1·775 ; Coeff. of Sk. = —0.186)

7. Calculate Karl Pearson's Coeff. of skewness.

Marks	: 10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of students :	185	77	34	180	136	23	50

($\bar{x}=39$; $z=47\cdot7$; $\sigma=18\cdot9$; Coeff. of Sk.=—0·46)

8. Find out mean deviation and quartile deviation from the following data. Also calculate Karl Pearson's coeff. of skewness.

	Above	Above	Above	Above	Above	Above	Above	Above
Wages (in Rs.)	200	210	220	230	240	250	260	270
No. of workers.	685	500	423	389	209	73	50	0

(Q.D=16·75 ; $\delta\bar{x}=16\cdot5$; Coeff. of Sk.=—0·46)

9. Karl Pearson's coeff. of skewness of a distribution is 0·40. Its standard deviation is 8 and mean is 30. Find the median and mode of the distribution.

(M=28·93 ; Z=26·8)

10. In a frequency distribution the coeff. of skewness based on quartiles is 0·6. If the sum of upper and lower quartiles is 100 and the median is 38 find the value of upper quartile.

($Q_3=70$)

11. In a distribution, the difference between two quartiles is 15 and their sum is 35. Find the coeff. of skewness if median is 20.

(Coeff. of Sk.=—0·33)

12. Calculate Bowley's Coeff. of skewness from the following :—

x :	10-15	15-20	20-25	25-30	30-35	35-40	40-45	45-50
f :	2	5	7	13	21	16	8	3

(Coeff. of Sk.=—0·05)

13. Following figures relate to the size of capital of companies. Calculate Bowley's Coeff. of Skewness.

Capital (in Rs. Lakhs)	1-5	6-10	11-15	16-20	21-25	26-30	31-35
No. of companies	20	27	29	38	48	53	70

($Q_1=14\cdot68$; M=23.47 ; $Q_3=30\cdot38$; Coeff. of Sk.=—0·119)

[**Hint** : Make class intervals exclusive]

14. Calculate appropriate measure of skewness for the following data :

Wages (Rs.)	Below 100	100-140	140-180	180-220	220-260	260-300	300-340	340 & above
No. of workers :	1	16	39	58	60	46	22	9

(**Hint** : Since open end classes are there, Bowley's method is appropriate.)

(Coeff. of Sk.=0·02)

15. The statistical constants given below relate to two distributions A and B. Comment on their dispersion and skewness.

	A	B
Median	19·6	24·4
Lower Quartile	13·5	15·6
Upper Quartile	26	37·8

($Q.D_A=6{\cdot}25$; $Q.D_B=11{\cdot}1$. Distribution B is more variable) Bowley's Coeff. of Sk. of **A=0·0024** ; Sk. of **B=0·019.** Distribution B is more skewed than A)

16. Calculate Bowley's Coeff. of skewness from the following data :

Mid value :	75	100	125	150	175	200	225	250
Frequency :	35	40	48	100	125	80	50	22

($Q_1=138$; M=167·9 ; $Q_3=195{\cdot}94$; Coeff. of Sk.=—0·032)

(Hint : Make class intervals from mid. values like 62·5—87·5 ; 87·5—112·5 ; 112·5—137·5 & so on)

17. **In a certain distribution the following results were obtained.**

$\bar{x}=45$; Median=48 and coeff. of $S_k=-0{\cdot}4$.

Estimate the value of standard deviation and co-efficient of variation.

($\sigma=22{\cdot}5$; C.V.=50%)

18. **For a group of 10 items $\Sigma x=452$, $\Sigma x^2=24270$ and Mode=43·7. Find the Karl Pearson's co-efficient of Skewness.**

($S_k=0{\cdot}08$)

19. **A frequency distribution gives the following results :**

C.V.=5 ; $\sigma=2$; Karl Pearson's co-efficient of $S_k=0{\cdot}5$. Find the mean and mode of the distribution.

(Mean=40, Mode=39)

20. **Calculate Bowley's co-efficient of Skewness from the following data :**

Weight (in kg.)	*No. of Students*	*Weight (in kg.)*	*No. of Students*
93— 97	2	113—117	14
98—102	5	118—122	6
103—107	12	123—127	3
108—112	17	128—132	1

(0·0075)

(Hint : ***Convert the class intervals to exclusive form.*****)**

21. **From the information given below calculate Karl Pearson's co-efficient of Skewness and also quartile (Bowley's) co-efficient of Skewness.**

Measure	*Mean*	*Median*	*S.D.*	Q_3	Q_1
Place A	150	142	30	195	62
Place B	140	155	55	260	80

(Place A $Sk_P = 0 \cdot 8$; $Sk_B = -0 \cdot 203$)

Place B $Sk_P = -0 \cdot 82$; $Sk_B = 0 \cdot 167$)

22. The following facts are gathered from a firm before and after an industrial dispute :

	Before Dispute	*After Dispute*
Mean wages (Rs.)	185	190
Median wages (Rs.)	182	180
Modal wages (Rs.)	176	160
Quartiles (Rs.)	175 and 192	175 and 195
S.D. (Rs.)	13	19
Persons Employed (No.)	600	550

Compare the position of the firm before and after the dispute by making use of the given data.

(Hint : Compare mean, total wage bill, mode, C.V. and coefficient of skewness before and after the dispute.)

— • —

8

Correlation Analysis

Introduction. So far we have studied how to make analysis of data involving only one variable. In practice we come across many problems involving two or more variables which may be related to each other. For example, there exists relation between height and weight of persons, price and demand of any commodity, age of husband and that of wife, length and breadth of leaves, pressure and volume of gas, level of income and level of consumption etc. The degree and direction of relationship between all such variables can be measured with the help of correlation analysis.

Meaning of Correlation. Correlation means a possible connection, **association, relationship or interdependence between two or more sets of phenomenon. Two variables are said to be correlated if the change in one** variable is followed by a change in other variable also. It is said to be the amount of similarity, in direction and degree, of variations in corresponding pairs of observations of two variables. In the words of **Connor**, "*If two or more quantities vary in sympathy so that movement in the one tend to be accompanied by corresponding movements in the other (s) then they are said to be correlated.*"

Hence we can say that correlation is concerned with the association of two or more series and it attempts to determine the degree and direction of relationship between variables.

Types of Correlation

Positive and Negative Correlation. Correlation between two variables is said to be positive or direct if both of them move in the same direction *i.e.*, an increase in one variable is followed by on increase in other or a decrease in one variable is followed by a decrease in the other. For example, relation between rainfall and sale of raincoats, level of income and level of consumption, weight and height, price and supply of the commodity etc. Let us take a numerical example of positive relationship between price and supply of commodity :

Price (in Rs.)	100	150	200	250	220	210
Supply (in Quintals)	4	6	8	10	9	7

Similarly, correlation between two variables is said to be negative or inverse if they move in the opposite direction *i.e.*, an increase in one variable is followed by the decrease in the other variable or a

decrease in one variable is followed by the increase in other variable. For example, relation between price and demand, temperature and sale of woollen clothes, volume and pressure of a gas etc. show negative correlation. Let us take a numerical example :

Price (in Rs.)	100	110	120	130	125	122	100
Demand (in Quintals)	5	4	3	1	3	4	5

2. Linear and Non-linear Correlation. Correlation is said to be linear if the amount of change in one variable bears a constant ratio to the amount of change in the other variable. Let us consider the following data :

Price (Rs.)	100	120	140	160	180	200
Supply (Quintals)	2	4	6	8	10	12

Thus, for Rs. 20 change in price, there is a constant change to the extent of 2 quintals in supply. The ratio between price and supply is 10 : 1 and it is constant. Linear correlation is generally found in physical sciences.

The correlation between two variables is said to be non-linear if the amount of change in one variable does not bear a constant ratio to the amount of change in the other variable. In social and economic data we generally come across non-linear correlation.

Note. In this chapter we shall confine ourselves to the measurement of linear relationship only. The measurement of non-linear correlation is beyond the scope of this book.

3. Simple, Partial and Multiple Correlation. When correlation is studied between two variables only, then it is said to be simple correlation. When there are three or more variables, the degree and direction of relationship between them can be studied by either partial correlation or multiple correlation technique. In the case of partial correlation, the relationship is studied between two variables and all other variables are ignored. On the other hand, in the case of multiple correlation, the relationship is studied between one variable and the combined effect of all other variables. In this book we confine ourselves to the study of **simple correlation** only.

Significance of the study of Correlation. The study of correlation is very useful because of the following reasons :—

(*i*) Correlation helps us to measure the degree and direction of relationship between variables.

(*ii*) If we know that the variables are related, we can estimate the value of one variable with the given values of the other variable. It is **done by using regression analysis.**

(*iii*) Since the degree of relationship between variables is expressed by a coefficient which is a pure number, we can compare the degree of

association between different sets of variables expressed in different units.

(*iv*) The effect of correlation is to reduce the range of uncertainty. The prediction based on correlation analysis is likely to be more reliable and near to reality.

Correlation and Causation. Correlation does not establish any cause and effect relationship between the variables. Even a very high degree of correlation between two variables does not suggest that two variables are causally related, *i.e.*, variation in one variable is the cause of variation in the other variable. The measure of correlation is only a measure of covariation. A high degree of correlation between variables may exist because of any one, or a combination of the following reasons :

(*i*) **Correlation may be due to pure chance.** Sometimes we may get a very high degree of correlation between variables which, in general, are not at all related to each other. For example, we may get a high degree of correlation between number of pens in a student's pocket and number of his brothers. Such a correlation arises purely by chance. It is also called nonsense correlation or spurious correlation.

(*ii*) **Both the variables may be influenced by one or more other variables.** Two variables may show correlation not because of any direct relation between them but due to the relation of each variable to some third variable. For example, we may find a high degree of correlation between yield per hectare of rice and potato. From this we cannot say that the variables are related as cause and effect. The correlation may be due to a number of other factors like rainfall, use of fertilisers, degree of mechanisation etc. which influence both the variables.

(*iii*) **Both the variables may be mutually influencing each other.** A high degree of correlation between variables may be due to their mutual dependence on each other. In such cases it is very difficult to say which variable is the cause of variation in other. We normally come across such situations in business and economic data. To take an example, we know that as price of any commodity increases, its demand decreases. So in this case price is the cause and demand is the effect. But it is also possible that demand may increase due to factors like population explosion. This increased demand will have the effect of increasing price. So now demand is the cause and price is the effect. Therefore, even if we find a high degree of correlation between price and demand, we cannot say which variable is the cause and which variable is the effect because of their mutual dependence on each other.

From the above discussion we find that the presence of correlation between two variables is not a conclusive proof of causation. It simply indicates the existence of association between them.

Degree of Correlation. The degree of relationship between variables is generally expressed by a coefficient which is known as the coefficient of correlation (r). It is a pure number independent of units of

original data. The value of r ranges between ± 1. When it is equal to $+1$ there is perfect positive relation between the variables. When its value is -1 there is perfect negative relation between the variables. $r=0$ shows no correlation between the variables. In other words, it shows that the variables are independent.

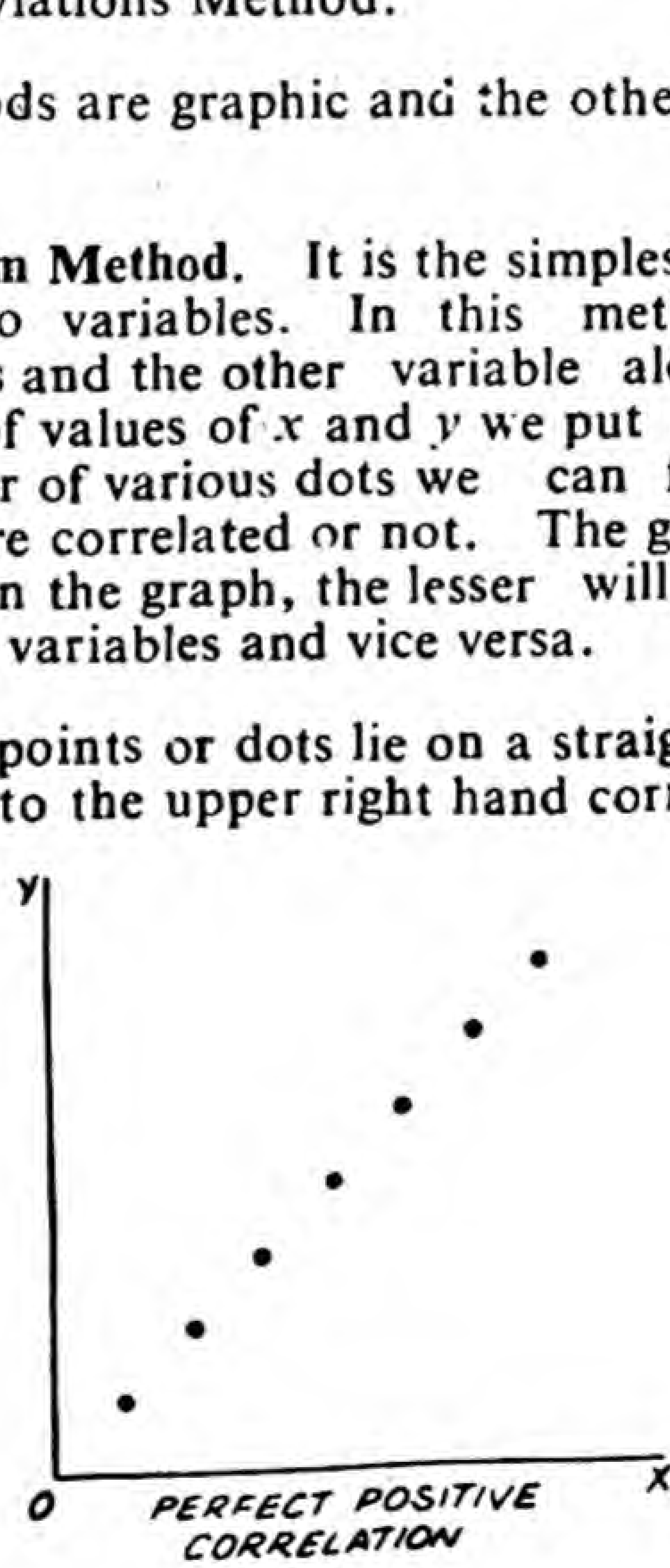
Methods of studying Correlation. The commonly used methods for studying simple linear correlation are :

1. Scatter Diagram Method
2. Graphic Method
3. Karl Pearson's Coefficient of Correlation.
4. Spearman's Rank Correlation Coefficient.
5. Concurrent Deviations Method.

The first two methods are graphic and the other three are algebraic in nature.

1. Scatter Diagram Method. It is the simplest method of studying relationship between two variables. In this method, one variable is represented along x-axis and the other variable along y-axis in an xy plane. For each pair of values of x and y we put a dot in the plane. By looking at the scatter of various dots we can form an idea as to whether the variables are correlated or not. The greater the scatter of various dots or points in the graph, the lesser will be the degree of correlation between the variables and vice versa.

Case I. If all the points or dots lie on a straight line rising from lower left hand corner to the upper right hand corner, as shown in the



PERFECT POSITIVE CORRELATION

figure, then there exists a perfect positive correlation between the variables. The coefficient of correlation is equal to $+1$.

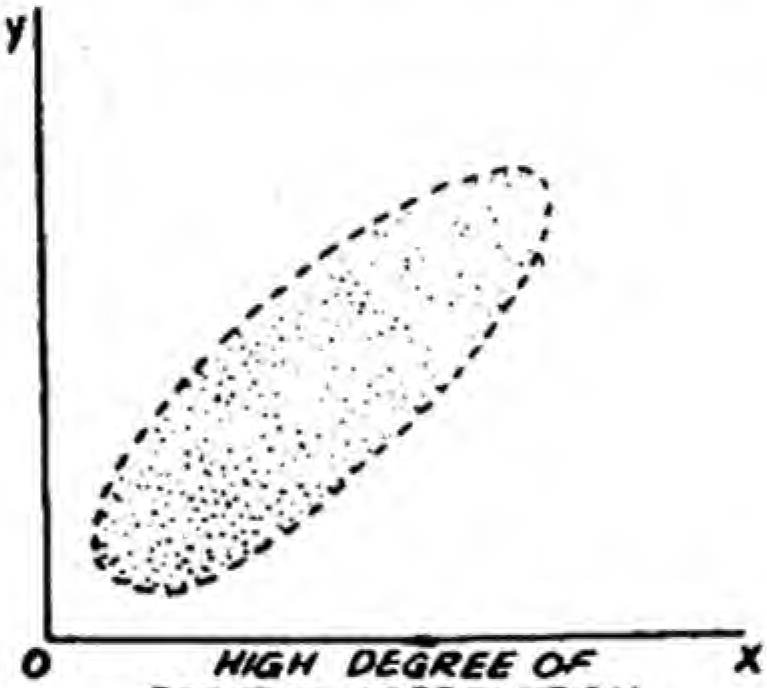
Case II. If all the points lie on a straight line falling from upper left hand corner to the lower right hand corner, as shown in the figure,

then the correlation is said to be perfectly negative. The coefficient of

PERFECT NEGATIVE CORRELATION

correlation in this case is equal to —1.

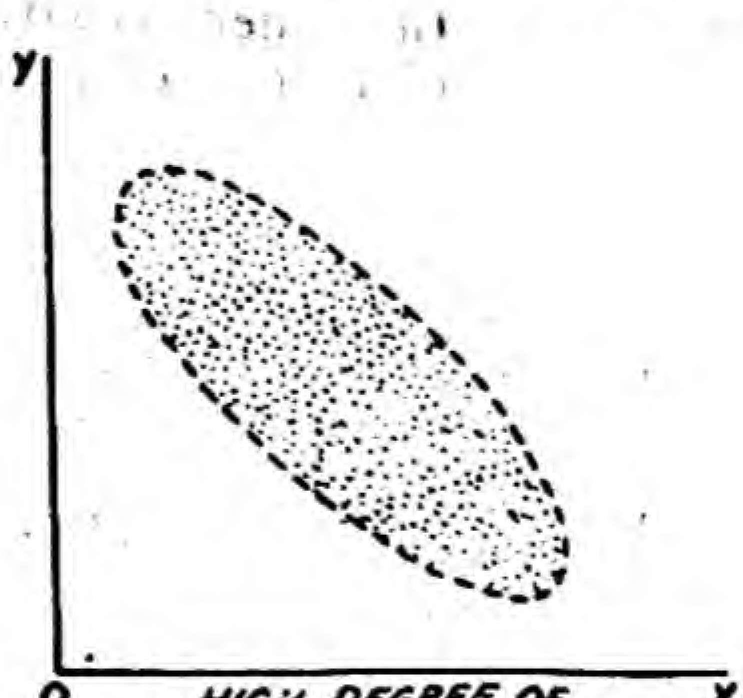
Case III. If the plotted points fall in a narrow band rising from



HIGH DEGREE OF POSITIVE CORRELATION

lower left hand corner to upper right hand corner, as shown in the figure, then there is said to exist a high degree of positive correlation.

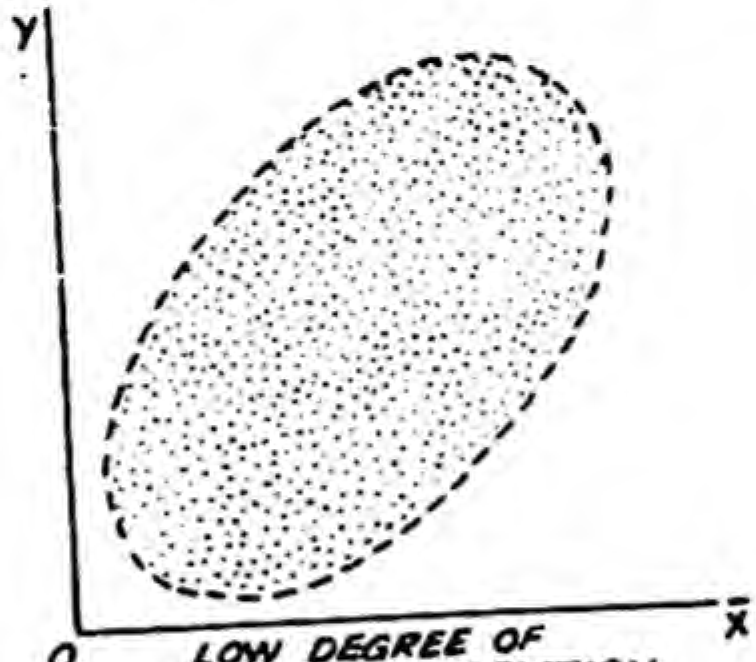
Case IV. If the plotted points fall in a narrow band falling from



HIGH DEGREE OF NEGATIVE CORRELATION

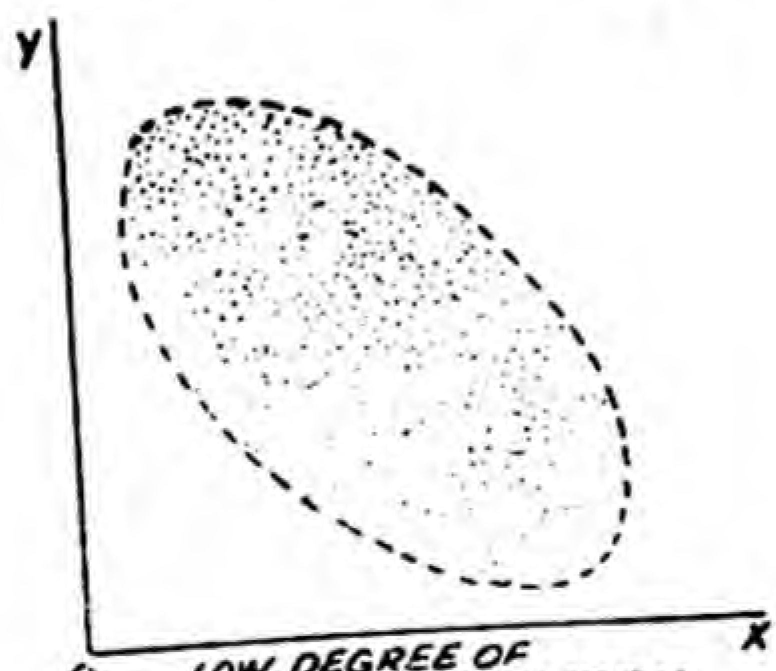
upper left hand corner to lower right hand corner, as shown in the figure, then there is said to exist a high degree of negative correlation.

Case V. If the plotted points fall in a wider band rising from lower left hand corner to the upper right hand corner, as shown in the figure, then there is said to exist a low degree of positive correlation.



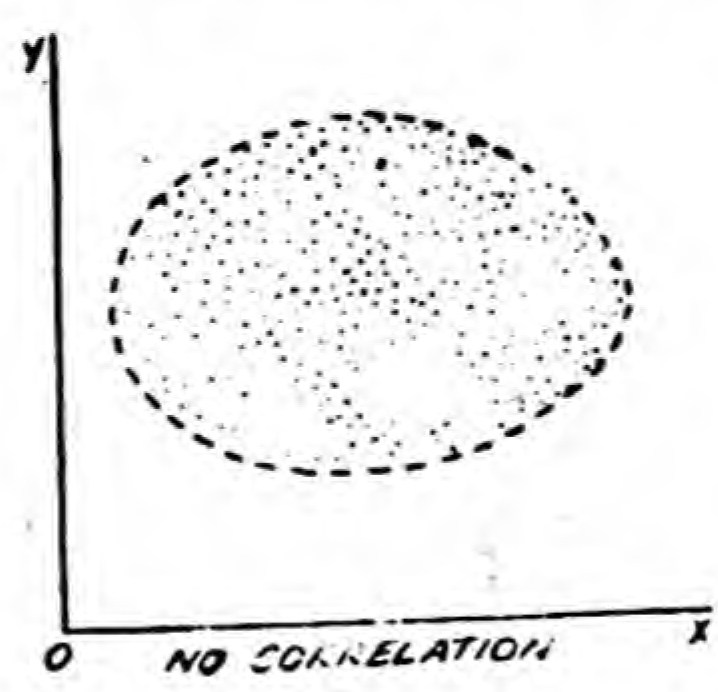
LOW DEGREE OF POSITIVE CORRELATION

Case VI. If the plotted points fall in a wider band falling from upper left hand corner to the lower right hand corner, as shown in figure, then there is said to exist a low degree negative correlation.



LOW DEGREE OF NEGATIVE CORRELATION

Case VII. If the plotted points are scattered in an haphazard manner, as shown in the figure, then there is said to exist no correlation between the variables. The coefficient of correlation is equal to zero.



NO CORRELATION

Merits and Demerits of Scatter Diagram Method

Merits.

(*i*) It is the simplest method of studying correlation.

(*ii*) It is easier to understand as compared to complex mathematical methods of calculating the coefficient of correlation.

(*iii*) It is not affected by the presence of extreme items in the data.

(*iv*) It is useful in detecting abnormal points, if any, in the data.

Demerits. This method indicates only the direction of relationship between the variables *i.e.*, whether it is positive or negative or correlation is high or low. This method does not enable us to determine the exact degree of relationship between the variable. For this purpose the mathematical methods are generally used.

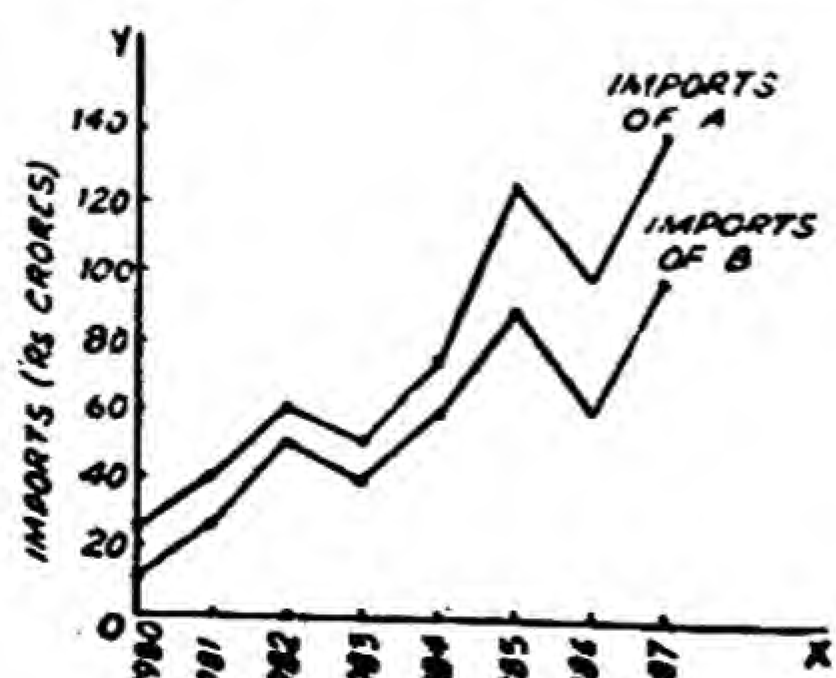
The scatter diagram method is generally used when we intend to have a quick and rough idea about the nature of relationship between two variables.

2. Graphic Method. In this method the individual values of the two variables are plotted on a graph paper. So we shall get two curves-one for variable x and the other for variable y. From a careful study of direction and closeness of the two curves we can know about the presence or absence of correlation between the two variables. If both the curves move in the same direction, then correlation is positive. On the other hand,if the curves move in the opposite direction, then correlation is negative. If the two curves are very near to each other, then the degree of correlation is very high. If the curves are at a distance, then there is a low degree of correlation between the variables. Let us consider the following example.

Example 1. The following data relate to the imports of two commodities A and B during 1980—1987. Find graphically whether imports of two commodities are related.

Year	1980	1981	1982	1983	1984	1985	1986	1987
Imports of A (Rs. Crores)	25	40	60	50	75	125	100	140
Imports of B (Rs. Crores)	10	25	50	40	60	90	60	100

Sol. Graph Showing Imports of Commodities A and B.



The graph shows that both the curves are moving in the same direction and they are very near to each other. Thus we find a very high degree of positive correlation between imports of commodites A and B.

Remarks. This method is generally used when data is given over a period of time. Like the scatter diagram method, this method also gives the direction of relationship between variables. It does not give us a numerical value of extent of relationship between variables.

3. **Karl Pearson's Coefficient of Correlation.** It is a mathematical method for measuring the magnitude of linear relationship between two variables. It was suggested by Karl Plearson, a great Biometrician and Statistician, and is the widely used method of calculating the degree of correlation between two variables. The formula for calculating correlation coefficient (r) is as follows :—

$$r=\frac{\Sigma(x-\bar{x})(y-\bar{y})}{n\,\sigma_x\,\sigma_y}\quad\ldots\qquad\ldots(\mathrm{I})$$

where $\Sigma(x-\bar{x})(y-\bar{y})$ is the sum of the product of deviations of x and y series from their respective means. n, σ_x and σ_y are respectively the number of pairs of items, standard deviation of x series and the standard deviation of y series. r is the Pearson's coefficient of correlation. It is a pure number free from units of the data. The value of 'r' lies between ± 1. The above formula can also be written as

$$r=\frac{\text{covariance}\,(x,y)}{\sigma_x\,.\,\sigma_y}\quad\ldots\ldots\qquad\ldots(\mathrm{II})$$

where covariance $(x, y)=\dfrac{\Sigma(x-\bar{x})(y-\bar{y})}{n}$,

The formula I can be simplified as follows

$$r=\frac{\Sigma(x-\bar{x})(y-\bar{y})}{n.\sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}\sqrt{\frac{\Sigma(y-\bar{y})^2}{n}}}\qquad \because\ \sigma_x=\sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$$

$$\text{or}\qquad r=\frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2\,\Sigma(y-\bar{y})^2}}\rightarrow(\mathrm{III})\qquad \because\ \sigma_y=\sqrt{\frac{\Sigma(y-\bar{y})^2}{n}}$$

For practical purposes formula III is more useful.

Note. It should be noted that the above formulae can be used only when actual means $\bar{x}$ and $\bar{y}$ are integers. If $\bar{x}$ and $\bar{y}$ are in fractions, then it becomes very difficult to calculate $\Sigma(x-\bar{x})^2$, $\Sigma(y-\bar{y})^2$ and $\Sigma(x-\bar{x})(y-\bar{y})$. In this case short cut methods are preferred.

Example 2. Calculate Karl Pearson's coefficient of correlation from the following data.

x :	10	12	15	14	18	20	16
y :	20	22	25	21	24	27	15

Sol.

x	y	$x-\bar{x}$ $x-15$	$y-\bar{y}$ $y-22$	$(x-\bar{x})^2$	$(y-\bar{y})^2$	$(x-\bar{x})(y-\bar{y})$
10	20	-5	-2	25	4	10
12	22	-3	0	9	0	0
15	25	0	3	0	9	0
14	21	-1	-1	1	1	1
18	24	3	2	9	4	6
20	27	5	5	25	25	25
16	15	1	-7	1	49	-7
$\Sigma x=$ 105	$\Sigma y=$ 154			$\Sigma(x-\bar{x})^2$ $=70$	$\Sigma(y-\bar{y})^2$ $=92$	$\Sigma(x-\bar{x}).$ $(y-\bar{y})=35$

$$\bar{x}=\frac{\Sigma x}{n}=\frac{105}{7}=15 \quad ; \qquad \bar{y}=\frac{\Sigma y}{n}=\frac{154}{7}=22.$$

$$\sigma_x=\sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} \qquad \sigma_y=\sqrt{\frac{\Sigma(y-\bar{y})^2}{n}}=\sqrt{\frac{92}{7}}$$

$$=\sqrt{\frac{70}{7}}$$

$$=\sqrt{10} \qquad =\sqrt{13\cdot14}$$

$$r_{xy}=\frac{\Sigma(x-\bar{x})(y-\bar{y})}{n.\,\sigma_x.\,\sigma_y} \qquad =\frac{35}{7(\sqrt{10})(\sqrt{13\cdot14})}$$

$$=\frac{5}{(3\cdot162)(3\cdot625)} \qquad =\frac{5}{11\cdot462}$$

$$=0\cdot436$$

Now let us solve the same numerical by using

$$r_{xy}=\frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2\cdot\Sigma(y-\bar{y})^2}}$$

$$=\frac{35}{\sqrt{70\times92}} \qquad =\frac{35}{\sqrt{6440}}$$

$$=\frac{35}{80\cdot249}=0\cdot436.$$

(ii)

So we find the same value of coefficient of correlation by using any formula.

Example 3. The following results are obtained between two series from their respective means. Compute the coefficient of correlation.

	x-Series	y-Series
Number of items	7	7
Sum of squares of deviations from mean	28	76

Sum of products of deviations of x and y series from their respective means $=46$.

Sol. Given $n=7$, $\Sigma(x-\bar{x})^2=28$

$\Sigma(y-\bar{y})^2=76$; $\Sigma(x-\bar{x})(y-\bar{y})=46$.

We know that

$$r=\frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2.\Sigma(y-\bar{y})^2}}$$

Substituting the values, we have

$$r=\frac{46}{\sqrt{28\times76}}=\frac{46}{\sqrt{2128}}$$

$$=\frac{46}{46\cdot13}=0\cdot997.$$

Short Cut Method. In this method deviations are not taken from actual means but they are taken from assumed means. This method is generally followed when actual means are in fractions. If A is the assumed mean for x series and B is the assumed mean for y series, then the coefficient of correlation is given by

$$r_{xy}=\frac{\frac{\Sigma\, dx\, dy}{n}-\left(\frac{\Sigma dx}{n}\right)\left(\frac{\Sigma dy}{n}\right)}{\sqrt{\frac{\Sigma dx^2}{n}-\left(\frac{\Sigma dx}{n}\right)^2}\ \sqrt{\frac{\Sigma dy^2}{n}-\left(\frac{\Sigma dy}{n}\right)^2}}$$

Where $dx=x-\text{A}$; $dy=y-\text{B}$. The above formula, on simplification, can also be written in the following two forms :

(*i*) $$r_{xy}=\frac{\Sigma dxdy-\frac{(\Sigma dx)(\Sigma dy)}{n}}{\sqrt{\Sigma dx^2-\left(\frac{\Sigma dx}{n}\right)^2}\ \sqrt{\Sigma dy^2-\left(\frac{\Sigma dy}{n}\right)^2}}$$

(*ii*) $$r_{xy}=\frac{n\,\Sigma dxdy-(\Sigma dx)(\Sigma dy)}{\sqrt{n\,\Sigma dx^2-(\Sigma dx)^2}\ \sqrt{n\Sigma dy^2-(\Sigma dy)^2}}$$

All these formulae give the same value of the coefficient of correlation.

Example 4. Calculate the coefficient of correlation between the marks obtained by 10 students in economics and statistics.

Marks in Economics : 49, 59, 65, 45, 52, 60, 70, 62, 56, 49

Marks in Statistics : 61, 75, 70, 55, 62, 60, 80, 69, 65, 61.

Sol.

Marks in Economics x	*Marks in Statistics* y	$A=60$ $dx=x-A$	$B=70$ $dy=y-B$	dx^2	dy^2	$dxdy$
49	61	−11	−9	121	81	99
59	75	−1	5	1	25	−5
65	70	5	0	25	0	0
45	55	−15	−15	225	225	225
52	62	−8	−8	64	64	64
60	60	0	−10	0	100	0
70	80	10	10	100	100	100
62	69	2	−1	4	1	−2
56	65	−4	−5	16	25	20
49	61	−11	−9	121	81	99
		$\Sigma dx=-33$	$\Sigma dy=-42$	$\Sigma dx^2=677$	$\Sigma dy^2=702$	$\Sigma dxdy=600$

$$r_{xy}=\frac{\frac{\Sigma dxdy}{n}-\left(\frac{\Sigma dx}{n}\right)\left(\frac{\Sigma dy}{n}\right)}{\sqrt{\frac{\Sigma dx^2}{n}-\left(\frac{\Sigma dx}{n}\right)^2}\sqrt{\frac{\Sigma dy^2}{n}-\left(\frac{\Sigma dy}{n}\right)^2}}$$

$$r_{xy}=\frac{\frac{600}{10}-\left(\frac{-33}{10}\right)\left(\frac{-42}{10}\right)}{\sqrt{\frac{677}{10}-\left(\frac{-33}{10}\right)^2}\sqrt{\frac{702}{10}-\left(\frac{-42}{10}\right)^2}}$$

$$=\frac{60-13\cdot86}{\sqrt{67\cdot7-10\cdot89}\sqrt{70\cdot2-17\cdot64}}=\frac{46\cdot14}{\sqrt{56\cdot81}\quad\sqrt{52\cdot56}}$$

$$=\frac{46\cdot14}{7\cdot5372\times7\cdot2498}=\frac{46\cdot14}{54\cdot643}=0\cdot844$$

Example 5. Calculate the coefficient of correlation for x and y series using short cut method.

x :	10	12	8	14	9	6	10	11
y :	8	7	5	12	10	13	15	4

33×42   42×33

Sol.

x	y	$A=10$ $dx=x-A$	$B=10$ $dy=y-B$	dx^2	dy^2	$dxdy$
10	8	0	−2	0	4	0
12	7	2	−3	4	9	−6
8	5	−2	−5	4	25	10
14	12	4	2	16	4	8
9	10	−1	0	1	0	0
6	13	−4	3	16	9	−12
10	15	0	5	0	25	0
11	4	1	−6	1	36	−6
		$\Sigma dx=0$	$\Sigma dy=-6$	$\Sigma dx^2=42$	$\Sigma dy^2=112$	$\Sigma dxdy=-6$

$$r_{xy}=\frac{n\Sigma dxdy-(\Sigma dx)(\Sigma dy)}{\sqrt{n\Sigma dx^2-(\Sigma dx)^2}\ \sqrt{n\Sigma dy^2-(\Sigma dy)^2}}$$

$$\text{or } r_{xy}=\frac{8(-6)-0(-6)}{\sqrt{8(42)-0}\ \sqrt{8(112)-(-6)^2}}$$

$$=\frac{-48}{\sqrt{336}\ \sqrt{896-36}}=\frac{-48}{\sqrt{336}\ \sqrt{860}}$$

$$=\frac{-48}{18\cdot33\times29\cdot326}=\frac{-48}{537\cdot54}=-0\cdot089.$$

Calculation of correlation coefficient by using actual data :

If the given observations are small which can be easily squared, then we donot take deviations. Instead we use actual data and apply the formula

$$r_{xy}=\frac{\frac{\Sigma xy}{n}-\left(\frac{\Sigma x}{n}\right)\left(\frac{\Sigma y}{n}\right)}{\sqrt{\frac{\Sigma x^2}{n}-\left(\frac{\Sigma x}{n}\right)^2}\ \sqrt{\frac{\Sigma y^2}{n}-\left(\frac{\Sigma y}{n}\right)^2}}$$

$$\text{or } r_{xy}=\frac{\Sigma xy-\frac{(\Sigma x)(\Sigma y)}{n}}{\sqrt{\Sigma x^2-\frac{(\Sigma x)^2}{n}}\ \sqrt{\Sigma y^2-\frac{(\Sigma y)^2}{n}}}$$

$$\text{or } r_{xy}=\frac{n\Sigma xy-(\Sigma x)(\Sigma y)}{\sqrt{n\Sigma x^2-(\Sigma x)^2}\ \sqrt{n\Sigma y^2-(\Sigma y)^2}}$$

All these formulae give the same value of the coefficient of correlation.

Example 6. Find the cofficient of correlation for the data given in example 5. You need not take devations either from actual means or assumed means.

Sol.

x	y	x^2	y^2	xy
10	8	100	64	80
12	7	144	49	84
8	5	64	25	40
14	12	196	144	168
9	10	81	100	90
6	13	36	169	78
10	15	100	225	150
11	4	121	16	44
$\Sigma x=80$	$\Sigma y=74$	$\Sigma x^2=842$	$\Sigma y^2=792$	$\Sigma xy=734$

Let us use the formula $r_{xy}=\dfrac{\Sigma xy-\dfrac{(\Sigma x)(\Sigma y)}{n}}{\sqrt{\Sigma x^2-\dfrac{(\Sigma x)^2}{n}}\sqrt{\Sigma y^2-\dfrac{(\Sigma y)^2}{n}}}$

$$\text{or } r_{xy}=\frac{734-\dfrac{80\times 74}{8}}{\sqrt{842-\dfrac{(80)^2}{8}}\sqrt{792-\dfrac{(74)^2}{8}}}$$

$$=\frac{734-740}{\sqrt{842-800}\,.\,\sqrt{792-684\cdot 5}}=\frac{-6}{\sqrt{42}\,.\,\sqrt{107\cdot 5}}$$

$$=\frac{-6}{6\cdot 48\times 10\cdot 368}=\frac{-6}{67\cdot 168}=-0\cdot 089. \qquad \text{(Same answer).}$$

Example 7. The coefficient of correlation between two variables x and y is 0·48. Their covariance is 36. The variance of x is 16. Find the standard deviation of y.

Sol.

We know that $r_{xy}=\dfrac{\text{cov.}(x,y)}{\sigma_x \cdot \sigma_y}$

where cov. $(x,y)=\dfrac{\Sigma(x-\bar{x})(y-\bar{y})}{n}=36$ (Given)

Also $\sigma_x=\sqrt{\text{Variance}}=\sqrt{16}=4$; $r_{xy}=0\cdot 48$.

Putting these values in the above formula

$$0\cdot 48=\frac{36}{4\cdot\sigma_y}$$

or $1\cdot 92\sigma_y=36$ or $\sigma_y=\dfrac{36}{1\cdot 92}=18\cdot 75.$

Example 8. The following table gives the distribution of the total population and those who are wholly or partially blind among them. Find out if there is any relation between age and blindness.

Age	*No. of Persons (in thousand)*	*Blind*	*Age*	*No. of Persons (in thousand)*	*Blind*
0—10	100	55	40—50	24	36
10—20	60	40	50—60	11	22
20—30	40	40	60—70	6	18
30—40	36	40	70—80	3	15

Sol.

In order to make comparison possible, let us determine the number of blinds in terms of a common denominator, say, one lakh. The first figure remains the same i.e. 55. The second figure becomes :—

Out of 60,000 no. of blinds$=40$

Out of 1,00,000 no.of blinds$=\frac{40}{60000}\times 1,00,000=67$ approx.

Similarly, for next figure :—

Out of 40000, no. of blind$=40$

Out of 1,00,000, no. of blinds$=\frac{40}{40,000}\times 1,00,000$

$=100.$

and so on.

Age	*No. of Blind out of 1 lakh y*	*Mid. values x*	*A=35 dx=x—A*	*B=100 dy=y—B*	*dx²*	*dy²*	*dxdy*
0—10	55	5	—30	—45	900	2025	1350
10—20	67	15	—20	—33	400	1089	660
20—30	100	25	—10	0	100	0	0
30—40	111	35	0	11	0	121	0
40—50	150	45	10	50	100	2500	500
50—60	200	55	20	100	400	10000	2000
60—70	300	65	30	200	900	40000	6000
70—80	500	75	40	400	1600	160000	16000
			$\Sigma dx=40$	$\Sigma dy=683$	$\Sigma dx^2=4400$	$\Sigma dy^2=215735$	$\Sigma dxdy=26510$

$$r_{xy}=\frac{n\Sigma dxdy-(\Sigma dx)(\Sigma dy)}{\sqrt{n\Sigma dx^2-(\Sigma dx)^2}\sqrt{n\Sigma dy^2-(\Sigma dy)^2}}$$

$$=\frac{8\times 26510-40\times 683}{\sqrt{8\times 4400-(40)^2}\sqrt{8\times 215735-(683)^2}}$$

$$=\frac{212080-27320}{\sqrt{35200-1600}\sqrt{1725880-466489}}$$

$$=\frac{184760}{\sqrt{33600}\sqrt{1259391}}=\frac{184760}{183\cdot 3\times 1122\cdot 2}$$

$$=\frac{184760}{205699\cdot 2}=0\cdot 898$$

Example 9. You are given the following information relating to a frequency distribution of 10 observations.

$\bar{x}=5\cdot 5$; $\bar{y}=4\cdot 0$; $\Sigma x^2=385$; $\Sigma y^2=192$;

$\Sigma(x+y)^2=947$. Find the coeff. of correlation.

Sol. Given $\Sigma(x+y)^2=947$

or $\Sigma(x^2+y^2+2xy)=947$. Taking Σ inside,

we have

$$\Sigma x^2+\Sigma y^2+2\Sigma xy=947$$

Putting the values,

we have $385+192+2\Sigma xy=947$.

or $2\Sigma xy=947-385-192$

or $2\Sigma xy=370$ or $\Sigma xy=185$

Now,
$$r_{xy}=\frac{\frac{\Sigma xy}{n}-\left(\frac{\Sigma x}{n}\right)\left(\frac{\Sigma y}{n}\right)}{\sqrt{\frac{\Sigma x^2}{n}-\left(\frac{\Sigma x}{n}\right)^2}\sqrt{\frac{\Sigma y^2}{n}-\left(\frac{\Sigma y}{n}\right)^2}}$$

or
$$r_{xy}=\frac{\frac{\Sigma xy}{n}-(\bar{x})(\bar{y})}{\sqrt{\frac{\Sigma x^2}{n}-(\bar{x})^2}\sqrt{\frac{\Sigma y^2}{n}-(\bar{y})^2}}$$

$\because \quad \bar{x}=\frac{\Sigma x}{n}$

$\bar{y}=\frac{\Sigma y}{n}$

Putting these values

$$r_{xy}=\frac{\frac{185}{10}-5\cdot 5\times 4}{\sqrt{\frac{385}{10}-(5\cdot 5)^2}\sqrt{\frac{192}{10}-(4)^2}}=\frac{18\cdot 5-22}{\sqrt{38\cdot 5-30\cdot 25}\sqrt{19\cdot 2-16}}$$

$$=\frac{-3\cdot 5}{\sqrt{8\cdot 25}-\sqrt{3\cdot 2}}=\frac{-3\cdot 5}{2\cdot 87\times 1\cdot 79}$$

$$=\frac{-3\cdot 5}{5\cdot 137}=-0\cdot 681$$

Correction of Incorrect Value of Correlation Coefficient.

Karl Pearson's correlation coefficient can be corrected by making some modifications. We find the correct values of Σx, Σy, Σx^2, Σy^2, Σxy and n and apply the formula.

$$r_{xy}=\frac{\frac{\Sigma xy}{n}-\left(\frac{\Sigma x}{n}\right)\left(\frac{\Sigma y}{n}\right)}{\sqrt{\frac{\Sigma x^2}{n}-\left(\frac{\Sigma x}{n}\right)^2}\sqrt{\frac{\Sigma y^2}{n}-\left(\frac{\Sigma y}{n}\right)^2}}$$

Example 10. A computer while calculating the correlation coefficient between two variables x and y obtained the following constants :

$$n=30,\ \Sigma x=120,\ \Sigma x^2=600,\ \Sigma y=90,\ \Sigma y^2=250,\ \Sigma xy=356.$$

It was, however, later discovered that it had wrongly copied down a pair of observations $x=10$ and $y=12$ as $x=8$ and $y=10$. Find the correct correlation coefficient.

Sol. Let us find correct value of Σx, Σy, Σx^2, Σy^2, Σxy and n.

wrong value of $x=8$

wrong value of $y=10$

correct value of $x=10$

correct value of $y=12$

Correct $\Sigma x=120-8+10=122$

Correct $\Sigma y=90-10+12=92$

Correct $\Sigma x^2=600-(8)^2+(10)^2=600-64+100=636$

Correct $\Sigma y^2=250-(10)^2+(12)^2=250-100+144=294$

Correct $\Sigma xy=356-8\times 10+10\times 12=356-80+120=396$

Correct $n=30-1+1=30$

Applying formula.

$$r_{xy}=\frac{\frac{\Sigma xy}{n}-\left(\frac{\Sigma x}{n}\right)\left(\frac{\Sigma y}{n}\right)}{\sqrt{\frac{\Sigma x^2}{n}-\left(\frac{\Sigma x}{n}\right)^2}\sqrt{\frac{\Sigma y^2}{n}-\left(\frac{\Sigma y}{n}\right)^2}}$$

$$=\frac{\frac{396}{30}-\left(\frac{122}{30}\right)\left(\frac{92}{30}\right)}{\sqrt{\frac{636}{30}-\left(\frac{122}{30}\right)^2}\sqrt{\frac{294}{30}-\left(\frac{92}{30}\right)^2}}$$

$$=\frac{13\cdot 2-12\cdot 47}{\sqrt{21\cdot 2-16\cdot 53}\sqrt{9\cdot 8-9\cdot 4}}=\frac{0\cdot 73}{\sqrt{4\cdot 67}\sqrt{0\cdot 4}}$$

$$=\frac{0\cdot 73}{2\cdot 16\times 0\cdot 63}=\frac{0\cdot 73}{1\cdot 3356}=0\cdot 546$$

Correlation Coefficient for Grouped Data.

When the number of items of two variables X and Y are considerably large, the data can be classified into a two-way frequency distribution called a correlation table. After the values of the two variables are grouped into various classes (not necessarily the same for both the variables), Karl Pearson's coefficient of correlation can be calculated. The formula used can be either

$$r_{xy}=\frac{\Sigma fdxdy-\frac{(\Sigma fdx)(\Sigma fdy)}{\Sigma f}}{\sqrt{\Sigma fdx^2-\frac{(\Sigma fdx)^2}{\Sigma f}}\sqrt{\Sigma fdy^2-\frac{(\Sigma fdy)^2}{\Sigma f}}}$$

where $dx=X-A$ and $dy=Y-B$.

A, B are assumed means of X and Y series respectively. The values of A and B nead not be equal.

Or

$$r_{xy}=\frac{\Sigma fd'xd'y-\frac{(\Sigma fd'x)(\Sigma fd'y)}{\Sigma f}}{\sqrt{\Sigma fd'x^2-\frac{(\Sigma fd'x)^2}{\Sigma f}}\sqrt{\Sigma fd'y^2-\frac{(\Sigma fd'y)^2}{\Sigma f}}}$$

where $d'x=\frac{dx}{c}$ and $d'y=\frac{dy}{h}$.

c, h being common factors used in X and Y series respectively. The values of c and h need not be equal.

Note. The coefficient of correlation is not affected by the choice of the values of A, B, C and h It is due to the fact that the Karl Pearson's correlation coefficient is independent of both origin and scale.

Example 11. Calculate coefficient of correlation from the following data relating to marks in economics and statistics.

Marks in Economics

Marks in Statistics	10	20	30	Total
5	2	6	4	12
10	3	5	7	15
15	—	2	—	2
20	1	4	—	5
25	8	7	1	16
Total	14	24	12	50

Solution. Let us denote marks in economics by x and marks in statistics by y.

Calculation of Coefficient of Correlation

$fd'y^2$	$d'y^2$	$fd'y$	$k=5$ $d'y=\frac{dy}{k}$	$B=15$ $dy=y-B$	y \ x	10	20	30	Total
48	4	−24	−2	−10	5	2 $\boxed{4}$	6 $\boxed{0}$	4 $\boxed{-8}$	12
15	1	−15	−1	−5	10	3 $\boxed{3}$	5 $\boxed{0}$	7 $\boxed{-7}$	15
0	0	0	0	0	15	— $\boxed{0}$	2 $\boxed{0}$	—	2
5	1	5	1	5	20	1 $\boxed{-1}$	4 $\boxed{0}$	—	5
64	4	32	2	10	25	8 $\boxed{-16}$	7 $\boxed{0}$	1 $\boxed{2}$	16
$\Sigma fd'y^2 = 132$		$\Sigma fd'y = -2$			Total	14	24	12	50
					$A=20$ $dx=x-A$ $=x-20$	−10	0	10	
					$C=10$ $d'x=\frac{dx}{c}$	−1	0	1	
					$f.d'x$	−14	0	12	$\Sigma fd'x = -2$
					$d'x^2$	1	0	1	
					$f.d'x^2$	14	0	12	$\Sigma fd'x^2 = 26$
					$d'y.f.d'x$	−10	0	−13	$\Sigma fd'xd'y = -23$

$\Sigma fd'xd'y$

$= \Sigma d'y \,.\, f \,.\, d'x$
$= -23.$

Applying the formula

$$r_{xy}=\frac{\Sigma fd'xd'y-\frac{(\Sigma fd'x)(\Sigma fd'y)}{\Sigma f}}{\sqrt{\Sigma fd'x^2-\left(\frac{\Sigma fd'x}{\Sigma f}\right)^2}\sqrt{\Sigma fd'y^2-\left(\frac{\Sigma fd'y}{\Sigma f}\right)^2}}$$

$$=\frac{-23-\frac{(-2)(-2)}{50}}{\sqrt{26-\left(\frac{-2}{50}\right)^2}\sqrt{132-\left(\frac{-2}{50}\right)^2}}$$

$$=\frac{-23-0{\cdot}08}{\sqrt{26-0{\cdot}0016}\sqrt{132-0{\cdot}0016}}$$

$$=\frac{-23{\cdot}08}{\sqrt{25{\cdot}9984}\sqrt{131{\cdot}9984}}$$

$$=\frac{-23{\cdot}08}{5{\cdot}099\times 11{\cdot}489}=\frac{-23{\cdot}08}{58{\cdot}58}$$

$$=-0{\cdot}394$$

Example 12. Calculate Karl Pearson's coefficient of correlation from the following 2×2 table relating to marks of 25 students in two subjects Hindi and Statistics.

Marks in Hindi	Marks in Statistics			
	30—40	40—50	50—60	60—70
30—40	3	1	1	—
40—50	2	6	1	2
50—60	1	2	2	1
60—70	—	1	1	1

Solution. **Calculation of Karl Pearson's Correlation Coefficient.**

$fd'y^2$	$fd'y$	$d'y^2$	$h=10$ dy	$B=45$ dy	M.V. (y)	Marks in Statistics / Marks in Hindi	30—40	40—50	50—60	60—70	Total (f)
5	−5	1	−1	−10	35	30—40	3 $\boxed{3}$	1 $\boxed{0}$	1 $\boxed{-1}$	— $\boxed{-}$	5
0	0	0	0	0	45	40—50	2 $\boxed{0}$	6 $\boxed{0}$	1 $\boxed{0}$	2 $\boxed{0}$	11
6	6	1	1	10	55	50—60	1 $\boxed{-1}$	2 $\boxed{0}$	2 $\boxed{2}$	1 $\boxed{2}$	6
12	6	4	2	20	65	60—70	— $\boxed{-}$	1 $\boxed{0}$	1 $\boxed{2}$	1 $\boxed{4}$	3
$\Sigma fd'y^2 = 23$	$\Sigma fd'y = 7$					Total (f)	6	10	5	4	25
						M.V. (x)	35	45	55	65	
						$A=45$ $dx=x-A$	−10	0	10	20	
						$C=10$ $d'x$	−1	0	1	2	
						$d'x^2$	1	0	1	4	
						$fd'x$	−6	0	5	8	$\Sigma fd'x=7$
						$fd'x^2$	6	0	5	16	$\Sigma fd'x^2=27$

$\Sigma fd'xd'y$
$=3-1-1+2+2+2+4$
$=11$

Putting the values, we get

$$r_{xy}=\frac{\Sigma fd'xd'y-\dfrac{(\Sigma fd'x)(\Sigma fd'y)}{\Sigma f}}{\sqrt{\Sigma fd'x^2-\dfrac{(\Sigma fd'x)^2}{\Sigma f}}\sqrt{\Sigma fd'y^2-\dfrac{(\Sigma fd'y)^2}{\Sigma f}}}$$

$$=\frac{11-\dfrac{7\times 7}{25}}{\sqrt{27-\dfrac{(7)^2}{25}}\sqrt{23-\dfrac{(7)^2}{25}}}=\frac{11-1{\cdot}96}{\sqrt{27-1{\cdot}96}\sqrt{23-1{\cdot}96}}$$

$$=\frac{9{\cdot}04}{\sqrt{25{\cdot}04}\sqrt{21{\cdot}04}}=\frac{9{\cdot}04}{(5{\cdot}0)(4{\cdot}5869)}=\frac{9{\cdot}04}{22{\cdot}934}$$

$$=0{\cdot}394.$$

Assumptions of Karl Pearson's Coefficient of Correlation

(*i*) The relationship between x and y is linear. In other words, when plotted on a graph paper, the values of x and y tend to scatter along a straight line rather than along a curve.

(*ii*) The distributions of both x and y variables are similar in shape.

(*iii*) A large number of independent causes operate in each of two series to produce normal distribution.

(*iv*) Forces operating on each of the variable series are not independent of each other but are related in a causal fashion.

Properties of Karl Pearson's Coefficient of Correlation.

(*i*) It is a pure number independent of units of measurement.

(*ii*) The value of r lies between -1 and $+1$. In other words,

$$-1\leqslant r\leqslant 1$$

(*iii*) The coefficient of correlation is independent of origin as well as scale.

(*iv*) If two variables are related by a linear equation $y=a+bx$, then the coefficient of correlation is equal to $+1$.

(*v*) The coefficient of correlation is symmetrical in the sense that the value of r between x and y is same as the value of r between y and x. In other words, $r_{xy}=r_{yx}$.

(*vi*) The coefficient of correlation is the geometric mean of the two regression coefficients b_{xy} and b_{yx}. In other words,

$$r=\sqrt{b_{xy}\cdot b_{yx}}\ .$$

Merits and Demerits of Karl Pearson's Coefficient of Correlation.

Merits

(*i*) It is the most popular and reliable method of finding degree and direction of relationship between two variables.

(*ii*) It possesses many useful properties.

(*iii*) It is based on all the values.

(*iv*) It is capable of further algebraic treatment.

(*v*) It can be used for making comparisons because it is a pure number which is independent of the units of original data.

Demerits

(*i*) It assumes a linear relationship between the variables.

(*ii*) The value of r is unduly affected by extreme items.

(*iii*) When compared with other methods of measuring correlation, Karl Pearson's method is more time consuming and involves lengthy calculations.

Interpretation of Coefficient of Correlation. The following general rules can be used to interpret the coefficient of correlation :

(*i*) When $r=+1$, there is perfectly positive correlation between two variables.

(*ii*) When $r=-1$, there is perfectly negative relationship between the variables.

(*iii*) When $r=0$, there is no correlation between the variables. Variables are said to be independent.

(*iv*) When r lies between $+1$ and -1 there can be high or low degree of correlation depending upon the closeness of r to ± 1 or zero. If r is near to zero, correlation is of low degree and if r is near to ± 1 the degree of correlation is high.

Probable Error. It is one of the most important methods of interpreting the coefficient of correlation. With the help of probable error it is also possible to determine the reliability of the correlation coefficient. It can be used to determine the limits within which correlation in the population can be expected to lie. The formula for finding probable error is

$$\text{P.E. } (r) = 0{\cdot}6745 \ \frac{1-r^2}{\sqrt{n}},$$

where r is the correlation coefficient and n is the number of pairs of observations.

Interpretation. (*i*) If the value of r is less than the value of probable error, then correlation is not at all significant.

(*ii*) If the value of r is more than 6 times the value of probable error, then correlation is said to be highly significant. In other words, r is significant if $r > 6$ P.E. (r).

or
$$\frac{r}{\text{P.E. } (r)} > 6.$$

(*iii*) In all other cases, no interpretation can be made with this method.

(*iv*) By adding and subtracting the value of probable error from correlation coefficient we can obtain the upper and lower limits in which the correlation coefficient in the population can be expected to lie. Therefore, limits for population correlation coefficient are

$$r \pm \text{P.E. } (r).$$

Example 13. The value of r for a set of 10 pairs of observations was found to be 0·8. Using probable error method can you say that correlation is significant. Also find limits for population correlation coefficient.

Sol.

Given $n=10$ and $r=0{\cdot}8$.

we know that $$\text{P.E. }(r)=0{\cdot}6745\ \frac{1-r^2}{\sqrt{n}}$$

or $$\text{P.E. }(r)=0{\cdot}6745\ \frac{1-(0{\cdot}8)^2}{\sqrt{10}}$$

$$=\frac{0{\cdot}6745\times0{\cdot}36}{3{\cdot}16}=\frac{0{\cdot}2428}{3{\cdot}16}$$

$$=0{\cdot}076$$

Since the value of $r=0{\cdot}8$ is much more than 6 times the value of P.E. (r), correlation is said to be highly significant.

Limits for population correlation coefficient are given by

$$r\pm\text{ P.E }(r)$$

or $$0{\cdot}8\pm0{\cdot}076$$
$$0{\cdot}724-0{\cdot}876$$

Example 14. Find probable error and interpret the coeff. of correlation for data given in example 2.

Sol.

From example 2 we find that $r=0{\cdot}436$ and $n=7$.

Now $$\text{P.E }(r)=0{\cdot}6745\ \frac{1-r^2}{\sqrt{n}}$$

$$=0{\cdot}6745\ \frac{1-(0{\cdot}436)^2}{\sqrt{7}}=0{\cdot}6745\ \frac{1-0{\cdot}19}{2{\cdot}646}$$

$$=\frac{0{\cdot}6745\times0{\cdot}81}{2{\cdot}646}=\frac{0{\cdot}5463}{2{\cdot}646}=0{\cdot}206$$

Now $6\times\text{P.E }(r)=6\times0{\cdot}206=1{\cdot}236$.

The value of r is less than 6 times the value of probable error hence we cannot interpret the correlation coefficient.

Example 15. What will be the value of correlation coefficient if probable error for 16 pairs of items is 0·04 ?

Sol. Given P.E $(r)=0{\cdot}04$, $n=16$.

We know that

$$\text{P.E }(r)=0{\cdot}6745\ \frac{1-r^2}{\sqrt{n}}$$

or $$0{\cdot}04=0{\cdot}6745\frac{1-r^2}{\sqrt{16}}$$

$$\text{or} \quad 0{\cdot}04 = \frac{0{\cdot}6745\,(1-r^2)}{4}$$

$$\text{or} \quad \frac{0{\cdot}16}{0{\cdot}6745} = 1 - r^2 \quad \text{or} \quad r^2 = 1 - \frac{0{\cdot}16}{0{\cdot}6745}$$

$$\text{or} \quad r^2 = 1 - 0{\cdot}237$$

$$\text{or} \quad r^2 = 0{\cdot}763 \qquad \text{or} \quad r = \pm\sqrt{0{\cdot}763}$$

$$\text{or} \quad r = +0{\cdot}873$$

Example 16. For what value of n, the coefficient of correlation equal to 0·6 will be significant ?

Solution. We know that coefficient of correlation will be significant if

$$\frac{r}{\text{P.E.}(r)} > 6$$

$$\text{or} \quad \frac{r}{\dfrac{0{\cdot}6745\,(1-r^2)}{\sqrt{n}}} > 6$$

$$\text{or if} \quad \frac{r.\sqrt{n}}{0{\cdot}6745\,(1-r^2)} > 6$$

$$\text{or if} \quad \sqrt{n} > \frac{6 \times 0{\cdot}6745(1-r^2)}{r}$$

$$\text{or if} \quad n > \left[\frac{6 \times 0{\cdot}6745\,(1-r^2)}{r}\right]^2$$

$$\text{or if} \quad n > \left[\frac{6 \times 0{\cdot}6745(1-(0{\cdot}6)^2)}{0{\cdot}6}\right]^2$$

$$\text{or if} \quad n > \left[\frac{6 \times 0{\cdot}6745 \times 0{\cdot}64}{0{\cdot}6}\right]^2$$

or if $n >$ **18·63 (19 approximately)**

4. Spearman's Rank Correlation Coefficient. The Karl Pearson's coefficient of Correlation is based on the assumption that the population, from where samples are drawn, is normal. When this assumption is not fulfilled we cannot use Pearson's method and hence there arises need to study correlation by an alternative method. One such method is given by Charles Edward Spearman. It is called Rank Correlation method. In this method ranks are assigned to the values of both the variables. The procedure for giving ranks is that the highest value in each series is given first rank, next to the highest the second rank and so on. The following formula is used :

$$r_s = 1 - \frac{6\Sigma d^2}{n(n^2-1)} \qquad \text{or} \quad r_s = 1 - \frac{6\Sigma d^2}{n^3 - n}$$

where r_s is Spearman's Rank Correlation Coefficient, Σd^2 is the sum of squares of the difference of ranks of x and y series and n is the number of pairs of observations.

The value of rank Correlation coefficient also lies between ± 1. When r_s is $+1$, there is complete agreement in the order of ranks and correlation is perfectly positive. When r_s is -1, there is complete disagreement in the order of ranks and correlation is perfectly negative. $r_s=0$ implies no correlation between the variables being studied.

Rank correlation is extremely useful when the variables under consideration are qualitative in nature. For example, variables like honesty, beauty, friendship, etc. Numerical values cannot be assigned to such variables but they can be ranked. Hence Rank Correlation Coefficient is more appropriate as a measure of degree of relationship between such variables.

Calculation of Rank Correlation Coefficient :

(i) **When ranks are given.** We directly apply the formula given above and find the value of r_s.

Example 17. Find rank correlation coefficient for the following information.

Ranks of x :	5	8	7	1	4	6	2	3
Ranks of y :	4	6	3	8	7	2	1	5

Sol.

Ranks x	*Ranks* y	$d=R_x-R_y$	d^2
5	4	1	1
8	6	2	4
7	3	4	16
1	8	-7	49
4	7	-3	9
6	2	4	16
2	1	1	1
3	5	-2	4
$n=8$		$\Sigma d=0$	$\Sigma d^2=100$

$$r_s=1-\frac{6\Sigma d^2}{n(n^2-1)}$$

$$=1-\frac{6\times 100}{8(8^2-1)}=1-\frac{600}{8\times 63}=1-\frac{600}{504}$$

$$r_s=1-1{\cdot}19 \quad = -0{\cdot}19$$

Example 18. Ten competitors in a beauty contest are ranked by three judges in the following order :

1 st Judge	1,	5,	4,	8,	9,	6,	10,	7,	3,	2
2nd Judge	4,	8,	7,	6,	5,	9,	10,	3,	2,	1
3rd Judge	6,	7,	8,	1,	5,	10,	9,	2,	3,	4

Use Rank Correlation Coefficient to discuss which pair of judges has the nearest approach to the common tastes in beauty.

Sol.

In order to find the pair of judges having nearest approach to common tastes in beauty we compare rank correlation between the judgements of :

(*i*) Ist Judge and 2nd Judge (r_{12})

(*ii*) Ist Judge and 3rd Judge (r_{13})

(*iii*) 2nd Judge and 3rd Judge (r_{23})

The ranks of Ist, 2nd and 3rd judges are denoted by R_1, R_2 and R_3 respectively.

R_1	R_2	R_3	R_1-R_2 d_{12}	d^2_{12}	R_1-R_3 d_{13}	d^2_{13}	R_2-R_3 d_{23}	d^2_{23}
1	4	6	−3	9	−5	25	−2	4
5	8	7	−3	9	−2	4	1	1
4	7	8	−3	9	−4	16	−1	1
8	6	1	2	4	7	49	5	25
9	5	5	4	16	4	16	0	0
6	9	10	−3	9	−4	16	−1	1
10	10	9	0	0	1	1	1	1
7	3	2	4	16	5	25	1	1
3	2	3	1	1	0	0	−1	1
2	1	4	1	1	−2	4	−3	9
			$\Sigma d_{12}=0$	$\Sigma d^2_{12}=74$	$\Sigma d_{13}=0$	$\Sigma d^2_{13}=156$	$\Sigma d_{23}=0$	$\Sigma d^2_{23}=44$

Rank correlation between 1st and 2nd Judge :

Now $r_{12}=1-\dfrac{6\Sigma d^2_{12}}{n(n^2-1)}=1-\dfrac{6\times 74}{10(100-1)}$

$=1-\dfrac{444}{990}\qquad =1-0\cdot448=0\cdot552$

Rank correlation between 1st and 3rd Judge :

$r_{13}=1-\dfrac{6\Sigma d^2_{13}}{n(n^2-1)}=1-\dfrac{6\times 156}{10(100-1)}$

$=1-\dfrac{936}{990}=1-0\cdot945=0\cdot055$

Rank correlation between 2nd and 3rd judge :

$r_{23}=1-\dfrac{6\Sigma d^2_{23}}{n(n^2-1)}\quad =1-\dfrac{6\times 44}{10(100-1)}$

or $r_{23}=1-\dfrac{264}{990}=1-0\cdot267=0\cdot733.$

Since there is maximum positive correlation between the judgement of 2nd and 3rd judge, we conclude that 2nd and 3rd judges have the nearest approach to common tastes in beauty.

(*ii*) **When ranks are not given :** In such a case first of all ranks are given to the values of both the variables by following the procedure discussed earlier. Then same formula is used to calculate rank correlation coefficient,

Example 19. Calculate the ranks correlation coefficient between marks of 7 students in economics and statistics.

Marks in Economics x : 45, 65, 72, 34, 57, 60, 80

Marks in Statistics y : 60, 68, 70, 45, 64, 53, 75

Sol.

Marks in Economics x	Marks in Statistics y	R_x	R_y	$d=R_x-R_y$	d^2
45	60	6	5	1	1
65	68	3	3	0	0
72	70	2	2	0	0
34	45	7	7	0	0
57	64	5	4	1	1
60	53	4	6	—2	4
80	75	1	1	0	0
				$\Sigma d=0$	$\Sigma d^2=6$

$$r_{xy}=1-\frac{6\Sigma d^2}{n(n^2-1)}=1-\frac{6\times 6}{8(8^2-1)}$$

$$=1-\frac{36}{8\times 63}=1-\frac{36}{504}$$

$$=1-0{\cdot}071=0{\cdot}929$$

(*iii*) **Equal Ranks.** Sometimes, while assigning ranks we find that some values are same in a series. As per rule equal or common ranks should be assigned to such equal or repeated values. Their common ranks are the arithmetic mean of the ranks which these items would have got if they were slightly different from each other. The next item will get the rank next to the rank used in computing the common rank. Let us take an example. Suppose after giving first four ranks we find two equal values. These values should be given ranks 5 and 6. But we shall assign arithmetic mean of 5 and 6 *i.e.* $\frac{5+6}{2}=5{\cdot}5$ to each equal value as rank. The next item will get rank 7. The following adjustment is made in the formula :

$$r_s=1-\frac{6\left[\Sigma d^2+\frac{1}{12}(m^3-m)+\frac{1}{12}(m^3-m)+\ldots\ldots\right]}{n(n^2-1)}$$

where m is the number of times any value is repeated and $\frac{1}{12}(m^3-m)$ occurs as many times as there are groups of equal values.

Example 20. Calculate rank correlation coefficient for the following data :

x values :	15	48	22	48	26	25	14	78
y values :	40	17	38	35	35	32	20	35

Sol. Since actual values are given, we first assign ranks. In x series value 78 gets Ist rank. Now there are two equal values 48, 48. These get average of ranks 2 and 3 *i.e.*, 2·5 each. Next item 26 gets the 4th rank. Similarly, ranks are assigned for y series.

x values	*y values*	R_x	R_y	$d=R_x-R_y$	d^2
15	40	7	1	6	36·0
48	17	2·5	8	−5·5	30·25
22	38	6	2	4	16·0
48	35	2·5	4	−1·5	2·25
26	35	4	4	0	0
25	32	5	6	−1	1·0
14	20	8	7	1	1·0
78	35	1	4	−3	9·0
				$\Sigma d=0$	$\Sigma d^2=95{\cdot}50$

$$r_s = 1 - \frac{6\left[\Sigma d^2 + \frac{1}{12}(m^3 - m) + \frac{1}{12}(m^3 - m)\right]}{n(n^2 - 1)}$$

$$= 1 - \frac{6\left[95 \cdot 50 + \frac{1}{12}(2^3 - 2)^* + \frac{1}{12}(3^3 - 3)^{**}\right]}{8(8^2 - 1)}$$

$$= 1 - \frac{[6(95 \cdot 50 + 0 \cdot 5 + 2)]}{8 \times 63}$$

$$= 1 - \frac{6\,[98]}{504} = 1 - \frac{588}{504}$$

$$= 1 - 1 \cdot 167 = -0 \cdot 167$$

Correction of Incorrect Value of Rank Correlation Coefficient. Just as we can correct an incorrect value of Karl Pearson's correlation coefficient, in the same way we can correct an incorrect rank correlation coefficient. We find correct values of Σd^2 and n and use the formula.

Example 21. The coefficient of rank correlation of a beauty contest involving 12 candidates was calculated as 0·6. However, it was later on discovered that the difference in ranks of a pair was wrongly taken as 8 instead of 3. Find correct correlation coefficient.

Sol. Given $r_s = 0 \cdot 6$; $n = 12$
Wrong value of $d = 8$
Correct value of $d = 3$.

We know that $r_s = 1 - \dfrac{6\Sigma d^2}{n(n^2 - 1)}$

or $\quad 0 \cdot 6 = 1 - \dfrac{6\Sigma d^2}{12[(12)^2 - 1)]}$

or $\quad 0 \cdot 6 = 1 - \dfrac{\Sigma d^2}{2 \times 143}$

or $\quad \dfrac{\Sigma d^2}{286} = 1 - 0 \cdot 6$

or $\Sigma d^2 = (0 \cdot 4)(286) = 114 \cdot 4$. This is the wrong value of Σd^2.

Correct Σd^2 = wrong Σd^2 — (wrong d)^2 + (correct d)^2

$= 114 \cdot 4 - (8)^2 + (3)^2 = 114 \cdot 4 - 64 + 9$

$= 114 \cdot 4 - 55 = 59 \cdot 4$

Correct $n = 12 - 1 + 1 = 12$.

Correct correlation coeff. $\quad r_s = 1 - \dfrac{6\Sigma d^2}{n(n^2 - 1)}$

---

* In first series item 48 occurs 2 times $\quad \therefore \quad m = 2$

**In second series item 35 occurs 3 times $\quad \therefore \quad m = 3$

$$=1-\frac{6\times 59\cdot 4}{12(144-1)} \qquad =1-\frac{356\cdot 4}{12\times 143}$$

$$=1-\frac{356\cdot 4}{1716}=1-0\cdot 207=0\cdot 793.$$

Merits and Demerits of Rank Correlation.

Merits. (*i*) Rank Correlation Coefficient is simple to understand and easy to calculate as compared to Karl Pearson's method.

(*ii*) It is very useful for qualitative data.

(*iii*) It is a distribution free method because no assumption is made about the form of population from where sample data is taken.

Demerits. (*i*) The process of assigning ranks becomes very time consuming if the number of items is very large.

(*ii*) The rank correlation coefficient is not as accurate as Karl Pearson's correlation coefficient because instead of actual values ranks are used in the former method.

(*iii*) Rank correlation coefficient cannot be calculated for grouped data.

5. **Concurrent Deviations Method**. This method is the simplest of all the methods of studying correlation between two variables. This method is generally employed when we want to find the direction of change in the variables *i.e.*, whether they move in the same direction or in the opposite direction. The direction of change or deviation is recorded from the preceding value. The first value is taken as base. If the second value is more than the first value, then a + (positive) sign is placed before it. If the second value is less than first value, a— (negative) sign is placed before it. If the second value is equal to the first value, then the direction of change is indicated by zero. Same process is followed for all the values. The direction of change in x and y series is indicated by d_x and d_y respectively.

The deviations in the two variables are said to be concurrent if they have the same sign *i.e.*, either deviations of both variables are positive or both are negative. These concurrent deviations are denoted by C. The following formula is used :

$$r_c=\pm\sqrt{\pm\frac{2C-N}{N}},$$

where r_c is the Coefficient of Concurrent Deviations.

C is the number of concurrent deviations.

N is the number of pairs of deviations.

or N=number of pairs of items minus one.

The sign of r_c depends upon the sign of $\frac{2C-N}{N}$.

(*i*) If $\frac{2C-N}{N}$ is negative, we take minus sign inside and outside the under-root and co-efficient of concurrent deviations is also negative.

(*ii*) If $\frac{2C-N}{N}$ is positive, then co-efficient of concurrent deviations is also positive.

Like Karl Pearson's and Rank Correlation Coefficients the Coefficient of Concurrent Deviations also varies betweent $\pm$ 1. In other words,

$-1 \leqslant r_c \leqslant 1$. When $r_c=+1$, there is perfect positive correlation. When $r_c=-1$, there is perfect negative correlation.

Steps for Calculating Coefficient of Concurrent Deviations.

(*i*) Find direction of change in x and y series separately *i.e.*, find d_x and d_y.

(*ii*) Multiply d_x and d_y. The number of positive signs is denoted by C *i.e.*, concurrent deviations.

(*iii*) Apply the formula

$$r_c=\pm\sqrt{\pm\frac{2C-N}{N}}$$

Example 22. Calculate the coefficient of correlation by concurrent deviations method from the following data :

Weight (Kgm.)	50	55	60	58	65	62	70	75
Age (years)	22	23	27	28	26	25	27	30

Sol.

Weight (Kgm.) x	*Age (years)* y	dx	dy	$dx \cdot dy$
50	22			
55	23	+	+	+
60	27	+	+	+
58	28	—	+	—
65	26	+	—	—
62	25	—	—	+
70	27	+	+	+
75	30	+	+	+
				C=5

$$r_c=\pm\sqrt{\pm\frac{2C-N}{N}}$$

$$=\pm\sqrt{\pm\frac{2\times5-7}{7}}=+\sqrt{+\frac{3}{7}}$$

$$=\sqrt{0{\cdot}429}=0{\cdot}655.$$

Example 23. Calculate the coefficient of concurrent deviations

from the data given below :

Year	1975	1976	1977	1978	1979	1980	1981	1982	1983
Demand	160	164	171	183	170	190	192	195	180
Price	250	244	230	200	210	205	201	188	190

Sol.

Year	*Demand* (x)	*Price* (y)	*dx*	*dy*	*dx. dy*
1975	160	250			
1976	164	244	+	—	—
1977	171	230	+	—	—
1978	183	200	+	—	—
1979	170	210	—	+	—
1980	190	205	+	—	—
1981	192	201	+	—	—
1982	195	188	+	—	—
1983	180	190	—	+	—
					C=0

$$r_c = \pm\sqrt{\pm\frac{2C-N}{N}} = \pm\sqrt{\pm\frac{2\times 0-8}{8}}$$

$$= -\sqrt{-\frac{-8}{8}} = -\sqrt{\frac{8}{8}} = -1.$$

So there is perfectly negative relationship between demand and price.

Evaluation. Concurrent deviations method is very simple. This method is followed when we want quick results and also the number of pairs of observations is very large. But this method gives information only about the direction of relationship between variables and not the degree or extent of correlation. It is because no distinction is made between large and small deviations. Only direction or sign of deviations is noted.

If we want to have the proper knowledge of direction as well as degree of relationship between variables, then Karl Pearsons coefficient of correlation is generally recommended.

MISCELLANEOUS EXAMPLES

Example 24. Calculate Karl Pearson's coeffieient of correlation from the data given ahead :-

Marks	*Age in Years* 18	19	20	21	22
22·5	3	2	—	—	—
17·5	—	5	4	—	—
12·5	—	—	7	10	—
7·5	—	—	—	3	2
2·5	—	—	—	3	1

Solution. Let us denote age in years by X and marks by Y.

$$r_{xy}=\frac{\Sigma fdxd'y-\dfrac{(\Sigma fdx)(\Sigma fd'y)}{\Sigma f}}{\sqrt{\Sigma fdx^2-\dfrac{(\Sigma fdx)^2}{\Sigma f}}\sqrt{\Sigma fd'y^2-\dfrac{(\Sigma fd'y)^2}{\Sigma f}}}$$

or
$$r_{xy}=\frac{-38-\dfrac{9\times 6}{40}}{\sqrt{47-\dfrac{(9)^2}{40}}\sqrt{50-\dfrac{(6)^2}{40}}}=\frac{-38-1\cdot35}{\sqrt{47-2\cdot025}\sqrt{50-0\cdot9}}$$

$$=\frac{-39\cdot35}{\sqrt{44\cdot975}\sqrt{49\cdot01}}=\frac{-3\ldots\cdot35}{(6\cdot\ldots)(7)}$$

$$=\frac{-39\cdot35}{46\cdot90}=-0\cdot839.$$

Calculation of Correlation Coefficient

$fdxd'y$	$fd'y^2$	$fd'y$	$d'y^2$	$h=5$ $d'y$	$B=12{\cdot}5$ $dy=y-B$	Y \ X	18	19	20	21	22	Total
−16	20	10	4	2	10	22·5	3 [.12]	2 [−4]	—	—	—	5
−50	9	9	1	1	5	17·5	—	5 [−5]	4 [0]	—	—	9
0	0	0	0	0	0	12·5	—	—	7 [−0]	10 [0]	—	17
−7	5	−5	1	−1	−5	7·5	—	—	—	3 [−3]	2 [−4]	5
−10	16	−8	4	−2	−10	2·5	—	—	—	3 [−6]	1 [−4]	4
−38	50	6				Total	3	7	11	16	3	40
						A=20, $dx=x-A$	−2	−1	0	1	2	
						fdx	−6	−7	0	16	6	$\Sigma fdx=9$
						dx^2	4	1	0	1	4	
						fdx^2	12	7	0	16	12	$\Sigma fdx^2=47$

Example 25. A computer while calculating the correlation coefficient between two variables X and Y obtained the following constants :

$N=30$, $\Sigma x=120$, $\Sigma x^2=600$, $\Sigma y=90$, $\Sigma y^2=250$, $\Sigma xy=356$

It was later discovered that two pairs of observations were wrongly copied as

x	y
8	10
12	7

and correct values were

x	y
8	12
10	8

Find correct correlation coefficient.

Solution. Correct $\Sigma x=120-(8+12)+(8+10)=118$

Correct $\Sigma y=90-(10+7)+(12+8)=93$

Correct $\Sigma x^2=600-[(8)^2+(12)^2]+[(8)^2+(10)^2]$

$=600-64-144+64+100=556$

Correct $\Sigma y^2=250-[(10)^2+(7)^2]+[(12)^2+(8)^2]$

$=250-100-49+144+64=309$

Correct $\Sigma xy=356-[(8\times 10)+(12\times 7)]+[(8\times 12)+(10\times 8)]$

$=356-80-84+96+80=368$

Correct $n=30-2+2=30$

Putting these values in the formula

$$r\times y=\frac{\Sigma xy-\dfrac{(\Sigma x)(\Sigma y)}{n}}{\sqrt{\Sigma x^2-\dfrac{(\Sigma x)^2}{n}}\sqrt{\Sigma y^2-\dfrac{(\Sigma y)^2}{n}}}$$

$$=\frac{368-\dfrac{118\times 93}{30}}{\sqrt{556-\dfrac{(118)^2}{30}}\sqrt{309-\dfrac{(93)^2}{30}}}$$

$$=\frac{368-365\cdot 8}{\sqrt{556-464\cdot 1}\sqrt{309-288\cdot 3}}$$

$$=\frac{2\cdot 2}{\sqrt{91.9}\sqrt{20\cdot 7}}=\frac{2\cdot 2}{(9\cdot 586)(4\cdot 549)}$$

$$=\frac{2\cdot 2}{43\cdot 6067}=0\cdot 05.$$

Example 26. Obtain the sample coefficient of corretation from the following data :

x :	2·5	1·7	3·1	6·5	4·2
y :	—5·2	—3·5	4·1	—6·2	2·8

Solution. Calculation of correlation coefficient

x	$(x-\bar{x})$ $(x-3{\cdot}6)$	$(x-\bar{x})^2$	$\bar{y}$	$y-\bar{y}$ $y-(-1{\cdot}6)$ $y+1{\cdot}6$	$(y-\bar{y})^2$	$(x-\bar{x})(y-\bar{y})$
2·5	−1·1	1·21	−5·2	−3·6	12·96	3·96
1·7	−1·9	3·61	−3·5	−1·9	3·61	3·61
3·1	−0·5	0·25	4·1	5·7	32·49	− 2·85
6·5	2·9	8·41	−6·2	−4·6	21·16	−13·34
4·2	0·6	0·36	2·8	4·4	19·36	2·64
$\Sigma x=18$		$\Sigma(x-\bar{x})^2=13{\cdot}84$	$\Sigma y=-8$		$\Sigma(y-\bar{y})^2=89{\cdot}58$	$\Sigma(x-\bar{x})(y-\bar{y})=-5{\cdot}98$

$n=5$

$$\bar{x}=\frac{\Sigma x}{n}=\frac{18}{5}=3{\cdot}6 \qquad \bar{y}=\frac{\Sigma y}{n}=\frac{-8}{5}=-1{\cdot}6$$

$$r_{xy}=\frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{[\Sigma(x-\bar{x})^2][\Sigma(y-\bar{y})^2]}}=\frac{-5{\cdot}98}{\sqrt{(13{\cdot}84)(82{\cdot}58)}}$$

$$=\frac{-5{\cdot}98}{\sqrt{1239{\cdot}78}}$$

$$=\frac{-5{\cdot}98}{35{\cdot}2}=-0{\cdot}1698.$$

Example 27. Calculate the values of $y=(x-6)^6$ corresponding to $x=1, 2, 3, 4$ and 5 and obtain the Karl Pearson's correlation coefficient between x and y.

Solution.

(x)	$(x-6)$	$(x-6s)^2$	$(x-\bar{x})$	$(x-\bar{x})^2$	$(y-\bar{y})$	$(y-\bar{y})^2$	$(x-\bar{x})(y-\bar{y})$
1	−5	−3125	−2	4	−2240	5017600	4480
2	−4	−1024	−1	1	−139	19321	139
3	−3	− 243	0	0	642	412164	0
4	−2	− 32	1	1	853	727609	853
5	−1	− 1	2	4	884	781455	1768
$\Sigma x=15$		$\Sigma y=-4425$		$\Sigma(x-\bar{x})^2=10$		$\Sigma(y-\bar{y})^2=6958150$	7240

$$\bar{x} = \frac{\Sigma x}{n} = \frac{15}{5} = 3 \;;\quad \bar{y} = \frac{\Sigma y}{n} = \frac{-4425}{5} = -885$$

$$r_{xy} = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2 . \Sigma(y-\bar{y})^2}} = \frac{7240}{\sqrt{10 \times 6958150}}$$

$$= \frac{7240}{\sqrt{69581500}} = \frac{7240}{8341 \cdot 55} = 0 \cdot 8679.$$

Example 28. The marks of 12 students in two subjects A and B are given below. The two numbers within the brackets denote the marks of same student in subjects A and B respectively.

(40, 15), (60, 18), (45, 16), (40, 20), (45, 40), (50, 42)
(56, 43), (49, 41), (40, 42), (60, 20), (48, 55), (68, 47)

Use Spearman's formula to find rank correlation coefficient.

Solution.

Marks in Subject A	*Marks in Subject B*	*Ranks in A*	*Ranks in B*	$d = R_A - R_B$	d^2
40	15	11	12	−1·0	1·00
60	18	2·5	10	−7·5	56·25
45	16	8·5	11	−2·5	6·25
40	20	11	8·5	2·5	6·25
45	40	8·5	7	1·5	2·25
50	42	5	4·5	0·5	0·25
56	43	4	3	1·0	1·00
49	41	6	6	0	0
40	42	11	4·5	6·5	42·25
60	20	2·5	8·5	−6	36·00
48	55	7	1	6	36·00
68	47	1	2	−1	1·00
				0	$\Sigma d^2 = 188 \cdot 50$

$$r_s = 1 - \frac{6\left[\Sigma d^2 + \frac{1}{12}(m^3-m) + \frac{1}{12}(m^3-m) + \ldots\ldots\right]}{n(n^2-1)}$$

$$1 - \frac{6\left[188 \cdot 5 + \frac{1}{12}(3^3-3) + \frac{1}{12}(2^3-2) + \frac{1}{12}(2^3-2) + \frac{1}{12}(2^3-2 + \frac{1}{12}(2^3-2)\right]}{12[(12)^2-1]}$$

$$= 1 - \frac{6[188 \cdot 5 + 2 + 0 \cdot 5 + 0 \cdot 5 + 0 \cdot 5 + 0 \cdot 5]}{12 \times 143}$$

$$= 1 - \frac{6[192 \cdot 5]}{12 \times 143} = 1 - \frac{1156}{1716} = 1 - 0 \cdot 673$$

$$= 0 \cdot 327$$

Example 29. The coefficient of rank correlation between share prices and debenture prices is found to be 0·143. If the sum of squares of the difference in ranks is given to be 48, find the value of n.

Solution. We know that $r_s = 1 - \dfrac{6\Sigma d^2}{n(n^2-1)}$

Given $r_s = 0{\cdot}143$, $\qquad \Sigma d^2 = 48$,

Putting the values in above formula, we have

$$0{\cdot}143 = 1 - \frac{6\times 48}{n(n^2-1)}$$

or $\quad 0{\cdot}143 = 1 - \dfrac{288}{n^3-n}$ or $\dfrac{288}{n^3-n} = 1 - 0{\cdot}143$

or $288 = (0.857)(n^3-n)$

$\therefore \quad \dfrac{288}{0{\cdot}857} = n^3 - n$

$\therefore \quad n^3 - n = 336. \qquad$ or $\qquad n^3 - n - 336 = 0.$

or $\quad n^3 - 343 - n + 7 = 0.$

or $\quad n^3 - 343 - (n-7) = 0.$

or $\quad n^3 - 7^3 - (n-7) = 0$

or $\quad (n-7)(n^2+7n+49) - (n-7) = 0.$

$\qquad\qquad (\because \; a^3 - b^3 = (a-b)(a^2+ab+b^2))$

or $\quad (n-7)[n^2+7n+49-1] = 0$

$\quad (n-7)(n^2+7n+48) = 0$

$\therefore \quad n - 7 = 0 \qquad\qquad \left[\because \; n^2+7n+48=0 \text{ gives imaginary value of } n\right]$

or $\quad n = 7$

QUESTIONS

1. What do you mean by correlation ? What are the various types of correlation ?
2. What is meant by correlation ? Does it always signify cause and effect relationship between the two variables ? Explain with illustrations.
3. What is scatter diagram ? How does it help in studying correlation between two variables in respect of both its nature and extent ?
4. Distinguish, giving suitable examples, between :
   (i) Positive and Negative correlation.
   (ii) Linear and Non-linear correlation.
   (iii) Simple, partial and multiple correlation.
5. (a) Define Karl Pearson's Coefficient of Correlation. How would you interpret the sign and magnitude of a correlation coefficient ?

(*b*) Explain the meaning and significance of the concept of correlation.

6. What is Rank Correlation ? State the merits and demerits of Spearman's Rank Correlation method.

7. What are the assumptions, properties and significance of Karl Pearson's Coefficient of Correlation ?

8. What do you mean by Probable Error ? How does it help in interpreting the coefficient of correlation ?

9. What do you mean by Concurrent Deviation method of studying correlation ? In which circumstances is this method generally employed ?

10. Find the coefficient of correlation between marks of 10 students in economics and statistics. Use direct method.

Marks in economics :	45, 70, 65, 30, 90, 40, 50, 75, 85, 60
Marks in statistics :	35, 90, 70, 40, 95, 40, 60, 80, 80, 50

(r=0·903)

11. Calculate Karl Pearson's coefficient of correlation for the following data :

x :	1	2	3	4	6	7	8	9
y :	9	8	10	12	13	14	16	15.

(r=0·95)

12. Calculate the coefficient of correlation between birth rate and death rate (using short cut method)

Year :	1931	1941	1951	1961	1971	1981
Birth Rate :	24	26	32	33	35	30
Death Rate :	15	20	22	24	27	24

(r=0·92)

13. From the following data compute the coefficient of correlation between x and y.

	x Series	y Series
Number of items	15	15
Arithmetic mean	25	18
Squares of deviations from mean	136	138.

Sum of products of deviations of x and y series from their respective means is 122

(r=0·89)

14. It is known that covariance between x and y is equal to 353·5. The sum of squares of deviations of x and y series from their means are 3490 and 4390 respectively. Find Karl Pearson's coefficient of correlation if there are 10 pairs of observations.

(r=0·903)

15. Calculate the number of items for which $r=0{\cdot}5$,

$\Sigma(x-\bar{x})(y-\bar{y})=60,\ \sigma x=4,\ \Sigma(y-\bar{y})^2=90$

(N=10)

Hint. Apply $r=\dfrac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2 \cdot \Sigma(y-\bar{y})^2}}$

16. Calculate Karl Pearson's coefficient of correlation for the following data relating to price and demand of a certain commodity :

Price (in Rs.)	21,	22,	23,	24,	25,	26,	27,	28,	29,	30
Demand (in Thousand Units)	18,	19,	19,	16,	17,	16,	16,	15,	13,	11

($r=-0.91$)

17. Find Karl Pearsons correlation coefficient between age and playing habits of the following students :

Age (years)	15	16	17	18	19	20
No. of students	250	200	150	120	100	80
Regular Players	200	150	90	48	30	16

[($r=-0.989$)]

18. Karl Pearson's coefficient of correlation between two variables x and y is 0·28, their covariance is 7·6. If the variance of x is 6, find the standard deviation of y series.

($\sigma_y=9.048$)

19. Find the coefficient of correlation and Probable error for the following data :

Height of Fathers (inches)	64,	65,	66,	67,	68,	69,	70
Height of sons (inches)	66,	67,	65,	68,	70,	68,	72

($r=0.81$; P.E. $(r)=0.09$)

20. A student calculates the value of r as 0·7 when the number of items in the sample is 25. Find the limits within which population correlation coefficient can be expected to lie.

(Lower limit 0·631 ; Upper limit 0·768)

21. Calculate coefficient of correlation between x and y series using Karl Pearson's method. Also find Probable error.

x :	78,	89,	96,	69,	59,	79	68,	61
y :	125,	137,	156,	112,	107,	136,	123,	108.

[($r=0.95$; P.E. $(r)=0.02$)]

22. In order to find the correlation coefficient between two variables x and y from 25 pairs of observations, the following calculations were made :

$$\Sigma x=125,\ \Sigma y=100,\ \Sigma x^2=650,\ \Sigma y^2=460,\ \Sigma xy=508.$$

But two pairs (6, 14) and (8, 6) were wrongly taken while correct values were (8, 12) and (6, 8). Find correct value of correlation coefficient.

($r=0.67$)

23. Coefficient of correlation between x and y for 20 items is 0·3. Mean of x is 15 and that of y is 20. Standard deviations of x and y are 4

and 5 respectively. At the time of calculations one item 27 was wrongly taken as 17 in case of x series and 35 instead of 30 in case of y series. Find the correct correlation coefficient.

($r=0{\cdot}504$)

24. If $r=0{\cdot}6$ for a pair of 64 observations, find the probable error and also determine the limits for population correlation coefficient.

[(P.E(r)$=0{\cdot}054$; Limits are $0{\cdot}546$—$0{\cdot}654$)]

25. The rankings of ten students in two subjects A and B are as follows :

A : 3, 5, 8, 4, 7, 10, 2, 1, 6, 9

B : 6, 4, 9, 8, 1, 2, 3, 10, 5, 7.

Calculate rank correlation coefficient.

($r_s=-0{\cdot}297$)

26. Ten competitors in a beauty contest are ranked by three judges. Using rank correlation coefficient find the pair of judges having nearest approach to common tastes in beauty.

1st Judge	1,	6,	5,	10,	3,	2,	4,	9,	7,	8.
2nd Judge	3,	5,	8,	4,	7,	10,	2,	1,	6,	9
3rd Judge	6,	4,	9,	8,	1,	2,	3,	10,	5,	7

(1st and 3rd Judges)

27. Calculate the coefficient of correlation from the following data using rank correlation method :

x	75,	88,	95,	70,	60,	80,	81,	50
y	120,	134,	150,	115,	110,	140,	142,	100

($r_s=-0{\cdot}93$)

28. Calculate rank correlation coefficient for data relating to marks of 11 students in two subjects A and B.

Marks in subject A : 24, 29, 19, 14, 30, 19, 27, 30, 20, 28, 11
Marks in subject B : 37, 35, 16, 26, 23, 27, 19, 20, 16, 11, 21

($r_s=0{\cdot}0225$)

29. The coefficient of rank correlation of marks obtained by 10 students in economics and statistics was found to be 0·5. It was later discovered that the difference in ranks in two subjects obtained by one of the students was wrongly taken as 3 instead of 7. Find correct coefficient of rank correlation.

($r_s=0{\cdot}2576$)

30. The coefficient of rank correlation between age and height of 7 students was found to be —0·893. Find the sum of squares of difference of ranks.

($\Sigma d^2=106$)

31. Find the coefficient of concurrent deviations for the following data :

Year	1980	1981	1982	1983	1984	1985	1986
Supply	150	154	160	172	160	165	180
Price	200	180	170	160	190	180	172

($r_c=-1$)

32. Apply concurrent deviations method to find the coefficient of correlation for the following data.

Marks in statistics x : 50, 60, 62, 58, 70, 72, 40.

Marks in sports y : 40, 35, 45, 42, 50, 46, 48.

($r_c=0$)

33. Calculate rank correlation coefficient from the following data :

x : 92, 89, 87, 86, 83, 77, 71, 63, 53, 50.

y : 86, 83, 91, 77, 68, 85, 52, 82, 37, 57.

($r_s=0{\cdot}73$)

34. Calculate the coefficient of rank correlation for the following data :

x : 48, 34, 40, 12, 16, 16, 66, 25, 16, 56.

y : 14, 14, 23, 8, 13, 6, 20, 9, 9, 14.

($r_s=0{\cdot}806$)

35. In a set of 50 observations it was found that $r=0{\cdot}3$, $\bar{x}=10$, $\sigma_x=3$, $\bar{y}=6$, $\sigma_y=2$. It was later found that one value of $x=10$ and $y=6$ were inaccurate and were weeded out. What is the corrected correlation coefficient ?

($r=0{\cdot}3$)

36. A survey regarding the incomes and savings of 100 school teachers in a certain city provided the following data :

Income (in Rs.)	*Savings in Rs.* 50	100	150	200	*Total*
400	8	4	—	—	12
600	—	12	24	6	42
800	—	9	7	2	18
1000	—	—	10	5	15
1200	—	—	4	9	13
Total	8	25	50	17	100

Find correlation coefficient between income and savings.
($r_{xy}=0.523$)

37. Find the Karl Pearson's correlation coefficient between age and playing habits of the following students

Age (in years)	15	16	17	18	19	20
No. of Students	250	200	150	120	100	80
Regular players	200	150	90	48	30	12

($r_{xy}=-0.991$)

(Hint : Find percentage of regular players *e.g.* $\frac{200}{250}\times 100=80$; $\frac{150}{200}\times 100=75$ etc. and calculate cerrelation between age and these percentage.)

38. Find coefficient of correlation with the help of Karl Plearson's Method.

Marks in Statistics	*Marks in Mathematics* 5—15	15—25	25—35	35—45	45—55
2·5— 7·5	2	4	1	4	1
7·5—12·5	8	2	5	1	—
12·5—17·5	—	3	2	1	—
17·5—22·5	—	1	3	2	4
22·5—27·5	—	—	4	2	—

$r_{xy}=0.36$

39. Calculate correlation coefficient from the following results :

$n=100,\quad \Sigma x=140,\quad \Sigma y=150,\quad \Sigma(x-10)^2=180,$

$\Sigma(y-15)^2=2\ 5$ and $\Sigma(x-10)(y-15)=60.$

($r_{xy}=0.915$)

40. The coefficient of rank correlation between x and y was found to be 0·5 with 10 pairs of observations. It was later discovered that one wrong pair of observations whose difference of ranks was 6, was taken by mistake. It was later on weeded out. Find correct rank correlation coefficient for 9 pairs of observations.

($r_s=0.6125$)

9

Index Numbers

An index number is a statistical device designed to measure relative changes in the level of a phenomenon (variable or a group of variables) with respect to time, geographical location or other characteristics. The phenomenon or variables under consideration may be :

(*i*) The price of any commodity like wheat, rice, milk etc., or a group of commodities like consumer goods, producer goods, cosmetics etc.

(*ii*) The volume of industrial or agricultural production, exports, imports, trade etc.

(*iii*) The national income of the country, per capita income, foreign exchange reserves etc.

With the help of index numbers we can measure the average change in such phenomenon over a period of time or at different places. For example, suppose we want to study the general change in the price level of consumer goods during a period of time. These changes are not directly measurable as the price quotations of different commodities are expressed in different units e.g., prices of wheat, rice, pulses etc. are quoted in Rs. per quintal ; price of water is quoted in Rs. per gallon ; prices of milk, petrol, kerosene etc. are quoted in Rs. per litre ; price of cloth in Rs. per metre and so on. Again, the prices of some commodities may be increasing while those of other commodities may be decreasing over the time period under consideration. Again the rate of increase or decrease may be different for different commodities. Thus, it is not possible to directly measure the changes in prices and find the average rate of change. Index number enables us to arrive at a single representative figure which tells us about the average change in the price level of consumer goods. Thus, index numbers are called specialised types of average. They are also called specialised type of rates, ratios, percentages which give the general level of magnitude of a group of distinct but related variables in two or more situations.

In the present world, index numbers are widely used by economists, businessmen, governments, sociologists, health authorities etc. We generally find in the newspapers reports like "the Wholesale Price Index number was 270 in 1980-81 with base 1970-71." It means that there has been an overall increase in prices to the extent of 170 per cent in ten years 1970-71 to 1980-81. In other words, 17 per cent increase, on

an average, every year. The base year index is always equal to 100. Similarly, index of industrial production with 1970 as base was 154 in 1980 indicating an increase in production by 54 per cent during the decade 1970-80.

Uses of Index Numbers. Index numbers are today one of the most useful statistical devices. It is difficult to find any field of quantitative measurement where index numbers are not used. They are used in almost all sciences—natural, social and physical. The main uses of index numbers are discussed below :

1. Index numbers are economic barometers. Like barometers which are used in Physics and Chemistry to measure atmospheric pressure, index numbers are rightly called as economic barometers which measure the pressure of economic and business behaviour. In the words of *Simpson and Kafka*, "*Index numbers are used to take the pulse of the economy and they have come to be used as indicators of inflationary or deflationary tendencies.*" A careful study of index numbers of national income, prices, imports, exports, consumption, production—agricultural industrial etc. gives us a fairly good appraisal of the general trade, economic development and business activity of the country.

2. Index numbers are helpful in framing suitable policies. Many economic and business polices are based on the information provided by index number. For example, dearness allowance is fixed by considering the cost of Living Index Number. Similarly, requirements for raw materials, labour, electricity etc. are determined on the basis of indices of industrial and agricultural production. Import and Export indices dictate various policies in the field of foreign trade. The excise duty on the production and sale of a commodity is determined according to the index numbers of the consumption of the commodity from time to time and so on.

3. Index numbers measure the purchasing power of money. There exists an inverse relation between the value of money and the price level. By using price index we can very easily calculate the purchasing power of money at any place and time as follows :

$$\text{Purchasing power of money} = \frac{1}{\text{Price Index}}$$

For example, if the consumer Price Index in any year is 150, then the purchasing power of one rupee will be $\frac{1}{160} = 62{\cdot}5$ paisa.

4. Index numbers are very useful in deflating. We know that the purchasing power of money goes on changing with the change in price level. Index numbers are used to adjust the original data for price changes and thus convert nominal wages into real wages or nominal income into real income. To determine real wages, the nominal wages are divided by the price index.

5. **Index numbers are helpful in studying trends and tendencies.** Since index numbers study the relative changes in the level of any phenomenon over a period of time, they can be used to study the trend of the phenomenon in a time series data. For example, by examining the index number of the exports of India for last 10 to 12 years we can say about the trend of exports of India. Similarly, trends of national income, prices, consumption, balance of payments, terms of trade, wages, production etc. can be formed by using index numbers.

Types of Index Numbers. Broadly speaking, there are three types of index numbers :

1. Price Index Numbers.
2. Quantity Index Numbers.
3. Value Index Numbers.

1. Price Index Numbers. Price index numbers show the changes in prices of the commodities produced or consumed in a given period with reference to some base period. Price indices can be of two types :

(i) *Wholesale Price Index Numbers.* They show the changes in the general price level of a country.

(ii) *Ratail Price Index Numbers.* They reflect the changes in the retail prices of commodities like wheat, rice, milk, cosmetics etc. The cost of living index is a special kind of retail price index.

2. Quantity Index Numbers. These index numbers measure changes in the quantity of goods produced, consumed or distributed during the current period with reference to any base period.

3. Value Index Numbers. These index numbers are prepared to compare changes in the value of any phenomenon, in the current period with reference to any base period. For example, changes in total revenue (Price × Quantity) can be studied by constructing a value index number.

Problems in the Construction of Index Numbers. While constructing index numbers, various problems are faced by the statisticians. These are also called the preliminaries to the construction of index numbers. These are given below :

1. The Purpose of the Index Number. The first and foremost task before constructing any index number is to define in clear terms the objective or purpose of index numbers. There is no all-purpose index number. The knowledge about purpose of index number helps us to collect relevant data, select appropriate commodities, assign suitable weights and use proper techniques. Failure to decide clearly the purpose of the index would lead to confusion and wastage of time with no fruitful results. For example, suppose we want to study changes in cost of living. The class of people for whom index is to be constructed must be clearly defined so that we can start the task of preparing relevant index number accurately.

2. Selection of Commodities. After defining clearly the purpose of index, next problem is to make a selection of commodities which are to be included in the index. Commodities shall be relevant to the purpose of index. Their number should be neither too large nor too small. The selected commodities should be representative of the tastes, habits, customs etc. of the people and they must be of standard quality. Thus, in a consumer price index for people belonging to lower income groups, commodities like car, VCR, Colour television etc. should not be included. On the other hand, if the index number is for richer class of people, then index number will not be representative if we exclude above mentioned commodities. According to *R.L.A. Holmes*, "*An index number cannot include every commodity. Certain items must be chosen from a very wide field, and it follows that these chosen items must be representative, and not only must they be representative, but they must be consistent in quality.*"

3. Obtaining Price Quotations. After the commodities have been selected and their number decided, next problem is to obtain price quotations for these commodities. The data relating to prices of selected commodities may be collected from standard trade journals, reputed periodicals, newspapers and government publications. The collected data must be suitable to the purpose of the index. Again prices of commodities differ from place to place and at one place from shop to shop. For the purpose of obtaining price quotations representative markets should be selected and from each market those shops should be selected which are reputed and known for trading. After deciding the place and shops from where prices of commodities are to be taken, the next job is to appoint an authority who will supply the price quotations from time to time on regular basis. In order to check the accuracy of price quotations supplied by an agency the price quotations can be obtained from more than one agency.

4. Selection of Base Period. Whenever we construct an index number, we construct it with reference to some base period. The index for base period is always taken as 100. The following points need careful consideration regarding base period :

(*i*) *The base period should be a normal one.* The base period should be a year in which business and economic conditions were stable and there was no financial boom or crisis created by war, famine, epidemic, depression etc. However, it is sometimes very difficult to select a year which is normal in all respects. Accordingly, sometimes an average of two or more years is taken as a base period.

(*ii*) *The base period should not be too distant in the past.* In this rapidly changing world it is not advisable to make comparisons of today with any period in the last century. It is because things have changed to a very great extent. There has been considerable change in the customs, tastes, fashions, habits etc. of people in the last few decades. So base period should not be very far off from the current period. For example, when we construct index number for 1988, the base period

should not be 1940, which is too far. The items used now may not be available in that period, e.g. Computers or electronic goods were not available at that time. So base period should be a recent one.

(*iii*) *Fixed Base or Chain Base*. The base period can be either fixed or changing. When the base period is kept fixed for all current years it is called fixed-base period. In the case of chain base method, the values for the current period are linked with those of the preceding period and not with any fixed years. The choice between fixed or chain base depends on the purpose of the index.

5. Selection of Appropriate Weights. By weight we mean the relative importance given to different items. There are two types of index numbers (*i*) Unweighted index numbers (*ii*) Weighted index numbers. In the first category equal importance is given to all the items while in the second category weights are assigned to different items depending upon their importance. While constructing weighted price index numbers, the quantity is taken as weight and while constructing weighted quantity index numbers, prices of different commodities are taken as weights.

There are two methods of assigning weights : (*i*) *Implicit Weights* ; (*ii*) *Explicit Weights*. Implicit weight is a method of giving varying emphasis to different items by the number of times a given item is included in the index. For example, suppose wheat is to be given thrice as much importance as rice, then three varieties of wheat against one variety of rice would be included in the index. In the case of explicit weights, some outward evidence of importance of various items in the index is given. It can be the consumption figure, the production figure or the distribution figure which brings out the economic importance of each commodity.

Again, there can be either *quantity weights* or *value weights*. When *aggregative method* is employed for constructing index numbers, then quantity weights are used. On the other hand, when *average of relatives method* is used for constructing index numbers, then value weights are used.

6. Selection of Average. Another problem which generally arises in the construction of index numbers is the choice of average. In the construction of index numbers, first price relatives (prices in the current period as a ratio of prices in the base period) are obtained. These price relatives have to be averaged to find one representative value of index. For this purpose, arithmetic mean, median, mode, geometric mean, harmonic mean are available. But median, mode and harmonic mean are never used. The choice lies between arithmetic mean and geometric mean.

Theoretically, the geometric mean is considered to be the most appropriate average in the construction of index numbers because

(*i*) Since index numbers deal with ratios and relative changes, the geometric mean gives equal weights to the equal ratios of change.

(*ii*) Since geometric mean gives more importance to small items and less importance to large items, it is not unduly affected by extreme and violent fluctuations in the observations.

(*iii*) Index numbers based on geometric mean satisfy the *time reversal test* and the *factor reversal test*.

However, in practice, since it is very difficult to calculate geometric mean, the arithmetic mean is generally used for averaging price relatives. Thus, when a very high degree of accuracy and precision is required, the geometric mean is used otherwise normally arithmetic mean serves the purpose of averaging.

7. Selection of an Appropriate Formula. A large number of formulae have been developed by different statisticians for constructing index numbers. Thus there is one more problem of selecting an appropriate formula. The choice of the formula depends on (*i*) the purpose of index (*ii*) weighted or unweighted (*iii*) aggregative or average of relatives (*iv*) use of average *i.e.*, arithmetic mean or geometric mean and (*v*) the availablity of data. *Prof Irving Fisher* has suggested an index which is said to be an *ideal index*. It satisfies both the time reversai and the factor reversal tests and is based on geometric mean. However, its use also depends on the availability of appropriate data.

Various Notations and Terminology. Before studying the methods of constructing index numbers, it is important to know the meaning of various notations and terms used in their construction.

Base year. It is the year with reference to which the comparisons are made. It is denoted by suffix 'o'.

Current year. It is the year for which comparisons are required. It is denoted by suffix 1.

P_0 : Price of the commodity in the base year.

P_1 : Price of the commodity in the current year.

q_0 : Qnantity in the base year.

q_1 : Quantity in the current year.

P : Price relative expressing current year price as percentage of base year price.

$$P = \frac{P_1}{P_0} \times 100.$$

Po_1 : Price index number for the current year with reference to the base year.

P_{10} : Price index number for the base year with reference to the current year.

Qo_1 : Quantity index number for the current year with reference to the base year.

$Q_{1\,0}$: Quantity index number for base with reference to the current year.

Vo_1 : Value index number in the current year with reference to the base year.

Methods of constructing Price Index Numbers. Various methods of constructing price index numbers can be divided into two categories.

(*a*) Unweighted Price Index Numbers.

(*b*) Weighted Price Index Numbers.

Each of these types may further be divided into two heads :

(1) Aggregative Methods.
(2) Average of Relatives Methods.

Let us discuss these methods in detail.

Unweighted Price Index Numbers.

(*1*) *Simple Aggregative Method.* It is the simplest of all the methods of constructing price index numbers.

$$Po_1=\frac{\Sigma P_1}{\Sigma P_0}\times 100$$

Where ΣP_1 is sum of prices in the current period and ΣP_0 is the sum of prices in the base period.

Example 1. From the following data relating to five commodities prepare price index number in 1988 with 1980 as base using simple aggregative method.

Commodity	*Units*	*Price in (Rs.)* *1988*	*1980*
Wheat	Quintal	160	100
Milk	kg.	5	3
Cloth	metre	35	20
Bananas	dozen	7	4
Kerosene	tin	25	10

Sol. Construction of Price Index Number for 1988 with 1980 as base.

Commodity	*Unit*	*(Price in Rs.)* *1988* P_1	*Price* *1980* P_o
Wheat	Quintal	160	100
Milk	Kg.	5	3
Cloth	metre	35	20
Bananas	dozen	7	4
Kerosene	tin	25	10
		$\Sigma P_1=232$	$\Sigma P_o=137$

$$Po_1=\frac{\Sigma P_1}{\Sigma P_o}\times 100=\frac{232}{137}\times 100=169\cdot 34$$

Merits. (*i*) This method is simple to understand

(*ii*) Price index is very simple to compute.

Demerits. (*i*) This method ignores the relative importance of different commodities.

(*ii*) When commodities are quoted in different units, this method gives inappropriate results.

2. Simple Average of Relatives Method. In this method first of all the price relatives are obtained and then these price relatives are averaged using either arithmetic mean or geometric mean and hence index number is constructed. Price relative $P=\frac{P_1}{P_0}\times 100$. It expresses the price in current period as percentage of price in the base period.

(*i*) ***When Arithmetic Mean is used.***

$$P_{01}=\frac{\Sigma\left(\frac{P_1}{P_0}\times 100\right)}{N} \quad \text{or} \quad P_{01}=\frac{\Sigma P}{N}$$

$\Sigma\left(\frac{P_1}{P_0}\times 100\right)$ is the sum of price relatives and N is the number of commodities.

(*ii*) *When Geometric Mean is used.*

$$Po_1=\text{Anti log}\left\{\frac{\Sigma\left(\log\frac{P_1}{P_0}\times 100\right)}{N}\right\} \quad \text{or} \quad P_{01}=AL\left[\frac{\Sigma \log P}{N}\right]$$

Example 2. Using arithmetic mean calculate price index number in 1980 for the following data. You may apply simple average of relatives method.

Commodity	A	B	C	D	E
Price in 1980	15	20	10	25	40
Price in 1970	10	12	5	20	35

Sol. Construction of Price index number in 1980 taking 1970 as base year.

Commodity	*Price in 1980* P_1	*Price in 1970* P_0	$\frac{P_1}{P_0}$	$\frac{P_1}{P_0}\times 100$
A	15	10	$\frac{15}{10}=1\cdot 5$	$1\cdot 5\times 100=150\cdot 7$
B	20	12	$\frac{20}{12}=1\cdot 667$	$1\cdot 667\times 100=166\cdot 7$
C	10	5	$\frac{10}{5}=2\cdot 0$	$2\cdot 0\times 100=200\cdot 0$

D	25	20	$\frac{25}{20}=1\cdot25$	$1\cdot25\times100=125\cdot0$
E	40	35	$\frac{40}{35}=1\cdot142$	$1\cdot142\times100=114\cdot2$

$$\Sigma\left(\frac{P_1}{P_0}\times100\right)=755\cdot9$$

$$P_{01}=\frac{\Sigma\left(\frac{P_1}{P_0}\times100\right)}{N}=\frac{755\cdot9}{5}=151\cdot18$$

Example 3. Construct index number by simple average of relatives method for the data given in example 2. Use geometric mean for averaging.

Sol.

Commodity	*Price in 1980* P_1	*Prtce in 1970* P_o	$\frac{P_1}{P_0}\times 100$	*log* $\left(\frac{P_1}{P_0}\times100\right)$
A	15	10	150·0	2·1761
B	20	12	166·7	2·2219
C	10	5	200·0	2·3010
D	25	20	125·0	2·0969
E	40	35	114·2	2·0577

$$\Sigma\log\left(\frac{P_1}{P_0}\times100\right)=10\cdot8536$$

$$P_{01}=AL\left[\frac{\Sigma\log\left(\frac{P_1}{P_0}\times100\right)}{N}\right]$$

or $\quad P_{01}=AL\left(\frac{10\cdot8536}{5}\right)=AL\,(2\cdot1707)$

or $\quad P_{01}=148\cdot1$

Note. The value of price index by using geometric mean will always be less than the value of price index by using arithmetic mean. This is also clear from example numbers 2 & 3.

Merits of Simple Average of Relatives Method.

(*i*) This method is simple to understand and easy to calculate.

(*ii*) This method is not influenced by the units in which prices are quoted. This is because the price relatives are pure numbers and are free from original units.

(*iii*) Extreme items do not influence the index.

Demerits. (*i*) Weights to different commodities are not assigned according to their importance.

(*ii*) When geometric mean is used for averaging price relatives, this method becomes difficult to use.

Weighted Price Index Numbers. In these index numbers various commodities included in the index are assigned weights according to their relative importance. There are two methods of constructing **Weighted Price Index Numbers :**

1. Aggregative Methods.
2. Average of Relatives Method.

1. Weighted Aggregative Methods. Many weighted aggregative formulae have been suggested by different statisticians for constructing **price index numbers. Important among them are :—**

(*i*) Laspeyre's Method

(*ii*) Paasche's Method

(*iii*) Dorbish and Bowley's Method

(*iv*) Fisher's Ideal Method

(*v*) Kelly's Method.

(i) **Laspeyre's Method.** This method was devised by the German statistician and economist Etinne Laspeyre in 1871. It is a weighted aggregate price index number with base period quantity (q_0) as weight. Laspeyre's formula is as follows :

$$P_{01} = \frac{\Sigma P_1 q_o}{\Sigma P_0\, q_o} \times 100$$

Where $\Sigma\, P_1\, q_0$ is the sum of the product of prices in the current period and quantities in the base period of the commodities. $\Sigma\, P_0\, q_0$ is the sum of product of price and quantity of commodities in the base period.

The Laspeyre's index attempts to answer the question, "What is the change in the aggregate value of the base period's list of goods when valued at current period prices ?"

Example 4. From the following data construct Laspeyre's Price Index Number in 1988 with 1985 as base period.

Commodity	Price in 1985	Price in 1988	Quantity in 1985	Quantity in 1988
A	5	6	12	15
B	7	10	14	20
C	10	15	8	10
D	4	5	6	8
E	6	10	10	16

Sol. Since 1985 is base period, price and quantity in 1985 are

denoted by p_0 and q_0 respectively. Similarly, price and quantity in 1988 are denoted by p_1 and q_1 respectively.

Construction of Laspeyre's Price Index Number.

Commodity	Price in 1985 p_0	Price in 1988 p_1	Quantity in 1985 q_0	Quantity in 1988 q_1	$p_0 q_0$	$p_1 q_0$
A	5	6	12	15	60	72
B	7	10	14	20	98	140
C	10	15	8	10	80	120
D	4	5	6	8	24	30
E	6	10	10	16	60	100
					$\Sigma p_0q_0=322$	$\Sigma p_1q_0=462$

$$p_{01}=\frac{\Sigma p_1q_0}{\Sigma p_0 q_0}\times 100=\frac{462}{322}\times 100=143\cdot48$$

(ii) **Paasche's Method.** This method was devised by the German statistician and economist Hermann Paasche in 1874. It is the weighted aggregate price index number with current year quantity (q_1) as weight. The formula is given as follows :

$$p_{01}=\frac{\Sigma p_1q_1}{\Sigma p_0q_1}\times 100.$$

Where Σp_1q_1 is the sum of the product of the price and quantity of commodities in the current period and Σp_0q_1 is the sum of the product of price in the base period and quantity in the current period of the given commodities.

The Paasche's Index attempts to answer the question, "what is the change in the aggregate value of the current period's list of goods when valued at base period prices."

Example 5. From the data given in example 4 construct Paasche's Price Index in 1988 with 1985 as the base year.

Sol. Construction of Paasche's Price Index Number.

Commodity	Price in 1985 p_0	Price in 1988 p_1	Quantity in 1985 q_0	Quantity in 1988 q_1	p_1q_1	p_0q_1
A	5	6	12	15	90	75
B	7	10	14	20	200	140
C	10	15	8	10	150	100
D	4	5	6	8	40	32
E	6	10	10	16	160	96
					$\Sigma p_1q_1=640$	$\Sigma p_0q_1=443$

$$p_{01}=\frac{\Sigma p_1q_1}{\Sigma p_0q_1}\times 100=\frac{640}{443}\times 100=144{\cdot}47$$

(*iii*) **Dorbish and Bowley's Method.** In this method both base period and current period quantities are taken as weights. This method is nothing but the arithmetic mean of Laspeyre's and Paasche's methods.

L=Laspeyre's Index
P=Paasche's Index

$$p_{01}=\frac{L+P}{2}$$

or
$$p_{01}=\frac{\frac{\Sigma p_1q_0}{\Sigma p_0q_0}\times 100+\frac{\Sigma p_1q_1}{\Sigma p_0q_1}\times 100}{2}$$

or
$$p_{01}=\frac{\frac{\Sigma p_1q_0}{\Sigma p_0q_0}+\frac{\Sigma p_1q_1}{\Sigma p_0q_1}}{2}\times 100$$

For example, using values from examples 4 and 5. Dorbish and Bowley's Price Index is given by

$$p_{01}=\frac{\frac{462}{322}+\frac{640}{443}}{2}\times 100$$

$$=\frac{1{\cdot}4347+1{\cdot}4447}{2}\times 100=\frac{2{\cdot}8794}{2}\times 100=143{\cdot}97$$

(*iv*) **Fisher's Ideal Method.** Prof. Irving Fisher developed in 1927 an index number which is considered to be the ideal index number. Fisher's index number is based on the geometric mean of Laspeyre's and Paasche's weighted aggregative price index number. In other words,

$$p_{01}=\sqrt{L\times P}$$

or
$$p_{01}=\sqrt{\left(\frac{\Sigma p_1q_0}{\Sigma p_0q_0}\times 100\right)\left(\frac{\Sigma p_1q_1}{\Sigma p_0q_1}\times 100\right)}$$

$$=\sqrt{\frac{\Sigma p_1q_0}{\Sigma p_0q_0}\times\frac{\Sigma p_1q_1}{\Sigma p_0q_1}}\times 100.$$

Fisher's Index is said to be ideal because of the following reasons :

(*a*) It is free from bias.

(*b*) It is based on geometric mean which is theoretically best average for constructing index numbers.

(*c*) It satisfies both time reversal and factor reversal tests.

(*d*) In this method, the influence of both current year and base year quantities is taken into account.

e.g., using values from examples 4 and 5, Fisher's Ideal Index is given by

$$p_{01}=\sqrt{\frac{462}{322}\times\frac{640}{443}}\times 100$$

$$=\sqrt{(1\cdot4347)(1\cdot4447)}\times 100=\sqrt{2\cdot0727}\times 100$$

$$=1\cdot4397\times 100=143\cdot969$$

Example 6. Using Fisher's Ideal Index number construct a price index number in 1980 with 1975 as base year.

Commodity	*1975* Price	*1975* Quantity	*1980* Price	*1980* Quantity
I	12	10	10	15
II	10	12	9	10
III	15	14	16	15
IV	8	16	10	12
V	6	20	15	11

Sol. Construction of Fisher's Price Index Number

Commodity	*1975* Price p_0	*1975* Quantity q_0	*1980* Price p_1	*1980* Quantity q_1	p_0q_0	p_1q_1	p_0q_1	p_1q_0
I	12	10	10	15	120	150	180	100
II	10	12	9	10	120	90	100	108
III	15	14	16	15	210	240	225	224
IV	8	16	10	12	128	120	96	160
V	6	20	15	11	120	165	66	300
					$\Sigma p_0q_0=698$	$\Sigma p_1q_1=765$	$\Sigma p_0q_1=667$	$\Sigma p_1q_0=892$

$$p_{01}=\sqrt{\frac{\Sigma p_1q_0}{\Sigma p_0q_0}\times\frac{\Sigma p_1q_1}{\Sigma p_0q_1}}\times 100$$

$$=\sqrt{\frac{892}{698}\times\frac{765}{667}}\times 100=\sqrt{1\cdot2779\times 1\cdot1469}\times 100$$

$$=\sqrt{1\cdot4656}\times 100=1\cdot2106\times 100$$

$$=121\cdot06$$

Example 7. Construct price index numbers from the following data taking 1980 as base. Use (*i*) Laspeyre's Method (*ii*) Paasche's Method (*iii*) Bowley's Method (*iv*) Fisher's Method.

Commodities	*1980*		*1981*	
	Price	*Value*	*Price*	*Value*
A	10	140	12	144
B	12	144	15	105
C	15	300	16	96
D	20	300	25	200
E	25	100	30	90

Sol. In the data price and value are given both for base year and current year. Quantity can be calculated by dividing the value by corresponding price in each of base and current year. Value in 1980 is denoted by p_0q_0 and in 1981 by p_1q_1.

Construction of Price Index Numbers.

Commodity	*1980*			*1981*				
	Price (p_0)	*Value* (p_0q_0)	*Quantity* (q_0) $\left(\frac{p_0q_0}{p_0}\right)$	*Price* (p_1)	*Value* (p_1q_1)	*Quantity* $\left(\frac{p_1q_1}{p_1}\right)$	p_0q_1	p_1q_0
A	10	140	14	12	144	12	120	168
B	12	144	12	15	105	7	84	180
C	15	300	20	16	96	6	90	320
D	20	300	15	25	200	8	160	375
E	25	100	4	30	90	3	75	120
		$\Sigma p_0q_0 = 984$			$\Sigma p_1q_1 = 635$		$\Sigma p_0q_1 = 529$	$\Sigma p_1q_0 = 1163$

Laspeyre's Index $po_1 = \frac{\Sigma p_1q_0}{\Sigma p_0q_0} \times 100$

$$p_{01}=\frac{1163}{984}\times 100=118\cdot 19$$

Paasche's Index $p_{01}=\frac{\Sigma p_1q_1}{\Sigma p_0q_1}\times 100$

$$=\frac{635}{529}\times 100=120\cdot 03$$

Bowley's Index $p_{01}=\frac{L+P}{2}=\frac{118\cdot 19+120\cdot 03}{2}$

$$=\frac{238\cdot 22}{2}=119\cdot 11$$

Fisher's Ideal Index

$$p_{01}=\sqrt{L\times P}$$
$$=\sqrt{(118\cdot 19)(120\cdot 03)}=\sqrt{14186\cdot 34}$$
$$=119\cdot 10$$

(v) **Kelly's Method.** This method was given by Truman L. Kelly. It is also known as **Fixed Weight Aggregative Index.** In this method weights are fixed for all periods. Weights *i.e.*, quantities may refer to any period and not necessarily base period or current period. Kelly's price index is given by

$$p_{01}=\frac{\Sigma p_1q}{\Sigma p_0q}\times 100.$$

where q refers to the quantity used as weight. Sometimes the average of quantities consumed in two or three or more years may be taken as weights. If weight is the average of quantity in the base period (q_0) and current period (q_1), then in the above formula we use $q=\frac{1}{2}(q_0+q_1)$.

Example 8. The following figures relate to the prices and quantities of certain commodities. Construct a price index number in 1987 taking 1985 as base.

Commodity	A	B	C	D	E
Price (in Rs.) in 1985	40	50	52	55	64
Price (in Rs.) in 1987	50	60	70	80	90
Quantity Consumed	15	10	8	6	4

Sol. Since weight (quantity) is fixed, we use Kelly's Method.

Commodity	Price in 1985 (in Rs.) p_0	Price in 1987 (in Rs.) P_1	Quantity q	P_0q	P_1q
A	40	50	15	600	750
B	50	60	10	500	600
C	52	70	8	416	560
D	55	80	6	330	480
E	64	90	4	256	360
				Σp_0q $=2102$	Σp_1q $=2740$

$$p_{01}=\frac{\Sigma p_1q}{\Sigma p_0q}\times 100$$

$$=\frac{2740}{2102}\times 100=130{\cdot}35.$$

2. Weighted Average of Relatives Method. In this method, the price relatives are assigned appropriate weights to show the relative importance of commodities in the given group. The price index is obtained by taking weighted average. For this purpose, either artihmetic mean or geometric mean can be used.

When Arithmetic Mean is used for averaging

Price Index $p_{01}=\frac{\Sigma wp}{\Sigma w}$

Where $p=\frac{p_1}{p_o}\times 100$ and $w=p_o\,q_o$ *i.e.*, it is the value in the base period and is taken as weight.

When Geometric Mean is used for averaging.

Price Index $p_{01}=$ $\text{AL}\left(\frac{\Sigma w \log p}{\Sigma w}\right)$

Example 9. Construct weighted average of relatives price index numbers to the following data, taking 1980 as base year when

(*i*) Arithmetic Mean is used

(*ii*) Geometric Mean is used

Commodity	1980 Price (Rs.)	1980 Quantity	1981 Price (Rs.)
I	5	10	8
II	12	15	24
III	20	20	30
IV	28	25	35
V	10	30	12
VI	6	40	9

Sol. Construction of weighted Average of Relative Price Index Numbers. Price and quantity in 1980 are denoted by p_o and q_o respectively. Price in 1981 is denoted by P_1

Commodity	1980 p_o	1980 q_o	1981 p_1	$P=\frac{P_1}{p_o}\times 100$	$w=p_o.q_o$	$w.p$	$log\ p.$	$w.log$ P
I	5	10	8	$\frac{8}{5}\times 100=160$	50	8000	22 .041	110·20
II	12	15	24	$\frac{24}{12}\times 100=200$	180	36000	2·3010	414·18
III	20	20	30	$\frac{30}{20}\times 100=150$	400	60000	2·1761	870.44
IV	28	25	35	$\frac{35}{28}\times 100=125$	700	87500	2·0969	1467·83
V	10	30	12	$\frac{12}{10}\times 100=120$	300	36000	2·0792	623·76
VI	6	40	9.	$\frac{9}{6}\times 100=150$	240	36000	2·1761	522·26
					Σw = 1870	Σwp= 263500		Σw log$p$ =4008·6

(*i*) **When Arithmetic Mean is used.**

$$p_{o1}=\frac{\Sigma wp}{\Sigma w}=\frac{263500}{1870}=140{\cdot}91$$

(*ii*) **When Geometric Mean is used**

$$p_{o1}=\text{AL}\left(\frac{\Sigma w \log p}{\Sigma w}\right)=\text{AL}\left(\frac{4008{\cdot}67}{1870}\right)$$

$$=\text{AL}\ (2{\cdot}1436)=139{\cdot}2$$

One Commodity Price Index Numbers. So far we have studied various methods of constructing price index numbers when many commodities are involved. When there is only one commodity and data is given over a period of time, then one period is taken as base and price relatives are obtained. These price relatives are called index numbers.

Price Index=Price Relative$=\frac{p_1}{p_o}\times 100$. Where p_1 is the price in current period and p_o is the price in the base period.

Example 10. From the data given below calculate price index numbers taking (*i*) 1975 as base year (*ii*) 1978 as base year (*iii*) Average of Prices from 1976 to 1978 as base year.

Year	1975	1976	1977	1978	1979	1980	1981	1982
Price (Rs.)	5	7	9	10	12	14	16	18

Sol. Determination of Price Index Numbers.

Year	*Price (Rs.)*	*Base 1975=100 Price Index=* $\frac{p_1}{p_0}\times 100$	*Base 1978=100 Price Index=* $\frac{p_1}{p_0}\times 100$	*Base Average of Price in 1976, 1977, 1978* $=\frac{5+7+9}{3}=7$
1975	5	100	$\frac{5}{10}\times 100=50$	$\frac{5}{7}\times 100=71{\cdot}43$
1976	7	$\frac{7}{5}\times 100=140$	$\frac{7}{10}\times 100=70$	$\frac{7}{7}\times 100=100{\cdot}0$
1977	9	$\frac{9}{5}\times 100=180$	$\frac{9}{10}\times 100=90$	$\frac{9}{7}\times 100=128{\cdot}6$
1978	10	$\frac{10}{5}\times 100=200$	$\frac{10}{10}\times 100=100$	$\frac{10}{7}\times 100=142{\cdot}8$
1979	12	$\frac{12}{5}\times 100=240$	$\frac{12}{10}\times 100=120$	$\frac{12}{7}\times 100=171{\cdot}4$
1980	14	$\frac{14}{5}\times 100=280$	$\frac{14}{10}\times 100=140$	$\frac{14}{7}\times 100=200{\cdot}0$
1981	16	$\frac{16}{5}\times 100=320$	$\frac{16}{10}\times 100=160$	$\frac{16}{7}\times 100=228{\cdot}6$
1982	18	$\frac{18}{5}\times 100=360$	$\frac{18}{10}\times 100=180$	$\frac{18}{7}\times 100=257{\cdot}1$

Quantity Index Numbers. These index numbers measure the changes in the quantities of goods consumed, produced, or distributed during the current period with reference to any base period. Like price index numbers, the quantity index numbers can be either unweighted or weighted. In the case of weighted quantity index numbers either price or value (p_0q_0) is taken as weight depending on the use of aggregative method or the average of relatives method. Various formulae for constructing quantity index numbers can be derived from the corresponding price index numbers by using q in place of p and vice versa.

I. Unweighted Methods :

(*i*) *Simple Aggregative Method* :

$$Q_{01}=\frac{\Sigma q_1}{\Sigma q_0}\times 100,$$ where Σq_1 is the sum of quantity in the current period and Σq_0 is the sum of quantity in the base period.

(ii) *Simple Average of Relatives Method* :
Case (*a*) When arithmetic mean is used.

$$Q_{01}=\frac{\Sigma\left(\frac{q_1}{q_0}\times 100\right)}{N} \quad \text{or} \quad Q_{01}=\frac{\Sigma q}{N}$$

where $q=\frac{q_1}{q_0}\times 100$

Case (*b*) When geometric mean is used

$$Q_{01}=\text{AL}\,\frac{\left(\Sigma \log \frac{q_1}{q_0}\times 100\right)}{N}$$

or $Qo_1=\text{AL}\left(\frac{\Sigma \log q}{N}\right)$, where $q=\frac{q_1}{q_0}\times 100$

II. Weighted Methods.

(*i*) *Weighted Aggregative Methods.*

(*a*) Laspeyre's Quantity Index : $Qo_1=\frac{\Sigma q_1 P_0}{\Sigma q_0 P_0}\times 100$

(*b*) Paasche's Quantity Index : $Qo_1=\frac{\Sigma q_1 P_1}{\Sigma q_0 P_1}\times 100$

(*c*) Dorbish and Bowley's Index : $Qo_1=\frac{L+P}{2}$

$$\text{or} \quad Q_{01}=\frac{\frac{\Sigma q_1 P_0}{\Sigma q_0 P_0}+\frac{\Sigma q_1 P_1}{\Sigma q_0 P_1}}{2}\times 100$$

This index is the arithmetic mean of Laspeyre's and Paasche's Quantity Index.

(*d*) Fisher's Ideal Index : $Q_{01}=\sqrt{L\times P}$

$$\text{or} \quad Qo_1=\sqrt{\frac{\Sigma q_1 P_0}{\Sigma q_0 P_0}\times\frac{\Sigma q_1 P_1}{\Sigma q_0 P_1}}\times 100$$

This index is the geometric mean of Laspeyre's and Paasche's Quantity Index Numbers.

(*e*) **Kelly's Method** :

$$Q_{01}=\frac{\Sigma q_1 P}{\Sigma q_0 P}\times 100$$

where P is the fixed weight.

(*ii*) *Weighted Average of Relatives Method* :

(*a*) When arithmetic mean is used.

$$Q_{01}=\frac{\Sigma wq}{\Sigma w} \quad \text{where } q=\frac{q_1}{q_0}\times 100 \quad \text{and } w=P_0 q_0$$

(*b*) When geometric mean is used.

$$Q_{01}=\text{AL}\left[\frac{\Sigma w \log q}{\Sigma w}\right], \quad q=\frac{q_1}{q_0}\times 100 \quad \text{and } w=P_0 q_0$$

Example 11. From the following data construct quantity index number in 1981 with 1980 as base year. Use simple aggregative method.

Commodity	A	B	C	D	E
Quantity in 1980 (kg.)	10	12	15	18	20
Quantity in 1981 (kg.)	15	17	20	22	30

Sol. Construction of Quantity Index Number.

Commodity	*Quantity in 1980 (kg.)* (q_0)	*Quantity in 1981 (kg.)* (q_1)
A	10	15
B	12	17
C	15	20
D	18	22
E	20	30
	$\Sigma q_0 = 75$	$\Sigma q_1 = 104$

using Simple Aggregative Method

$$Q_{01} = \frac{\Sigma q_1}{\Sigma q_0} \times 100 \qquad = \frac{104}{75} \times 100 \qquad = 138{\cdot}67$$

Example 12. For the data given in example 11, construct simple average of relatives quantity index number in 1981 taking 1980 base. Use (*i*) Arithmetic Mean (*ii*) Geometric Mean.

Sol.

Commodity	1980 qo	1981 q_1	$\frac{q_1}{qo} \times 100$	$\log \left(\frac{q_1}{q_0}\right) \times 100$
A	10	15	$\frac{15}{10} \times 100 = 150{\cdot}0$	2·1761
B	12	17	$\frac{17}{12} \times 100 = 141{\cdot}67$	2·1511
C	15	20	$\frac{20}{15} \times 100 = 133{\cdot}33$	2·1249
D	18	22	$\frac{22}{18} \times 100 = 122{\cdot}22$	2 0871
E	20	30	$\frac{30}{20} \times 100 = 150{\cdot}0$	2·1761
			$\Sigma\left(\frac{q_1}{q_0} \times 100\right) = 697{\cdot}22$	$\Sigma \log\left(\frac{q_1}{q_0} \times 100\right) = 10{\cdot}7153$

(*i*) When Arithmetic Mean is used

$$Q_{01}=\frac{\Sigma\left(\frac{q_1}{q_0}\times 100\right)}{N}=\frac{697\cdot22}{5}=139\cdot44$$

(*ii*) When Geometric Mean is used

$$Q_{01}=AL\left(\frac{\Sigma \log\frac{q_1}{q_0}\times 100}{N}\right)$$

$$=AL\left(\frac{10\cdot7153}{5}\right)=AL(2\cdot1430)$$

$$=139\cdot0$$

Example 13. Calculate current year quantity indices using (*i*) Laspeyre's (*ii*) Paasche's (*iii*) Bowley's (*iv*) Fisher's methods.

Commodity	*Base year*		*Current year*	
	Price	*Quantity*	*Price*	*Quantity*
A	5	3	8	2
B	4	7	6	5
C	7	4	10	3
D	6	6	7	5

Sol. Computation of Weighted Aggregative Indices.

Commodity	*Base year*		*Current year*					
	Price	*Quantity*	*Price*	*Quantity*				
	P_o	q_0	P_1	q_1	q_0P_0	q_1P_1	q_1P_0	q_0P_1
A	5	3	8	2	15	16	10	24
B	4	7	6	5	28	30	20	42
C	7	4	10	3	28	30	21	40
D	6	6	7	5	36	35	30	42
					Σq_0P_0 $=107$	Σq_1P_1 $=111$	Σq_1P_0 $=81$	Σq_0P_1 $=148$

(*i*) Laspeyre's Index $Q_{01}=\frac{\Sigma q_1P_0}{\Sigma q_0P_0}\times 100=\frac{81}{107}\times 100=75\cdot7$

(*ii*) Paasche's Index $Q_{01}=\frac{\Sigma q_1P_1}{\Sigma q_0P_1}\times 100=\frac{111}{148}\times 100=75$

(*iii*) Bowley's Index

$$Q_{01}=\frac{\frac{\Sigma q_1P_0}{\Sigma q_0P_0}+\frac{\Sigma q_1P_1}{\Sigma q_0P_1}}{2}\times 100=\frac{\frac{81}{107}+\frac{111}{148}}{2}\times 100$$

$$=\frac{0\cdot757+0\cdot75}{2}\times 100=75\cdot35$$

(*iv*) Fisher's Ideal Index $Q_{01}=\sqrt{\frac{\Sigma q_1P_0}{\Sigma q_0P_0}\times\frac{\Sigma q_1P_1}{\Sigma q_0P_1}}\times 100$

$$=\sqrt{\frac{81}{107}\times\frac{111}{148}}\times 100=\sqrt{0\cdot757\times 0\cdot75}\times 100$$

$$=\sqrt{0\cdot56775}\times 100=0\cdot7534\times 100$$
$$=75\cdot34$$

Example 14. From the data given below compute quantity index numbers using weighted average of relatives method when (*i*) Arithmetic mean is used (*ii*) Geometric mean is used.

		Base Period		Current Period	
Commodity	*Units*	*Price*	*Quantity*	*Price*	*Quantity*
A	kg.	2	10	1	15
B	Litre	3	12	4	18
C	Dozen	1	15	2	30
D	Pairs	4	20	3	25
E	Metres	5	25	4	50

Sol. **Construction of Weighted Average of Relatives Quantity Index Numbers.**

Commodity	*Units*	P_0	q_0	P_1	q_1	$q=\frac{q_1}{q_0}\times 100$	$w=P_0q_0$	*w.q.*	*log q*	*w. log q*
A	Kg.	2	10	1	15	150	20	3000	2·1761	43·522
B	Litre	3	12	4	18	150	36	5400	2·1761	78·339
C	Dozen	1	15	2	30	200	15	3000	2·3010	34·515
D	Pairs	4	20	3	25	125	80	10000	2·0969	167·752
E	Metres	5	25	4	50	200	125	25000	2·3010	287·625
							$\Sigma w=$ 276	$\Sigma wq=$ 46400		$\Sigma w \log q$ $=611\cdot753$

(*i*) When Arithmetic Mean is used.

$$Qo_1=\frac{\Sigma wq}{\Sigma w}=\frac{46400}{276}=168\cdot116$$

(*ii*) When Geometric Mean is used.

$$Qo_1=\text{AL}\left(\frac{\Sigma w \log q}{\Sigma w}\right)=\text{AL}\left(\frac{611\cdot753}{276}\right)$$
$$=\text{AL}(2\cdot2165)=164\cdot6.$$

Value Index Numbers. The third category of index numbers which are generally used are value index numbers. These index numbers are prepared to compare the change in the total value of any phenomenon in the current period with reference to some base period. For example, the changes in total revenue (Price × Quantity) can be measured by constructing a value index. It is given by :

$$\text{Value Index}=\frac{\Sigma p_1q_1}{\Sigma p_0q_0}\times 100$$

$$\text{Or Value Index}=\frac{\Sigma v_1}{\Sigma v_0}\times 100$$

where v_1 is the value in the current period and v_0 is the value in the base period.

Value index numbers are not as common as price and quantity index numbers.

Index Number Tests

We have studied various formulae for the construction of index numbers. Thus there arises the question, "which formula is accurate ?" The accuracy of a formula, according to some theoretical statisticians, depends upon whether or not it meets certain mathematical tests. Whether these tests are logically valid or not is an open question. Not only can an index be considered ideal if if it meets these tests, but other indexes that do not meet them can be graded according to how closely they approximate them in actual practice. These tests include

(*i*) Unit Test

(*ii*) Time Reversal Test

(*iii*) Factor Reversal Test

(*iv*) Circular Test

(*i*) Unit Test

It is a very simple test and it requires that the index number formula should be independent of the units in which the prices or quantities of various commodities are quoted. All the formulae except the simple aggregative (unweighted) satisfy this test.

(*ii*) Time Reversal Test

This test was purposed by Prof. Irving Fisher. This is merely a test to determine whether a given method will work both ways in time, **forward and backward. In the words of Prof. Fisher,** ***"The formula for calculating an index number should be such that it gives the same ratio between one point of comparison and the other no matter which of the two is taken as the base or putting it antoher way, the index number reckoned forward should be reciprocal of the one reckoned backward."***

For example, from 1970 to 1971 sugar prices increase from rupees 3 to rupees 4 per kg, the price in 1971 would be 133·33 percent of the price in 1970, and the price in 1970 would be 75 percent of the price in 1971. One figure is the reciprocal of the other, *i.e.* their product

(1·3333 × 0·75) is Unity. Similarly, if a given method of index number construction shows the general price level in one year to be 133·33 percent of the level in the preceding year, it should work correctly when reversed. *i.e.* it should show that the price level in the first year was 75 percent of the price level in the second year.

Thus when the data for any two years are treated by the same method, but with the bases reversed, the two index numbers secured should be reciprocals of each other. Their product should always be unity, That is, we should have the relation.

$$P_{01} \times P_{10} = 1 \qquad \text{or} \qquad P_{01} = \frac{1}{P_{10}}$$

where P_{01} stands for price index in current period '1' with reference to base period '0' omitting the factor 100 and P_{10} stands for price index in the base period '1' with reference to current period '0' omitting the factor 100. If $P_{01} \times P_{10}$ is not equal to unity, then there is inherent bias in the formula. For this bias **Mudgett** in his book **Index Numbers** has used symbol E_1, where $E_1 = (P_{01} \times P_{10}) - 1$. This will be equal to zero when time reversal test is met. Time reversal test is satisfied by the following index number formulae.

1. Simple aggreagative index
2. Marshall-Edgeworth Formula
3. Kelly's fixed weight formula
4. Fisher's Ideal formula
5. Simple geometric mean of price relatives formula.
6. Weighted geometric mean of price relatives formula with fixed weights.

Simple Aggregative Index and Time Reversal Test

We know that $P_{01} = \frac{\Sigma p_1}{\Sigma p_0}$, (omitting 100)

Interchanging the time period, $P_{10} = \frac{\Sigma p_0}{\Sigma p_1}$

$$\therefore \qquad P_{01} \times P_{10} = \frac{\Sigma p_1}{\Sigma p_0} \times \frac{\Sigma p_0}{\Sigma p_1} = 1$$

Hence this method satisfies the time reversal test.

Marshall-Edgeworth Index and time Reversal Test

$$P_{01} = \frac{\Sigma p_1 q_0 + \Sigma p_1 q_1}{\Sigma p_0 q_0 + \Sigma p_0 q_1}, \; P_{10} = \frac{\Sigma p_0 q_1 + \Sigma p_0 q_0}{\Sigma p_1 q_1 + \Sigma p_1 q_0}$$

Therefore, $P_{01} \times P_{10} = \frac{\Sigma p_1 q_0 + \Sigma p_1 q_1}{\Sigma p_0 q_0 + \Sigma p_0 q_1} \times \frac{\Sigma p_0 q_1 + \Sigma p_0 q_0}{\Sigma p_1 q_1 + \Sigma p_1 q_0} = 1$

This method also satisfies the time reversal test.

Kelly's Fixed weight Index and Time Reversal Test

$$P_{01}=\frac{\Sigma p_1 q}{\Sigma p_0 q}, \quad P_{10}=\frac{\Sigma p_0 q}{\Sigma p_1 q}$$

$$\therefore \quad P_{01}\times P_{10}=\frac{\Sigma p_1 q}{\Sigma p_0 q}\times\frac{\Sigma p_0 q}{\Sigma p_1 q}=1$$

This method also satisfies the time reversal test.

Fisher's Ideal Formula and time Reversal Test

$$P_{01}=\sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0}\times\frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}}, \quad P_{10}=\sqrt{\frac{\Sigma p_0 q_1}{\Sigma p_1 q_1}\times\frac{\Sigma p_0 q_0}{\Sigma p_1 q_0}}$$

$$\therefore \quad P_{01}\times P_{10}=\sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0}\times\frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}\times\frac{\Sigma p_0 q_1}{\Sigma p_1 q_1}\times\frac{\Sigma p_0 q_0}{\Sigma p_1 q_0}}=\sqrt{1}=1$$

Thus Fisher's method also satisfies the time reversal test.

Simple geometric mean of price relatives formula and time reversal test.

we know that $P_{01}=\left[\pi\frac{p_1}{p_0}\right]^{1/n}$, where π means product

and $\quad P_{10}=\left[\pi\frac{p_0}{p_1}\right]^{1/n}$

Now $\quad P_{01}\times P_{10}=\left[\pi\frac{p_1}{p_0}\right]^{1/n}\times\left[\pi\frac{p_0}{p_1}\right]^{1/n}$

$$=\left[\pi\frac{p_1}{p_0}\times\pi\frac{p_0}{p_1}\right]^{1/n}=1$$

Similarly the test can be verified for the weighted geometric mean of price relatives index with fixed weights.

Laspeyre's Method and time Reversal test.

$$P_{01}=\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0}, \quad P_{10}=\frac{\Sigma p_0 q_1}{\Sigma p_1 q_1}$$

$$\therefore \quad P_{01}\times P_{10}=\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0}\times\frac{\Sigma p_0 q_1}{\Sigma p_1 q_1}\neq 1$$

Thus Laspeyres method does not satisfy time reversal test.

Paasche's Method and Time Reversal Test

$$P_{01}=\frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}, \quad P_{10}=\frac{\Sigma p_0 q_0}{\Sigma p_1 q}$$

$$\therefore \quad P_{01}\times P_{10}=\frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}\times\frac{\Sigma p_0 q_0}{\Sigma p_1 q_0}\neq 1$$

Thus this method also does not satisfy time reversal test.

Bowley's Method and Time Reversal Test

$$P_{01}=\frac{1}{2}\left[\frac{\Sigma p_1q_0}{\Sigma p_0q_0}+\frac{\Sigma p_1q_1}{\Sigma p_0q_1}\right],\quad P_{10}=\frac{1}{2}\left[\frac{\Sigma p_0q_1}{\Sigma p_1q_1}+\frac{\Sigma p_0q_0}{\Sigma p_1q_0}\right]$$

$$\therefore \quad P_{01}\times P_{10}=\frac{1}{2}\left[\frac{\Sigma p_1q_0}{\Sigma p_0q_0}+\frac{\Sigma p_1q_1}{\Sigma p_0q_1}\right]\times\frac{1}{2}\left[\frac{\Sigma p_0q_1}{\Sigma p_1q_1}+\frac{\Sigma p_0q_0}{\Sigma p_1q_0}\right]\neq 1$$

Thus Bowley's method also does not satisfy time reversal test.

(*iii*) Factor Reversal Test

A second fundamental test by means of which good index numbers can be detected is the factor reversal test. This test was also proposed by Prof. Irving Fisher. According to this test, the product of the price index and the quantity index should be equal to the corresponding value index.

In the words of Prof. Fisher, "*Just as our formula should permit the interchange of two times without giving inconsistent results, so it ought to permit interchanging the prices and quantities without giving inconsistent results i.e., the two results multiplied together should give the true value ratio, except for a constant of proportionality.*'

According to Croxton and Cowden, "*The factor reversal test may be stated in this way: If the p and q factors in a price (or quantity) index formula be interchanged, so that a quantity (or price) index formula is obtained, the product of the two indexes should give the true value ratio.*'

Symbolically, we should have (without factor 100)

$$P_{01}\times Q_{01}=\frac{\Sigma p_1q_1}{\Sigma p_0q_0}=V_{01}$$

where P_{01} stands for the price change for the current year on the base year, Q_{01} for the quantity change for the current year on the base year, p_1q_1 for the total value (Price × quantity) in the current year, p_0q_0 for the total value in the base year and V_{01} for the value index.

Fisher' formula satisfies the factor reveral test.

Simple Aggregative Method and Factor Reversal Test

$$P_{01}=\frac{\Sigma p_1}{\Sigma p_0},\ \text{(without factor 100)},\ Q_{01}=\frac{\Sigma q_1}{\Sigma q_0}$$

$$\therefore \quad P_{01}\times Q_{01}=\frac{\Sigma p_1}{\Sigma p_0}\times\frac{\Sigma q_1}{\Sigma q_0}\neq\frac{\Sigma p_1q_1}{\Sigma p_0q_0}$$

This method does not satisfy factor reversal test.

Laspeyre's Method and Factor Reversal Test

$$P_{01}=\frac{\Sigma p_1q_0}{\Sigma p_0q_0},\quad Q_{01}=\frac{\Sigma q_1p_0}{\Sigma q_0p_0}$$

$$\therefore \quad P_{01}\times Q_{01}=\frac{\Sigma p_1q_0}{\Sigma p_0q_0}\times\frac{\Sigma q_1p_0}{\Sigma q_0p_0}\neq\frac{\Sigma p_1q_1}{\Sigma p_0q_0}$$

This method also does not satisfy factor reversal test

Paasche's Method and Factor Reversal Test

$$P_{01}=\frac{\Sigma p_1q_1}{\Sigma p_0q_1},\quad Q_{01}=\frac{\Sigma q_1p_1}{\Sigma q_0p_1}$$

$$\therefore\quad P_{01}\times Q_{01}=\frac{\Sigma p_1q_1}{\Sigma p_0q_1}\times\frac{\Sigma q_1p_1}{\Sigma q_0p_1}\neq\frac{\Sigma p_1q_1}{\Sigma p_0q_0}$$

This method also does not satisfy Factor reversal test.

Dorbish and Bowley's Method and factor Reversal Test.

$$P_{01}=\tfrac{1}{2}\left[\frac{\Sigma p_1q_0}{\Sigma p_0q_0}+\frac{\Sigma p_1q_1}{\Sigma p_0q_1}\right],\ Q_{01}=\tfrac{1}{2}\left[\frac{\Sigma q_1p_0}{\Sigma q_0p_0}+\frac{\Sigma q_1p_1}{\Sigma q_0p_1}\right]$$

$$\therefore\quad P_{01}\times Q_{01}=\tfrac{1}{2}\left[\frac{\Sigma p_1q_0}{\Sigma p_0q_0}+\frac{\Sigma p_1q_1}{\Sigma p_0q_1}\right]\times\tfrac{1}{2}\left[\frac{\Sigma q_1p_0}{\Sigma q_0p_0}+\frac{\Sigma q_1p_1}{\Sigma q_0p_1}\right]\neq\frac{\Sigma p_1q_1}{\Sigma p_0q_0}$$

This method also does not satisfy the factor reversal test.

Marshall-Edgeworth Method and Factor Reversal Test.

$$P_{01}=\frac{\Sigma p_1q_0+\Sigma p_1q_1}{\Sigma p_0q_0+\Sigma p_0q_1},\quad Q_{01}=\frac{\Sigma q_1p_0+\Sigma q_1p_1}{\Sigma q_0p_0+\Sigma q_0p_1}$$

$$\therefore\quad P_{01}\times Q_{01}=\frac{\Sigma p_1q_0+\Sigma p_1q_1}{\Sigma p_0q_0+\Sigma p_0q_1}\times\frac{\Sigma q_1p_0+\Sigma q_1p_1}{\Sigma q_0p_0+\Sigma q_0p_1}\neq\frac{\Sigma p_1q_1}{\Sigma p_0q_0}$$

This method also does not satisfy factor reversal test.

Fisher's Ideal Method and Factor Reversal Test.

$$P_{01}=\sqrt{\frac{\Sigma p_1q_0}{\Sigma p_0q_0}\times\frac{\Sigma p_1q_1}{\Sigma p_0q_1}}.\quad Q_{01}=\sqrt{\frac{\Sigma q_1p_0}{\Sigma q_0p_0}\times\frac{\Sigma q_1p_1}{\Sigma q_0p_1}}$$

$$\therefore\quad P_{01}\times Q_{01}=\sqrt{\frac{\Sigma p_1q_0}{\Sigma p_0q_0}\times\frac{\Sigma p_1q_1}{\Sigma p_0q_1}\times\frac{\Sigma q_1p_0}{\Sigma q_0p_0}\times\frac{\Sigma q_1p_1}{\Sigma q_0p_1}}$$

$$=\frac{\Sigma p_1q_1}{\Sigma p_0q_0}$$

Hence only this method satisfies the factor reversal test.

Note. Since Fisher's index is the only index which satisfies both the time reversal test and factor reversal test, it is termed as Fisher's Ideal Index.

(*iv*) **Circular Test.**

This is another test for the adequacy of an index number. This was first suggested by Westergaard and highly favoured by C.M. Walsch who gave it the name of circular test. This test is an extension of the time reversal test. This requires the index to work in a circular manner and this property enables us to find the index numbers from period to period without referring back to the original base each time. In other

words, if an index number is computed for period '1' on the base period '0', another index number is computed for the period 2 on the base period 1, and still another index number is computed for period 3 on the base period 2, the product should be equal to 1 Symbolically,

$$P_{01} \times P_{12} \times P_{23} \times \ldots\ldots \times P_{n-1,\, n} \times P_{n,\, 0} = 1 \qquad \ldots(A)$$

where indices are considered without the factor 100

By time reversal test, we have

$$P_{0n} \times P_{n0} = 1$$

or $$P_{n0} = \frac{1}{P_{0n}}$$

Substituting this value in (A) we get

$$P_{01} \times P_{12} \times P_{23} \times \ldots\ldots \times P_{n-1,\, n} \times \frac{1}{P_{0n}} = 1$$

Hence by cross multiplication

$$P_{01} \times P_{12} \times P_{23} \times \ldots\ldots \times P_{n-1,\, n} = P_{0n}$$

Let us consider an example. If the 1980 index with base 1975 is 2, and 1975 index with 1970 as base is also 2, then the 1980 index with 1970 as base must be 4. In other words, we should get a consistent index for 1980 with base 1970 by multiplying the 1980 index with base 1975 by the corresponding index for 1975 with base 1970.

This test is met by practically none of the index number formulae. Exceptions are simple geometric mean of price relatives, simple aggregative method and weighted aggregative method with fixed weights. Let us see the application of this test to various index number formulae.

Simple Aggrigative Method and Circular Test.

According to circular Test $P_{01} \times P_{12} \times P_{20} = 1$

$$P_{01} = \frac{\Sigma p_1}{\Sigma p_0},\ P_{12} = \frac{\Sigma p_2}{\Sigma p_1},\ P_{20} = \frac{\Sigma p_0}{\Sigma p_2}$$

$$\therefore \quad P_{01} \times P_{12} \times P_{20} = \frac{\Sigma p_1}{\Sigma p_0} \times \frac{\Sigma p_2}{\Sigma p_1} \times \frac{\Sigma p_0}{\Sigma p_2} = 1$$

Therefore, this method satisfies the circular Test.

Laspeyre's Method and Circular Test.

$$P_{01} = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0},\ P_{12} = \frac{\Sigma p_2 q_1}{\Sigma p_1 q_1},\ P_{20} = \frac{\Sigma p_0 q_2}{\Sigma p_2 q_2}$$

$$\therefore \quad P_{01} \times P_{12} \times P_{20} = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_2 q_1}{\Sigma p_1 q_1} \times \frac{\Sigma p_0 q_2}{\Sigma p_2 q_2} \neq 1$$

This method does not satisfy the circular Test.

Paasche's Method and Circular Test.

$$P_{01}=\frac{\Sigma p_1q_1}{\Sigma p_0q_1},\ P_{12}=\frac{\Sigma p_2q_2}{\Sigma p_1q_2},\ P_{20}=\frac{\Sigma p_0q_0}{\Sigma p_2q_0}$$

$$\therefore \quad P_{01}\times P_{12}\times P_{20}=\frac{\Sigma p_1q_1}{\Sigma p_0q_1}\times\frac{\Sigma p_2q_2}{\Sigma p_1q_2}\times\frac{\Sigma p_0q_0}{\Sigma p_2q_0}\neq 1$$

This method also does not satisfy the circular Test

Dorbish and Bowley's Method and Circular Test

$$P_{01}=\frac{1}{2}\left[\frac{\Sigma p_1q_0}{\Sigma p_0q_0}+\frac{\Sigma p_1q_1}{\Sigma p_0q_1}\right],\ P_{12}=\frac{1}{2}\left[\frac{\Sigma p_2q_1}{\Sigma p_1q_1}+\frac{\Sigma p_2q_2}{\Sigma p_1q_2}\right]$$

$$P_{20}=\frac{1}{2}\left[\frac{\Sigma p_0q_2}{\Sigma p_2q_2}+\frac{\Sigma p_0q_0}{\Sigma p_2q_0}\right]$$

$$\therefore \quad P_{01}\times P_{12}\times P_{20}\neq 1$$

Hence this method also does not satisfy the circular Test.

Fisher's Ideal Method and Circular Test.

$$P_{01}=\sqrt{\frac{\Sigma p_1q_0}{\Sigma p_0q_0}\times\frac{\Sigma p_1q_1}{\Sigma p_0q_1}},\ P_{12}=\sqrt{\frac{\Sigma p_2q_1}{\Sigma p_1q_1}\times\frac{\Sigma p_2q_2}{\Sigma p_1q_2}}$$

$$P_{20}=\sqrt{\frac{\Sigma p_0q_2}{\Sigma p_2q_2}\times\frac{\Sigma p_0q_0}{\Sigma p_2q_0}}$$

$$\therefore \quad P_{01}\times P_{12}\times P_{20}$$

$$=\sqrt{\frac{\Sigma p_1q_0}{\Sigma p_0q_0}\times\frac{\Sigma p_1q_1}{\Sigma p_0q_1}\times\frac{\Sigma p_2q_1}{\Sigma p_1q_1}\times\frac{\Sigma p_2q_2}{\Sigma p_1q_2}\times\frac{\Sigma p_0q_2}{\Sigma p_2q_2}\times\frac{\Sigma p_0q_0}{\Sigma p_2q_0}}\neq 1$$

Therefore, even Fisher's Ideal method does not satisfy the circular Test.

Kelly's Method and Circluar Test.

$$P_{10}=\frac{\Sigma p_1q}{\Sigma p_0q},\ P_{12}=\frac{\Sigma p_2q}{\Sigma p_1q},\ P_{20}=\frac{\Sigma p_0q}{\Sigma p_2q}$$

$$\therefore \quad P_{01}\times P_{12}\times P_{20}=\frac{\Sigma p_1q}{\Sigma p_0q}\times\frac{\Sigma p_2q}{\Sigma p_1q}\times\frac{\Sigma p_0q}{\Sigma p_2q}=1$$

Hence Kelly's method involving fixed weights satisfies the circular test.

Example 15. Compute Fisher's Ideal Price Index Number from the following data. Also show that Fisher's Ideal Index staisfies the Time reversal and Factor reversal tests.

	Base year		*Current year*	
Commodity	*Price*	*Quantity*	*Price*	*Quantity*
A	4	10	5	12
B	6	8	7	10
C	10	5	12	4
D	3	12	4	15
E	5	7	5	8

Sol.

Commodity	P_0	q_0	P_1	q_1	$P_1 q_0$	P_0q_0	P_1q_1	P_0q_1
A	4	10	5	12	50	40	60	48
B	6	8	7	10	56	48	70	60
C	10	5	12	4	60	50	48	40
D	3	12	4	15	48	36	60	45
E	5	7	5	8	35	35	40	40
					$\Sigma P_1q_0=249$	$\Sigma P_0q_0=209$	$\Sigma P_1q_1=278$	$\Sigma P_0q_1=233$

Fisher's Price Index Number

$$P_{01}=\sqrt{\frac{\Sigma P_1q_0}{\Sigma P_0q_0}\times\frac{\Sigma P_1q_1}{\Sigma P_cq_1}}\times 100$$

$$=\sqrt{\frac{249}{209}\times\frac{278}{233}}\times 100=\sqrt{1{\cdot}19\times 1{\cdot}193}\times 100$$

$$=\sqrt{1{\cdot}41967}\times 100=1{\cdot}1915\times 100$$

$$=119{\cdot}15$$

Time Reversal Test is satisfied when $Po_1\times p_{10}=1$ when 100 is **Ignored.**

Fisher's Formula and Time Reversal Test

$$Po_1=\sqrt{\frac{\Sigma P_1q_0}{\Sigma P_0q_0}\times\frac{\Sigma P_1q_1}{\Sigma P_0q_1}},\quad P_{10}=\sqrt{\frac{\Sigma P_0q_1}{\Sigma P_1q_1}\times\frac{\Sigma P_0q_0}{\Sigma P_1q_0}}$$

$$\therefore\ P_{01}\times P_{10}=\sqrt{\frac{\Sigma P_1q_0}{\Sigma P_0q_0}\times\frac{\Sigma P_1q_1}{\Sigma P_0q_1}\times\frac{\Sigma P_0q_1}{\Sigma P_1q_1}\times\frac{\Sigma P_0q_0}{\Sigma P_1q_0}}$$

$$=\sqrt{\frac{249}{209}\times\frac{278}{233}\times\frac{233}{278}\times\frac{209}{249}}$$

$$=\sqrt{1}=1.$$ **Hence Proved.**

Factor Reversal Test is satisfied **when** $P_{01}\times Q_{01}=\frac{\Sigma P_1q_1}{\Sigma P_0q_0}$ **(when 100 is ignored from each index)**

Fisher's Formula and Factor Reversal Test

$$P_{01}=\sqrt{\frac{\Sigma P_1q_0}{\Sigma P_0q_0}\times\frac{\Sigma P_1q_1}{\Sigma P_0q_1}}\qquad Qo_1=\sqrt{\frac{\Sigma q_1P_0}{\Sigma q_0P_0}\times\frac{\Sigma q_1P_1}{\Sigma q_0P_1}}$$

$$\therefore\quad P_{01}\times Q_{01}=\sqrt{\frac{\Sigma P_1q_0}{\Sigma P_0q_0}\times\frac{\Sigma P_1q_1}{\Sigma P_0q_1}\times\frac{\Sigma q_1P_0}{\Sigma q_0P_0}\times\frac{\Sigma q_1P_1}{\Sigma q_0P_1}}$$

$$=\sqrt{\frac{249}{209}\times\frac{278}{233}\times\frac{233}{209}\times\frac{278}{249}}$$

$$=\sqrt{\frac{278\times 278}{209\times 209}}=\frac{278}{209}=\frac{\Sigma P_1q_1}{\Sigma P_0q_0}.\quad \text{Hence Proved}$$

Thus, Fisher's Formula satisfies both the tests.

Example 16. Construct Fisher's Ideal quantity index number to the following data. Also see if it satisfies Time reversal test and factor reversal test.

	1980		1981	
Item	***Price***	***Value***	***Price***	***Value***
A	5	50	4	48
B	8	48	7	49
C	6	18	5	20
D	4	16	4	16

Solution. Since value = Price × Quantity

or $\text{Quantity}=\dfrac{\text{Value}}{\text{Price}}$.

So first Quantities will be determined from value figures.

Construction of Fisher's Quantity Index Number

	1980		1981					
Item	p_0	q_0	p_1	q_1	p_0q_0	p_1q_1	p_0q_1	p_1q_0
A	5	10	4	12	50	48	60	40
B	8	6	7	7	48	49	56	42
C	6	3	5	4	18	20	24	15
D	4	4	4	4	16	16	16	16
					132	133	156	113

$$^{F}Q_{01}=\sqrt{\frac{\Sigma q_1p_0}{\Sigma q_0p_0}\times\frac{\Sigma q_1p_1}{\Sigma q_0p_1}}\times 100$$

$$= \sqrt{\frac{156}{132} \times \frac{133}{113}} \times 100 = \sqrt{\frac{20748}{14916}} \times 100$$

$$= \sqrt{1 \cdot 390989} \times 100 = 1 \cdot 1794 \times 100 = 117 \cdot 94$$

Time Reversal Test is satisfied if

$$Q_{0 \atop 1} \times Q_{1 \atop 0} = 1 \qquad \text{(without 100)}$$

i.e. $$\sqrt{\frac{\Sigma q_1 p_0}{\Sigma q_0 p_0} \times \frac{\Sigma q_1 p_1}{\Sigma q_0 p_1} \times \frac{\Sigma q_0 p_1}{\Sigma q_1 p_1} \times \frac{\Sigma q_0 p_0}{\Sigma p_1 q_0}}$$

$$= \sqrt{\frac{156}{132} \times \frac{133}{113} \times \frac{113}{133} \times \frac{132}{156}} = \sqrt{1} = 1$$

Factor Reversal Test is satisfied if $Q_{0 \atop 1} \times P_{0 \atop 1} = \frac{\Sigma p_1 q_1}{\Sigma p_1 q_0}$

L.H.S. $$Q_{0 \atop 1} \times P_{0 \atop 1} = \sqrt{\frac{\Sigma q_1 p_0}{\Sigma q_0 p_0} \times \frac{\Sigma q_1 p_1}{\Sigma q_0 p_1} \times \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}}$$

$$= \sqrt{\frac{156}{132} \times \frac{133}{113} \times \frac{113}{132} \times \frac{133}{156}} = \sqrt{\frac{(133)^2}{(132)^2}}$$

or $$Q_{0 \atop 1} \times p_{0 \atop 1} = \frac{133}{132}$$

R.H.S. $$\frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}$$

$$= \frac{133}{132}$$

Hence L.H.S. = R.H.S.

Thus, Fisher's Index satifies both time reversal and factor reversal tests.

The Chain Index Numbers.

So far we have studied the construction of index numbers by using a fixed base, *i.e.* keeping the base period fixed throughout the computation of the index number. Contrasted with it is the chain base or the shifting base method in which relatives for each year are calculated upon the prices of the preceding year. Thus, the base year is not fixed but changes from year to year. For example, for 1980, 1979 will be the base; for 1981, 1980 will be the base and so on. The relatives obtained are known as Link Relatives or Link Index Numbers. These link relatives are chained together by successive multiplication to get a chain index.

Steps for Constructing Chain Index.

(*i*) First we find link relatives by expressing price for each year as percentage of preceding year

$$\text{Link relative} = \frac{\text{Current year price}}{\text{Previous year price}} \times 100$$

(*ii*) These link relatives are chained together by successive multiplication to form a chain index. The following formula is used.

$$\text{Chain index for current year} = \frac{\text{Current year link relative} \times \text{chain index of preceding year}}{100}$$

The chain index of the first year is taken as 100. This process is explained in following example

17 From the following data of wholesale prices of cotton construct Index Number by chain base method.

Year	Price	Year	Price
1964	75	1969	70
1965	50	1970	69
1966	65	1971	75
1967	60	1972	84
1968	72	1973	80

Solution.

Calculations for Chain Index

Year	Price	Link Relatives	Chain Base Index Numbers	Fixed Base Index Numbers (1964=100)
1964	75	100	100	100
1965	50	$\frac{50}{75} \times 100 = 66{\cdot}67$	$\frac{66{\cdot}67 \times 100}{100} = 66{\cdot}67$	$\frac{50}{75} \times 100 = 66{\cdot}67$
1966	65	$\frac{65}{50} \times 100 = 130$	$\frac{130 \times 66{\cdot}67}{100} = 86{\cdot}67$	$\frac{65}{75} \times 100 = 86{\cdot}67$
1967	60	$\frac{60}{65} \times 100 = 92{\cdot}31$	$\frac{92{\cdot}31 \times 86{\cdot}67}{100} = 80$	$\frac{60}{75} \times 100 = 80$
1968	72	$\frac{72}{60} \times 100 = 120$	$\frac{120 \times 80}{100} = 96$	$\frac{72}{75} \times 100 = 96$
1969	70	$\frac{70}{72} \times 100 = 97{\cdot}22$	$\frac{97{\cdot}22 \times 96}{100} = 93{\cdot}33$	$\frac{70}{75} \times 100 = 93{\cdot}33$
1970	69	$\frac{69}{70} \times 100 = 98{\cdot}57$	$\frac{98{\cdot}57 \times 93{\cdot}33}{100} = 92$	$\frac{69}{75} \times 100 = 92$

1971	75	$\frac{75}{69} \times 100 = 108{\cdot}69$	$\frac{108{\cdot}69 \times 92}{100} = 100$	$\frac{75}{75} \times 100 = 100$
1972	84	$\frac{84}{75} \times 100 = 112$	$\frac{112 \times 100}{100} = 112$	$\frac{84}{75} \times 100 = 112$
1973	80	$\frac{80}{84} \times 100 = 95{\cdot}24$	$\frac{95{\cdot}24 \times 112}{100} = 106{\cdot}67$	$\frac{80}{75} \times 100 = 106{\cdot}67$

18. From the following annual average prices of three commodities given in rupees per unit, find chain index numbers based on 1977.

Commodities	1977	1978	1979	1980	1981
x	8	10	12	15	15
y	10	2	15	18	20
z	6	9	12	15	18

Sol. Calculation of chain Indices.

Commodities	1977	1978	1979	1980	1981
x	100	$\frac{10}{8} \times 100 = 125$	$\frac{12}{10} \times 100 = 120$	$\frac{15}{12} \times 100 = 125$	$\frac{15}{15} \times 100 = 100$
y	100	$\frac{2}{10} \times 100 = 20$	$\frac{15}{2} \times 100 = 750$	$\frac{18}{15} \times 100 = 120$	$\frac{20}{18} \times 100 = 111{\cdot}1$
z	100	$\frac{9}{6} \times 100 = 150$	$\frac{12}{9} \times 100 = 133{\cdot}3$	$\frac{15}{12} \times 100 = 120$	$\frac{18}{15} \times 100 = 120$
Total of link relative	300	295	1003·3	365	331·1
Average of link relative	100	98·3	334·4	121·67	110·37
Chain index (1977=100)	100	$\frac{98{\cdot}3 \times 100}{100} = 98{\cdot}3$	$\frac{334{\cdot}4 \times 98{\cdot}3}{100} = 328{\cdot}7$	$\frac{121{\cdot}67 \times 328{\cdot}7}{100} = 399{\cdot}9$	$\frac{110{\cdot}37 \times 399{\cdot}9}{100} = 441{\cdot}37$

Converting Chain Base Index to Fixed Base Index : The following formula is used for converting index numbers with chain base to fixed base index numbers.

$$\text{Current Year Fixed Base Index} = \frac{\text{Current Year Chain Base Index} \times \text{Previous Year Fixed Base Index}}{100}$$

The following example will explain the procedure clearly

19. From chain base index numbers given below prepare fixed base index number.

1975	1976	1977	1978	1979	1980
92	102	104	98	103	101·

Sol. Calculation of fixed base index number from chain base index numbers.

Year	1975	1976	1977	1978	1979	1980
Chain Base Index No.	92	102	104	98	103	101
Fixed Base Index No.	92	$\frac{102\times92}{100}$ $=93{\cdot}8$	$\frac{104\times93{\cdot}8}{100}$ $=97{\cdot}6$	$\frac{98\times97{\cdot}6}{100}$ $=95{\cdot}6$	$\frac{103\times95{\cdot}6}{100}$ $=98{\cdot}5$	$\frac{101\times98{\cdot}5}{100}$ $=99{\cdot}5$

Converting Fixed Base Index to Chain Based Index.

We can easily convert the fixed base index numbers to chain base index numbers using the following formula.

$$\text{Chain Base Index} = \frac{\text{Current year's fixed base index}}{\text{Previous year's fixed base index}} \times 100$$

20 From the fixed base index numbers given below find chain base index numbers.

Year	1976	1977	1978	1979	1980	1981
Index No.	200	220	240	250	280	300

Sol. Computation of Chain Base Index Numbers.

Year	Index No.	Chain Base Index No.
1976	200	100
1977	220	$\frac{220}{200}\times100=110$
1978	240	$\frac{240}{220}\times100=109{\cdot}09$
1979	250	$\frac{250}{240}\times100=104{\cdot}17$
1980	280	$\frac{280}{250}\times100=112$
1981	300	$\frac{300}{280}\times100=107{\cdot}14.$

Difference between Chain Base Method and Fixed Base Method.

1. In the chain base method, there is no fixed base. Base period is fixed under fixed base method-

2. Immediately preceding period is taken as base under chain base method while base period is arbitrarily chosen under fixed base method.

3. Comparison between two adjacent time periods is possible under chain base method through link relatives. However, such comperison is not possible under fixed base method.

4. The calculation of chain base index is difficult while the calculation of fixed base index is very simple.

5. The chain base index requires imformation about ail the periods. The subsequent chain indices cannot be computed if data for any one year is missing. In the case of fixed base method this problem does not arise.

6. Any error is likely to affect the entire series of chain index numbers. In the case of fixed base method, the error is confined to the index of that year only.

7. Chain base indices are free, to a great extent, from seasonal variations. On the other hand, fixed base index numbers contain seasonal variations.

8. In the chain base method new items can be introduced and cbsolete items can be deleted without needing the recalculation of the entire series. In the fixed method any alteration in the list of commodites requires the recasting of the entire series of index numbers.

9. Weights can be adjusted frequently in chain base method while the fixed base method does not allow the frequent adjustment of weights.

10. The chain base index numbers are suitable for short periods only. They depict only the short term tendency of data. On the other hand, fixed base index numbers are useful for long periods because they indicate the long term tendency of data.

Merits and Demerits of Chain Base Method

Merits 1: In economics and business, we are generally concerned with making comparsions with the perevious period and not with any distant past so chain base indices are useful.

2. This method allows the introduction of new items and deletion of old ones.

3 Index numbers obtained by this method are free from seasonal variations.

4. This method provides a more direct comparison between successive years than the fixed base method and makes it easy to change the base year when desired.

5. Revised weighting can be assigned at any time.

Demerits. The long range comparisons by this method are not strictly valid. Chain base indices are comparatively difficult to compute. If there is only one series of observations, then the fixed base indices and chain base indices will always be the same. In such case we should prefer the fixed base method since it requires much less calculations as compared with the chain base method.

Cost of Living Index Numbers

Cost of living or consumer price index numbers are designed to study the effect of changes in prices on the people as consumers. The

Labour Bureau in their publication "A Guide to Consumer Price Index Numbers" have explained that **the cost of living index number is intended to show over time the average change in the prices paid by the consumers belonging to the population group proposed to be covered by the index for a fixed list of goods and services consumed by them**. The need for constructing cost of living indices arises because of the fact that the general index numbers fail to give an idea of the effect of the change in the general price level on the cost of living of different classes of people. Due to the wide variations in the tastes, habits, fashions, customs of different sections or classes of people, their consumption patterns of various commodities also differ widely from class to class or group to group (rich, poor and middle income group). Even within the same group wide variations are found from region to region or time to time. **Due to this, the price changes affect different groups of people differently.** Hence, to study the effect of rise or fall in the prices of various commodities consumed by a particular group or class of people on their cost of living, the **cost of living index numbers** are constructed separately for different classes of people or groups or sections of the society and also for different geographical areas like town, city, rural area, urban area, hilly area etc. The cost of living index numbers enable us to measure the average increase or decrease in the cost of maintaining a given standard of living from year to year. As such, they analyse the effect of price changes on the standard of living. Since wages and salaries in most countries are adjusted in accordance with the consumer price index, these index numbers have a great significance.

Utility of Consumer Price Indices

1. Cost of living index numbers are used to determine the purchasing power of money and for computing real income from the nominal or money income. In other words,

$$\text{Purchasing Power of Money} = \frac{1}{\text{Cost of living index}}$$

and

$$\text{Real Income} = \frac{\text{Nominal Income}}{\text{cost of living index}}$$

Thus, changes in purchasing power of money or real income or real wages can be measured with the help of cost of living indices.

2. The most widely publicised use of consumer price index is for the automatic adjustment of wages under "escalator clauses" in collective bargaining agreements. Dearness allowance of employees is also linked with the consumer price indices.

3. Consumer price indices are used in framing wage policy, price policy, taxation policy and general economic polices.

4. Consumer price indices are also used for analysing markets for particular kinds of goods and services.

5. Consumer price indices are used for deflating income and value series in national accounts.

Main steps in the Construction of Cost of Living Index Numbers.

It would be an endless task to describe all the details of computation of the Consumer Price Index. The main steps involved in the construction of a cost of living index can be summarised as follows :

(*i*) Decision about the class of people for whom the index is meant.

The first step that is taken for the construction of cost of living index number series is to decide the class of people for which the index numbers are to be compiled, i.e., whether the index relates to rich, poor middle class, labour class, industrial workers, agricultural workers, central government employees etc. The scope or coverage of index must be clearly defined at the very outset. For example, an index of industrial workers covers all the workers employed in clearly defined and recognised industrial units. If it is desired to know the cost of living index of sports goods workers only, it must be clearly stated. In addition to the class of people, the geographical area such as rural area, urban area, city or town or locality of a town, etc., should also be clearly defined. For example, if we are constructing a cost of living index number for sports goods workers, is it all the workers say, in Jalandhar or in a particular locality of Jalandhar or in a particular unit ? Thus the identification of units with regard to their location is of utmost importance to know clearly the peculiar features of the index.

(*ii*) Conducting Family Budget Enquiry.

The next step is to conduct a family budget enquiry covering the class of people for whom the index is desired. The family budget or expenditure enquiry reveals the amount that an average family in the class of people selected depends on different items of consumption. The enquiry is made by selecting a random sample of adequate number of representative families from the class of people for whom the index is prepared. The Lottery Method may be used to select the families. The enquiry should be conducted in a normal period, i.e., a period free from boom, depression, war, famine etc. Since data is to be collected about the expenditure, which is the product of price and quantity, the commodities selected should be representative of the class of people for whom index is desired. The commodities are broadly classified into the following five major groups.

(*a*) Food

(*b*) Clothing

(*c*) Fuel and lighting

(*d*) House rent

(*e*) Miscellaneous.

Each of these major groups is further subdivided into smaller groups termed as sub-groups. For example, the broad head 'Food' may be sub-divided into wheat, rice, pulses, sugar, potatoes, meat, milk, fruits, spices, pan supari and tobacco etc. Similary, the broad head 'clothing' may cover clothing, bedding, footwear etc. The last group 'miscellaneous' includes items such as medicine and medical charges, amusement,

education, newspaper, gifts, charities, servants, dhobi and soap, transport, household requisites, personal care and effects and so on. It, however, does not include non-consumption money transaction, such as payments towards provident fund, insurance premiums, purchase of saving certificates and bonds etc. It should be noted that a cost of living index number should include only those commodities which are generally used by the class of people concerned, which are not subject to wide variation in quality nor to seasonal alterations in supply, and for which regular and comparable quotations of prices are obtainable. After conducting the family budget enquiry, an average budget is drawn up. It is considered standard budget for that class people.

(*iii*) **Obtaining Price Quotations.**

Since cost of living indices are constructed from the point of view of consumers, it becomes essential to collect retail prices. The prices are to be collected both for the base period and the current period. The collection of retail prices is the most difficult task because retail prices vary from place to place and even at one place from shop to shop and even at one shop from customer to customer. As such the index number constructed by the use of retail prices cannot be used for different places nor for different classes of people at one place. The following points are taken into consideration while collecting price quotations :

(1) The retail prices should relate to a fixed list of items determined by average or standard budget. For each item quality should be fixed by means of suitable specification.

(2) The price quotations for the selected commodities should be obtained from local markets where the class of people reside or from super bazars, fair-price shops or from departmental stores from where they usually make their purchases.

(3) Retail prices should be those charged to consumers inclusive of taxes for actual cash sales. Prices charged for damaged, stale or otherwise imperfect goods should be ignored.

(4) If during a period of rationing or price control exhorbitant prices are charged openly to the groups to which the index applies, such prices should be taken into consideration along with the controlled prices.

(5) Account should be taken of discounts given automatically to all customers. Similarly, 'Sale', 'Cut' or reduced price should be mentioned where they are charged.

(6) Since prices form the most important element of consumer price indices attention should also be paid to the methods of price collection and price collection personnel. Price quotations are generally obtained through persons specially appointed for the purpose, but sometimes price quotations are also obtained by post or through published price lists. The secondary sources may be either standard

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trade journals, or publications of government, Central Bank, or reports of companies etc. where special agents are employed, it is essential to give them intensive training. The collected prices may be checked by obtaining duplicate prices through different agents or by making actual purchase of the goods priced.

(iv) Averaging the Price Quotations

After obtaining the retail price quotations, an average price should be computed for each item included in the enquiry. Average price for each commodity is determined both for the base period and current period. The method of averaging price quotations should be such as to yield unbiased estimates of the average of prices as being actually paid by the group as a whole.

(v) Application of Suitable Weights.

To convert these average prices into index numbers the average price or their relatives must, of necessity, be weighted. The need for weighing arises because the relative importance of various items for different classes of people is not the same. That is why a cost living index is always a weighted index. The various items that are included in each consumption group of the average budget are assigned weights in proportion to their importance within that group, on the basis of the figures of either expenditure or quantity of consumption in the average budget.

Methods of Constructing Cost of Living Index Number

The cost of living index number can be constructed by the following methods,

(i) Aggregate Expenditure Method.

(ii) Family Budget Method.

(i) **Aggregate Expenditure Method** : The following steps are taken to construct index number.

(a) Estimate the quantities consumed by the particular class of people in the base year. These base year quantities (q_0) will constitute the weights.

(b) Multiply prices of commodities in the Current year (p_1) by quantities consumed in the base year (q_0).

(c) The sum of the products in step (b) is nothing but the aggregate expenditure on buying those commodities in the current year (*i.e.* Σp_1q_0).

(d) Similarly, obtain the aggregate expenditure for the base year (*i e.* Σp_0q_0).

(e) Divide the aggregate expenditure of the current year by the aggregate expenditure for the base year.

(f) Multiply the quotient obtained in step (e) by 100. The result is the consumer price index. Symbolically,

$$\text{Consumer Price Index} = \frac{\Sigma p_1q_0}{\Sigma p_0q_0} \times 100$$

This formula is the same as Laspeyres's method of determining index numbers.

(*ii*) **Family Budget Method** : This is also called the method of weighted relatives. In this method the cost of living index is obtained on taking the weighted average of price relatives, the weights being the values of quantities consumed in the base period. The following steps are taken :

(*a*) Scrutinize the family budgets of a large number of people.

(*b*) Estimate the aggregate expenditure of an average family on various items consumed by the particular class of people. The aggregate expenditure on different items shall constitute the weights (*i.e* $w=p_0q_0$).

(*c*) Current year's prices are converted into price relatives on the basis of base year prices $\left(i.e.\ P=\frac{p_1}{p_0}\times 100 \right)$.

(*d*) Multiply the price relatives by value weights

$$\left(i.e.\ P.W.=\frac{p_1}{p_0}\times 100\times p_0q_0 \right)$$

Find the sum of the products of P and W (*i.e.* ΣPW).

(*e*) Divide the sum obtained in step (*d*) by the sum of the value weights $\left(i.e.\ \frac{\Sigma PW}{\Sigma W} \right)$. This is called the consumer price index.

Symbolically,

$$\text{Consumer Price Index}=\frac{\Sigma PW}{\Sigma W},\ \text{where } P=\frac{p_1}{p_0}\times 100 \text{ and } W=p_0q_0$$

Note. It should be noted that the cost of living index numbers obtained by both the methods are the same. This is because of the fact that the second formula is exactly similar to first formula. In other words,

$$\frac{\Sigma PW}{\Sigma W}=\frac{\Sigma p_1q_0}{\Sigma p_0q_0}\times 100.$$

Substituting the values of P and W on the left hand side, we have

$$\frac{\Sigma PW}{\Sigma W}=\frac{\Sigma\frac{p_1}{p_0}\times 100\times p_0q_0}{\Sigma p_0q_0}$$

$$=\frac{\Sigma p_1q_0}{\Sigma p_0q_0}\times 100 \text{ which is equal to the right hand side.}$$

Example 16. Construct the cost of living index number by (*i*) Aggregate Expenditure Method and (*ii*) Family Budget Method.

Articles	Q. consumed in base Year	Units	Price in base Year (Rs.)	Price in current Year (Rs.)
Wheat	5 mds.	md.	20	22
Rice	6 mds.	md.	15	16

Pulses	2 mds.	md.	25	25
Ghee	2 kg.	kg.	12	15
Sugar	1 md.	md.	20	25
Oil	4 kg.	kg.	6	8
Salt	10 kg.	kg.	10	12
Cloth	30 yds.	yds.	3	4
Fire wood	10 mds.	mds.	1	1·2
Rent	1 house	house	50	8

Sol. Calculation of Cost of living Index No.

Article	Qt. consumed (q_0)	Units	p_0	p_1	p_1q_0	$p_0q_0=W$	$P=p_1/p_0\times 100$	P.W.
Wheat	5 mds.	mds.	20	22	110	100	110	11000
Rice	6 ,,	,,	15	16	96	90	106·67	9600·3
Pulses	2 ,,	,,	25	25	50	50	100	5000
Ghee	2 kg.	kg.	12	15	30	24	125	3000
Sugar	1 md.	md.	20	25	25	20	125	2500
Oil	4 kg.	kg.	6	8	32	24	133·33	3199·92
Salt	10 ,,	,,	10	12	120	100	120	12000
Cloth	30 yds.	yds.	3	4	120	90	133·3	11999·7
Fire-wood	10 mds.	mds.	1	1·2	12	10	120	1200
Rent	1 house	house	50	8	8	50	16	800
					603	558	1089·3	60299·92

By Aggregate Expenditure Method :

$$\text{Cost of living index No.}=\frac{\Sigma p_1q_0}{\Sigma p_0q_0}\times 100=\frac{603}{558}\times 100=108\cdot 04$$

By Family Budget Method

$$\text{Cost of living index No.}=\frac{\Sigma PW}{\Sigma W}=\frac{60299\cdot 92}{558}=108\cdot 06.$$

Example 17. An enquiry into the budget of the middle class families in Bombay gave the following information.

Expenses on	Food 35%	Rent 15%	clothing 20%	Fuel 10%	Miscell. 20%
Prices in (1984 in Rs.)	150	50	100	20	60
Prices in (1985 in Rs.)	174	60	125	25	90

What changes in the cost of living figures of 1985 have taken place as compared to 1984 ?

Sol. **Calculation of Cost of living Index Number.**

Expenses on	W	Prices in 1984 (p_0)	Prices in 1985 (p_1)	P= $p_1/p_0 \times 100$	PW
Food	35	150	174	116	4060
Rent	15	50	60	120	1800
Clothing	20	100	125	125	2500
Fuel	10	20	25	125	1250
Miscell.	20	60	90	150	3000
	$\Sigma W=100$				$\Sigma PW=12610$

$$\text{Cost of living index No.}=\frac{\Sigma PW}{\Sigma W}=\frac{12610}{100}=126\cdot10$$

Example 18. Given below are the prices and weights of five commodities for the year 1980, 1981, 1982.

Commodities	weight	Price in Rupees 1980	1981	1982
A	20	12·00	18·00	24·00
B	15	3·00	6·00	15·00
C	10	12·50	18·75	25·00
D	40	10·00	30·00	50·00
E	15	4·50	9·00	13·50

Calculate weighted price index numbers for 1981 and 1982, taking 1980 as base, using any method.

Sol. **Construction of Cost of Living Index.**

Commodities	Weight	1980 Price p_0	1981 Price p_1	P= $p_1/p_0 \times 100$	PW	1982 Price p_2	P'= $p_2/p_0 \times 100$	P'W
A	20	12·00	18·00	150	3000	24·00	200	4000
B	15	3·00	6·00	200	3000	15·00	500	7500
C	10	12·50	18·75	150	1500	25·00	200	2000
D	40	10·00	30·00	300	12000	50·00	500	20000
E	15	4·50	9·00	200	3000	13·50	300	4500
Total	$\Sigma W=100$				$\Sigma PW=22500$			$\Sigma P'W=38000$

$$\text{Cost of living Index for 1980}=\frac{\Sigma PW}{\Sigma W}=\frac{22500}{100}=225$$

$$\text{Cost of living Index for 1982}=\frac{\Sigma P'W}{\Sigma W}=\frac{38000}{100}=380$$

Example 19. In calculating a certain cost of living index number, the following weights were used. Food 15, clothing 3, rent 4, fuel and light 2, miscellaneous 1. Calculate the index number for a data when the average percentage increases in prices of items in various groups over the base period were 32, 54, 47, 78 and 58 respectively.

Suppose a business executive was earning Rs. 2050 in the base period, what should be his salary in the current period if his standard of living is to remain the same ?

Sol. Construction of cost of Living Index Number.

Group	Average % increase in Price	Group Index (I)	Weight (W)	I.W.
Food	32	132	15	1980
Clothing	54	154	3	462
Rent	47	147	4	588
Fuel and light	78	178	2	356
Miscellaneous	58	158	1	158
			$\Sigma W=25$	$\Sigma IW=3544$

$$\text{Cost of Living Index}=\frac{\Sigma IW}{\Sigma W}=\frac{3544}{25}=141\cdot76$$

The same standard of living will be maintained if the business executive gets $\frac{2050\times141\cdot76}{100}=\text{Rs. } 2906\cdot08$

Example 20. Calculate the cost of Living Index Number using the weighted Geometric Mean.

Group	Food	Fuel and Lighting	Clothing	House Rent	Miscellaneous
Index Number	320	150	200	150	225
Weights	10	2	2	2	4

Sol. Calculation of Cost of Living Index.

Group	Index number (I)	Weight (W)	log I	(log I)·W
Food	350	10	2·5441	25·4410
Fuel and lighting	150	2	2·1761	4·3522
Clothing	200	2	2·3010	4·6020
House rent	150	2	2·1761	4·3522
Miscellaneous	225	4	2·3522	9·4088
		$\Sigma W=20$		$\Sigma(\log I)W=48\cdot1562$

$$\text{Cost of living index}=AL\left[\frac{\Sigma(\log I)W}{\Sigma W}\right]=AL\left[\frac{48\cdot1562}{20}\right]$$
$$=AL[2\cdot4078]=255\cdot8$$

MISCELLANEOUS EXAMPLES

Example 21. From the following information regarding the price of three commodities, construct index number taking average prices as base.

Year	*Unit*	*Commodity A*	*Commodity B*	*Commodity C*
I	Price per Rupee	10 kg.	4 kg.	2 kg.
II	,, ,, ,,	8 kg.	3 kg.	3 kg.
III	,, ,, ,,	7 kg.	3 kg.	3 kg.

Solution. Since prices are given in terms of quantity (in kg.) per rupee, let us first convert prices into rupees for 40 kg. These prices are then averaged for each commodity. This average will be used to find price relatives.

Computation of Index Number

Base Average Price (in Rupees)=100

			First Year		*Second Year*		*Third Year*	
Commodities	*Unit*	*Average Price*	*Price*	*Price Relative*	*Price*	*Price Relative*	*Price*	*Price Relative*
A	40 kg.	4·9**	4*	81·6	5	102·0	5·7	116·3
B	40 kg.	12·2	10	82·0	13·3	109·0	13·3	109·0
C	40 kg.	15·5	20	129·0	13·3	85·8	13·3	85·8
Total				292 6		296·8		311·1
Index Number				$\frac{292\cdot6}{3}=97\cdot5$		$\frac{296\cdot8}{3}=98\cdot9$		$\frac{311\cdot1}{3}=103\cdot7$

Example 22. The following table gives the annual income of a teacher and the general index number of price during 1977-85. Prepare the index number to show the changes in the real income of the teacher.

Year	*Income (Rs.)*	*Price Index*	*Year*	*Income (Rs.)*	*Price Index*
1977	360	100	1982	640	290
1978	420	104	1983	680	300
1979	500	115	1984	720	320
1980	550	160	1985	750	330
1981	600	280			

*Price of 10 kg. of commodity A=Re. 1.

$\therefore$ Price of 40 kg.$=\frac{1}{10}\times40=$Rs. 4. Similarly Prices of all other commodities are calculated in different years

**4·9 is the average of 4, 5, 5·7 *i.e.* $\frac{4+5+5\cdot7}{3}$. Other averages are also calculated in the same way.

Solution. *Construction of Real Income Index showing changes in the Real Income of teacher.*

Year	Income Rs.	Price Index	Real Income	Real Income Index
1977	360	100	$\frac{360}{100}\times 100=360$	100
1978	420	104	$\frac{420}{104}\times 100=403{\cdot}85$	$\frac{403{\cdot}85}{360}\times 100=112{\cdot}18$
1979	500	115	$\frac{500}{115}\times 100=434{\cdot}78$	$\frac{434{\cdot}78}{360}\times 100=120{\cdot}77$
1980	550	160	$\frac{550}{160}\times 100=343{\cdot}75$	$\frac{343{\cdot}75}{360}\times 100=95{\cdot}49$
1981	600	280	$\frac{600}{280}\times 100=214{\cdot}28$	$\frac{214{\cdot}28}{360}\times 100=59{\cdot}52$
1982	640	290	$\frac{640}{290}\times 100=220{\cdot}69$	$\frac{220{\cdot}69}{360}\times 100=61{\cdot}30$
1983	680	300	$\frac{680}{300}\times 100=226{\cdot}67$	$\frac{226{\cdot}67}{360}\times 100=62{\cdot}96$
1984	720	320	$\frac{720}{320}\times 100=225{\cdot}0$	$\frac{225{\cdot}0}{360}\times 100=62{\cdot}5$
1985	750	330	$\frac{750}{330}\times 100=227{\cdot}27$	$\frac{227}{360}\times 100=63{\cdot}13$

Examdle 23. Construct Fisher's Ideal quantity index number to the following data. Also see if it satisfies Time reversal test and factor reversal test.

	1980		1981	
Item	*Price*	*Value*	*Price*	*Value*
A	5	50	4	48
B	8	48	7	49
C	6	18	5	20
D	4	16	4	16

Solution. Since value=Price×Quantity

or $\quad \text{Quantity}=\frac{\text{Value}}{\text{Price}}$.

So first Quantities will be determined from value figures.

Construction of Fisher's Quantity Index Number

Item	1980 p_0	q_0	1981 p_1	q_1	$p_0\ q_0$	$p_1\ q_1$	$p_0\ q_1$	$p_1\ q_0$
A	5	10	4	12	50	48	60	40
B	8	6	7	7	48	49	56	42
C	6	3	5	4	18	20	24	15
D	4	4	4	4	16	16	16	16
					132	133	156	113

$$\overset{F}{Q_{0}}{}_{1} = \sqrt{\frac{\Sigma q_1p_0}{\Sigma q_0p_0} \times \frac{\Sigma q_1p_1}{\Sigma q_0p_1}} \times 100$$

$$= \sqrt{\frac{156}{132} \times \frac{133}{113}} \times 100 = \sqrt{\frac{20748}{14916}} \times 100$$

$$= \sqrt{1\cdot390989} \times 100 = 1\cdot1794 \times 100 = 117\cdot94$$

Time Reversal Test is satisfied if

$$Q_{\substack{0\\1}} \times Q_{\substack{1\\0}} = 1 \qquad \text{(without 100)}$$

i.e. $$\sqrt{\frac{\Sigma q_1p_0}{\Sigma q_0p_0} \times \frac{\Sigma q_1p_1}{\Sigma q_0p_1} \times \frac{\Sigma q_0p_1}{\Sigma q_1p_1} \times \frac{\Sigma q_0p_0}{\Sigma p_1q_0}}$$

$$= \sqrt{\frac{156}{132} \times \frac{133}{113} \times \frac{113}{133} \times \frac{132}{156}} = \sqrt{1} = 1$$

Factor Reversal Test is satisfied if $Q_{\substack{0\\1}} \times P_{\substack{0\\1}} = \frac{\Sigma p_1q_1}{\Sigma p_1q_0}$

L.H.S. $$Q_{\substack{0\\1}} \times P_{\substack{0\\1}} = \sqrt{\frac{\Sigma q_1p_0}{\Sigma q_0p_0} \times \frac{\Sigma q_1p_1}{\Sigma q_0p_1} \times \frac{\Sigma p_1q_0}{\Sigma p_0q_0} \times \frac{\Sigma p_1q_1}{\Sigma p_0q_1}}$$

$$= \sqrt{\frac{156}{132} \times \frac{133}{113} \times \frac{113}{132} \times \frac{133}{156}} = \sqrt{\frac{(133)^2}{(132)^2}}$$

or $Q_{\underset{1}{0}} \times p_{\underset{1}{0}} = \frac{133}{132}$

R.H.S. $\frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}$

$$= \frac{133}{132}$$

Hence L.H.S. = R.H.S.

Thus, Fisher's Index satifies both time reversal and factor reversal tests.

Example 24. In 1976 the average price of a commodity was 20% more than in 1975, but 20% less than in 1974 and it was 50% more than in 1977. Construct price relatives, using 1975 as base.

Solution. Let us suppose that price of the commodity in 1976 = 100. So, as per given conditions prices in other years will be

(i) Price in $1975 = \frac{100}{120} \times 100 = 83{\cdot}33$

Price in $1974 = \frac{100}{80} \times 100 = 125{\cdot}0$

Price in $1977 = \frac{100}{150} \times 100 = 66{\cdot}67$

Computation of Price Relatives
Base 1975 = 100

Year	*Price*	*Price Relatives*
1974	125	$\frac{125}{83{\cdot}33} \times 100 = 150$
1975	83.33	100
1976	100	$\frac{100}{83{\cdot}33} \times 100 = 120$
1977	66.67	$\frac{66{\cdot}67}{83{\cdot}33} \times 100 = 80$

Limitations of Index Numbers

1. Since index numbers are based on sample data, they do not take into account each and every item available in the market. All the errors of sampling are introduced in index numbers.

2. There is possibility of the introduction of errors at each stage of the construction of index numbers. For example, there can be error in the (*i*) selection of commodities, (*ii*) selection of base period (*iii*) collection of data, (*iv*) system of weighting, (*v*) choice of formula and (*vi*) use of average.

3. An index is an average. Therefore, index numbers suffer from the limitations which are present in any average.

4. With the change in the tastes, fashions, habits, traditions etc., the consumption pattern of people also change. Accordingly, index numbers (which require that the items and their qualities should remain same over a period of time) may not be able to keep pace with the change in the nature and quality of the commodities and hence may not be really representative one.

5. There is no all purpose index number which is applicable for all phenomenon. One index number can be used for one purpose only.

Despite these limitations, it can safely be said that if an index number is not deliberately distorted it will show correctly, at least the trend of the phenomenon which it is measuring. But indices constructed for one purpose should not be used in other place where they may not be fully appropriate and may give fallacious conclusion.

QUESTIONS

1. What are index numbers ? What problems are faced in constructing price index numbers ?
2. What are index numbers ? State their limitations. Why are index numbers called Economic Barometers of a country ?
3. "Index numbers are devices for measuring differences in the magnitude of a group of related variables". Discuss the statement and point out the important uses of index numbers.
4. Discuss the steps involved in the construction of index numbers.
5. What is implied by "weighting" in the process of index number construction ? Why is it necessary ? What are the commonly proposed weighting schemes ?
6. Show the difference between Simple and Weighted Index Numbers. Explain "Weighted Aggregative Method" and "Weighted Average of Relatives Method."
7. What is Fisher's Ideal Index ? Why it is called ideal ?
8. What do you mean by time reversal test and factor reversal test ? Which index number formula satisfies these tests.
9. What are the various types of index numbers ? What problems are faced in constructing price index numbers ?
10. From the following data construct an index number for 1983 taking 1980 as base by simple aggregative method.

Commodities	Prices in 1980	1983
A	70	95
B	46	80
C	130	215
D	190	380.

($P_{01}=176{\cdot}6$)

11. Construct an index number for 1987 taking 1986 as base year.

Commodity	A	B	C	D	E
Price in 1986 (Rs.)	50	40	80	110	20
Price in 1987 (Rs.)	70	60	90	120	20.

($P_{01}=120$)

12. The following data gives prices of commodities in 1980 and 1985. Calculate a price index based on price relatives using (*i*) Arithmetic mean (*ii*) Geometric mean.

Commodity	A	B	C	D	E	F
Price in 1980 (Rs.)	45	60	20	50	85	120
Price in 1985 (Rs.)	55	70	30	75	90	130.

((*ii*) **$P_{01}=125{\cdot}517$** ; (*ii*) **$P_{01}=124{\cdot}3$**)

13. Compute Index numbers from the following data using : (*i*) Laspeyre's (*ii*) Paasche's (*iii*) Fisher's Ideal Formula.

Commodity	*Base year* Price	*Base year* Quantity	*Current year* Price	*Current year* Quantity
A	10	12	12	15
B	7	15	5	20
C	5	24	9	20
D	16	5	14	5.

(*i*) **$P_{01}=118{\cdot}8$** ; (*ii*) **$P_{01}=112{\cdot}8$** ; (*iii*) **$P_{01}=115{\cdot}8$**)

14. Compute by Fisher's formula the Quantity Index number from the data given below

Commodity	*1984* Price (Rs.)	*1984* Value (Rs.)	*1988* Price (Rs.)	*1988* Value (Rs.)
A	5	50	4	48
B	8	48	7	49
C	6	18	5	20

($Q_{01}F=120{\cdot}8$)

Hint : Since quantity is not given, first find it by using quantity

$$=\frac{\text{Value}}{\text{Price}}$$

15. From the following data prove that Fisher's Ideal Index satisfies both the time reversal test and factor reversal test :

	Base year		*Current year*	
Commodity	*Price*	*Quantity*	*Price*	*Quantity*
A	6	50	10	56
B	2	100	2	120
C	4	60	6	60
D	10	30	12	24

(Fisher's Price Index $P_{01}=136$)

16. Construct quantity index numbers from the following data using (*i*) Laspeyre's formula (*ii*) Paasche's formula (*iii*) Fisher's formula.

	1975		*1980*	
Commodity	*Price*	*Quantity*	*Price*	*Quantity*
A	5	10	4	12
B	8	6	7	7
C	6	3	5	4
D	4	5	6	3

(*i*) $Q_{01}=111{\cdot}76$; (*ii*) $Q_{01}=106{\cdot}29$; (*iii*) $Q_{01}=108{\cdot}99$.

17. Compute a price index using (*a*) Simple aggregative method (*b*) Simple average of relatives method using (*i*) Arithmetic mean (*ii*) Geometric mean.

Commodity	A	B	C	D	E	F
Price in 1987 (Rs.)	20	30	10	25	40	50
Price in 1988 (Rs.)	25	30	15	35	45	55

[(a) $P_{01}=117{\cdot}14$; (b) (i) $P_{01}=122{\cdot}92$ (ii) $P_{01}=121{\cdot}7$)]

18. From the following data compute price index by applying weighted average of price relatives method using.

(*a*) Arithmetic mean,

(*b*) Geometric mean.

Commodity	P_0 (*Rs.*)	Q_0	P_1 (*Rs.*)
Sugar	3·0	20 kg	4·0
Flour	1·5	40 kg.	1·6
Milk	1·0	10 lt.	1·5

(*a*) $P_{01}=122{\cdot}31$ (*b*) $P_{01}=121{\cdot}3$

19. Construct Kelly's fixed weight price index number in 1985 with 1980 as base year

Commodity	A	B	C	D	E
Quanitity Consumed :	50	35	55	45	15
Prices (in Rs.) in 1980 :	32	30	16	40	35
Prices (in Rs.) in 1985 :	40	42	24	52	42

($P_{01}=132{\cdot}53$)

20. Construct index number for each year from the following average annual wholesale prices of cotton with 1944 as base

Year	Wholesale Prices (Rs.)	Year	Wholesale Prices (Rs.)
1944	75	1949	70
1945	50	1950	69
1946	65	1951	75
1947	60	1952	84
1948	72	1953	80

(100, 66·67, 86·67, 80, 96, 93·33, 92, 100, 112, 106·67)

21. From the following data calculate price index numbers for 1980 with 1970 as base by (*i*) Laspeyre's (*ii*) Paasche's (*iii*) Fisher's Ideal Method.

	1970		*1980*	
Commodities	*Price*	*Quantity*	*Price*	*Quantity*
A	20	8	40	6
B	50	10	60	5
C	40	15	50	15
D	20	20	20	25

[(*i*) $\overset{L}{P}_{01}=124{\cdot}7$; (*ii*) $\overset{P}{P}_{01}=121{\cdot}77$; (*iii*) $\overset{F}{P}_{01}=123{\cdot}2$)]

22. From the following data calculate quantity index numbers in 1985 with quantity in 1980 as base.

Commodity	A	B	C	D	E
Units	kg.	kg.	kg.	kg.	kg.
Quantity in 1980	12	15	18	20	30
Quantity in 1985	24	30	27	25	30

Use (*i*) Simple aggregative method (*ii*) Simple average of relatives method applying (*a*) Arithmetic mean (*b*) Geometric mean.

[(*ii*) $Q_{01}=143{\cdot}16$ (*ii*) (*a*) $Q_{01}=155$ (*b*) $Q_{01}=149{\cdot}6$)]

23. From the following data calculate, by Fisher's formula, Quantity Index Number

	Base Year		Current Year	
Commodity	*Price*	*Value*	*Price*	*Value*
A	5	50	4	48
B	8	48	7	49
C	6	18	5	20
D	4	16	4	16

(Q_{01}=117·94)

24. Compute a suitable price index number for the following data and see if your index satisfies the time reversal test and factor reversal test.

	Base Year		Current Year	
Commodity	*Price*	*Quantity*	*Price*	*Quantity*
A	4	3	6	2
B	5	4	6	4
C	7	2	6	2
D	2	3	1	5

(Fisher's Index is suitable. P_{01}=105·67. It satisfies both the tests)

25. An enquiry into the budgets of middle class families in a certain city gave the following information.

Expenses	Food 35%	Fuel 10%	Clothing 20%	Rent 15%	Miscell 20%
Prices in 1975 (in Rs.)	150	25	75	30	40
,, ,, 1976 (in Rs.)	145	23	65	30	45

What is cost living index No. of 1976 as compared with that of 1975.

(97·86)

26. The prices of rice, wheat and pulses for a few successives years are given below :

	Weight	1983	1984	1985	1986	1987
Rice	5	100	110	123	129	138
Wheat	3	92	100	124	116	103
Pulses	1	58	60	73	74	68

Compute Index no. of cereal prices with the above data employing the weight given in the table and taking 1983 as base year.

(108·84, 127·25, 127·87, 127·01)

27. Compute the cost of Living Index Number using (*i*) Aggregative Expenditure and (*ii*) Family Budget Method from the following information :

Commodity	Wheat	Rice	Pulses	Ghee	Sugar	Oil	Fuel	Clothing
Consumption in base year	200	50	50	20	40	50	60	40
Price in base year (Rs.)	1·00	3·00	4·00	20·00	2·50	10·00	2·00	15.00
Price in Current Year (Rs.)	1·20	3·50	5·00	30·00	5·00	15·00	2·50	18·00

(135·9, 135·9)

28. **Calculate the cost of living index for following data.**

Commodity	Price: Base Period	Price: Current Period	Quantity in Base Period
A	6	8	50
B	2	3	100
C	5	6	60
D	10	12	30

(129·09)

29. **In the construction of a certain cost of Living Index number, the following group index numbers were found. Calculate the Cost of Living Index Number by using**

   (*i*) **The weighted arithmetic mean**

   (*ii*) **The weighted geometric mean.**

Group	Food	Fuel & Light	Clothing	House Rent	Miscellaneous
Index Number	352	200	230	160	190
Weights	48	10	8	12	15

(274·26)

30. **The cost of living index for a group of persons is 274. Determine the missing value for the data given below :**

Group	Food	Fuel & lighting	Clothing	Rent	Miscell.
Index No.	350	220	230	x	190
Weight	5	1	1	1	2

(x=160)

31. **Prepare index numbers of prices for three years with average price as base from the data given below :**

Year	*Unit*	*Commodities* A	B	C
I	Price per Rupee	10 kg.	4 kg.	3 kg.
II	Price per Rupee	9 kg.	3·5 kg.	3 kg.
III	Price per Rupee	9 kg.	3 kg.	2·5 kg.

(Year I=91 ; Year II=99 ; Year III=110)

32. Construct chain index numbers from the link relatives given below :

Year	1975	1976	1977	1978	1979
Link Index	100	105	95	115	102

(100, 105, 99·75, 114·71, 117)

33. Construct chain base indices from the following data :

Years :	1980	1981	1982	1983	1984	1985	1986	1987
Price :	50	55	60	70	5	80	85	100

(100, 110, 119·99, 139·99, 149·98, 159·98, 169·97, 199·97)

34. Compute chain index numbers with 1976 prices as base from the following table giving the average whole sale prices of commodities A, B and C

	Average whole sale prices (in Rs.)				
Commodity	1976	1977	1978	1979	1980
A	20	16	28	35	21
B	25	30	24	36	45
C	20	25	30	24	30

(100, 108·33, 135·41, 160·23, 165·57)

35. From the following fixed base index numbers find out chain base index numbers :

Year	1984	1985	1986	1987	1988
Fixed Base Index	425	446	457	480	496

(100, 105, 102·4 105, 103·3)

36. From the chain base index numbers given below find fixed base index numbers.

Year	1985	1986	1987	1988	1989
Chain Base Index	80	110	120	90	140

(80, 88, 105·6, 95·04, 133·06)

37. Write short notes on the following :
   (i) Chain index numbers.
   (ii) Uses of Index numbers.
   (iii) Types of Index numbers.
   (iv) Problems in the construction of index numbers.